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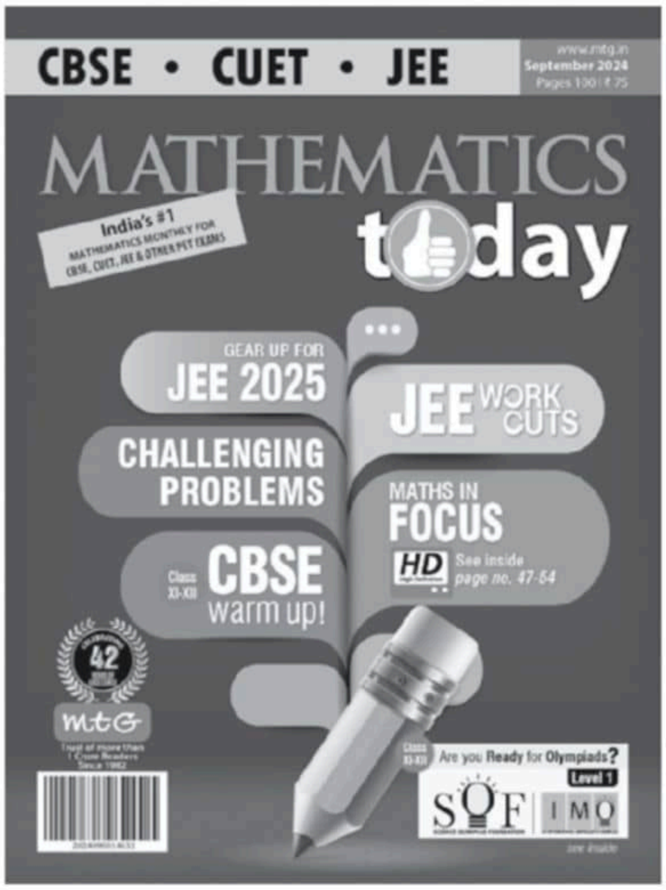


# MATHEMATICS today

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# GEAR UP for JEE 2025

## MULTIPLE CHOICE QUESTIONS

**1.** Let $f(x) = ||x - 1| - 1|$, $x \in [0, 2]$. Range of the function is

(a) $[0, 2]$ (b) $[0, 1]$
(c) $R$ (d) Not a function

**2.** In a triangle $ABC$, $\angle B = \frac{\pi}{3}$ and $\angle C = \frac{\pi}{4}$. Let $D$ divides $BC$ internally in the ratio $1 : 3$, then $\frac{\sin \angle BAD}{\sin \angle CAD}$ is equal to

(a) $\frac{1}{\sqrt{6}}$ (b) $\frac{1}{3}$ (c) $\frac{1}{\sqrt{3}}$ (d) $\sqrt{\frac{2}{3}}$

**3.** In a triangle $ABC$, $2ac \sin \frac{1}{2}(A - B + C) =$

(a) $a^2 + b^2 - c^2$ (b) $c^2 + a^2 - b^2$
(c) $b^2 - c^2 - a^2$ (d) $c^2 - a^2 - b^2$

**4.** The lines $2x - 3y = 5$ and $3x - 4y = 7$ are diameters of a circle of area 154 square units. Then the equation of this circle is

(a) $x^2 + y^2 + 2x - 2y = 62$
(b) $x^2 + y^2 + 2x - 2y = 47$
(c) $x^2 + y^2 - 2x + 2y = 47$
(d) $x^2 + y^2 - 2x + 2y = 62$

**5.** If $\alpha$ and $\beta$ are the roots of $ax^2 + bx + c = 0$, then the equation whose roots are $\frac{1}{a\alpha + b}, \frac{1}{a\beta + b}$ is

(a) $cax^2 - bx + 1 = 0$ (b) $cax^2 + bx + 1 = 0$
(c) $cax^2 + bx - 1 = 0$ (d) $cax^2 - bx - 1 = 0$

**6.** Which of the following functions is periodic ?

(a) $f(x) = x - [x]$, where $[x]$ denotes the greatest integer less than or equal to the real number $x$
(b) $f(x) = \sin\left(\frac{1}{x}\right), x \neq 0, f(0) = 0$
(c) $f(x) = x \cos x$ (d) None of these

**7.** The coefficient of $x^9$ in the expansion of $(1 + x + x^2 + x^3)^3 (1 - x)^6$ is

(a) $-7$ (b) $7$ (c) $9$ (d) $-9$

**8.** Let $f(x) = \sin x$ and $g(x) = \ln |x|$. If the ranges of the composition functions $fog$ and $gof$ are $R_1$ and $R_2$ respectively, then

(a) $R_1 = \{u : -1 \le u < 1\}, R_2 = \{v : -\infty < v < 0\}$
(b) $R_1 = \{u : -\infty < u < 0\}, R_2 = \{v : -1 \le v \le 0\}$
(c) $R_1 = \{u : -1 < u < 1\}, R_2 = \{v : -\infty < v < 0\}$
(d) $R_1 = \{u : -1 \le u \le 1\}, R_2 = \{v : -\infty < v \le 0\}$

**9.** If $\lim_{x \to 0} \frac{a \sin x - bx + cx^2 + x^3}{2x^2 \ln(1+x) - 2x^3 + x^4} = L$, where $L$ is finite and non-zero, then $L =$

(a) $\frac{1}{20}$ (b) $\frac{3}{20}$ (c) $\frac{3}{40}$ (d) $\frac{5}{41}$

**10.** The expression $3\left[\sin^4\left(\frac{3\pi}{2} - \alpha\right) + \sin^4(3\pi + \alpha)\right]$ $-2\left[\sin^6\left(\frac{\pi}{2} + \alpha\right) + \sin^6(5\pi - \alpha)\right]$ is equal to

(a) 0 (b) 1
(c) 3 (d) $\sin 4\alpha + \cos \alpha$

**11.** The first 12 letters of English alphabet are written down at random in a row. The probability, that there are exactly 4 letters between $A$ and $B$, is

(a) $\frac{7}{33}$ (b) $\frac{5}{66}$ (c) $\frac{7}{66}$ (d) $\frac{1}{13}$

**12.** Let $S_1, S_2, .....$ be squares such that for each $n \ge 1$, the length of a side of $S_n$, equals the length of a diagonal of $S_{n+1}$. If the length of a side of $S_1$ is 10 cm, then for which of the following values of $n$ is the area of $S_n$ less than 1 sq. cm?

(a) 7 (b) 6 (c) 9 (d) 5

**13.** If $(a^4 - 2a^2b^2 + b^4)^{x-1} = (a-b)^{2x}(a+b)^{-2}$, $a > 0$, $b > 0$, then $x =$

(a) $\frac{\log a}{\log b}$ (b) $\frac{\log b}{\log a}$

(c) $\frac{\log(a+b)}{\log|a-b|}$ (d) $\frac{\log|a-b|}{\log(a+b)}$

**14.** If $f(x)=\cos(\log x)$, then $f(x)f(y)-\frac{1}{2}\left[f\left(\frac{x}{y}\right)+f(xy)\right]$ has the value

(a) $-1$ (b) $\frac{1}{2}$

(c) $-2$ (d) None of these

**15.** If $f(x)$ is a polynomial such that $f(x)\,f(y) = f(xy) - f(x) - f(y)\ \forall\ x, y$ and $f(2) = 7$, then $f(-2) =$

(a) 8 (b) 10 (c) $-7$ (d) $-9$

**16.** The largest interval lying in $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ for which the function, $f(x)=4^{-x^2}+\cos^{-1}\left(\frac{x}{2}-1\right)+\log(\cos x)$ is defined, is

(a) $\left[-\frac{\pi}{4}, \frac{\pi}{2}\right)$ (b) $\left[0, \frac{\pi}{2}\right)$

(c) $[0, \pi]$ (d) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

**17.** Let $A=\begin{bmatrix}1 & \frac{1}{51}\\ 0 & 1\end{bmatrix}$. If $B=\begin{bmatrix}1 & 2\\ -1 & -1\end{bmatrix}A\begin{bmatrix}-1 & -2\\ 1 & 1\end{bmatrix}$, then the sum of all the elements of the matrix $\sum_{n=1}^{50} B^n$ is equal to

(a) 50 (b) 100 (c) 75 (d) 125

**18.** The expression $\left(x+\left(x^3-1\right)^{\frac{1}{2}}\right)^5+\left(x-\left(x^3-1\right)^{\frac{1}{2}}\right)^5$ is a polynomial of degree

(a) 5 (b) 6 (c) 7 (d) 8

**19.** Let $a=\lim_{x\to 1}\left(\frac{x}{\ln x}-\frac{1}{x\ln x}\right)$; $b=\lim_{x\to 0}\frac{x^3-16x}{4x+x^2}$; $c=\lim_{x\to 0}\frac{\ln(1+\sin x)}{x}$ and $d=\lim_{x\to -1}\frac{(x+1)^3}{3(\sin(x+1)-(x+1))}$, then the matrix $\begin{bmatrix}a & b\\ c & d\end{bmatrix}$ is

(a) Idempotent (b) Involutary

(c) Identity (d) Nilpotent

**20.** Consider the set $A$ of all determinants of order 3 with entries 0 or 1. Let $B$ be the subset of $A$ consisting of all determinants with value 1. Let $C$ be the subset of $A$ consisting of all determinants with value $-1$, then

(a) $C$ is empty

(b) $B$ has as many elements as $C$

(c) $A = B \cup C$

(d) $B$ has twice as many elements as $C$.

**21.** If tangents are drawn from the origin to the curve $y = \sin x$, then their points of contact lie on the curve

(a) $x - y = xy$ (b) $x + y = xy$

(c) $x^2 - y^2 = x^2y^2$ (d) $x^2 + y^2 = x^2y^2$

**22.** For positive integers $n_1$ and $n_2$, the values of the expression $(1+i)^{n_1}+(1+i^3)^{n_1}+(1+i^5)^{n_2}+(1+i^7)^{n_2}$ when $i=\sqrt{-1}$ is real if and only if

(a) $n_1 = n_2 + 1$ (b) $n_1 = n_2 - 1$

(c) $n_1 = n_2$ (d) $n_1 > 0, n_2 > 0$

**23.** If $(x+y)^2\frac{dy}{dx} = a^2$, $y = 0$ when $x = 0$, then $y = a$ if $\frac{x}{a} =$

(a) 1 (b) tan 1

(c) tan 1 + 1 (d) tan 1 – 1

**24.** The triangle formed by the tangent to the curve $f(x) = x^2 + bx - b$ at the point (1, 1) and the coordinate axes, lies in the first quadrant if its area is 2, then the value of $b$ is

(a) $-1$ (b) 3 (c) $-3$ (d) 1

**25.** The value of ${}^{47}C_4+\sum_{j=1}^{5}{}^{(52-j)}C_3$ is equal to

(a) ${}^{47}C_5$ (b) ${}^{52}C_5$

(c) ${}^{52}C_4$ (d) None of these

## NUMERICAL VALUE TYPE

**26.** For $x > 0$, let $f(x)=\int_1^x\frac{\ln t}{1+t}dt$, then the value of $f(e)+f\left(\frac{1}{e}\right)$ is _______.

**27.** Let the angles of a triangle $ABC$ be in A.P. and let $b:c=\sqrt{3}:\sqrt{2}$. The value of angle $A$ (in degrees) is _______.

**28.** The possible values of $b > 0$, so that the area of the bounded region enclosed between the parabolas $y = x - bx^2$ and $y=\frac{x^2}{b}$ is maximum, is _______.

**29.** Three normals are drawn from the point $(c, 0)$ to the curve $y^2 = x$. One normal is always the $x$–axis. The value of $c$ for which the other two normals are perpendicular to each other is _______.

**30.** The value of $\lim_{x\to 0}\sqrt{\dfrac{x-\sin x}{x+\cos^2 x}}$ is equal to _______.

**31.** Let $\overrightarrow{OA}=\vec{a}, \overrightarrow{OB}=10\vec{a}+2\vec{b}$ and $\overrightarrow{OC}=\vec{b}$ where $O$, $A$ and $C$ are non-collinear points. Let $p$ denote the area of the quadrilateral $OABC$ and let $q$ denote the area of the parallelogram with $OA$ and $OC$ as adjacent sides. If $p = kq$, then $k$ = _______.

**32.** Let $a$ and $b$ be the roots of the equation $x^2 - 10cx - 11d = 0$ and those of $x^2 - 10ax - 11b = 0$ are $c, d$ then the value of $a + b + c + d$, when $a, b, c, d$ are all distinct is _______.

**33.** A student is allowed to select at most $n$ books from a collection of $(2n + 1)$ books. If the total number of ways in which he can select at least one book is 63, the value of $n$ is _______.

**34.** If two events $A$ and $B$ are such that $P(A^C) = 0.3$, $P(B) = 0.4$ and $P(A \cap B^C) = 0.5$ then $P(B/(A \cup B^C))$ = _______.

**35.** A straight line $L$ with negative slopes passes through the point $(8, 2)$ and cuts the positive coordinate axes at points $P$ and $Q$. The absolute minimum value of $OP + OQ$, as $L$ varies, where $O$ is the origin is _______.

**36.** Tangents are drawn from $P(6, 8)$ to the circle $x^2 + y^2 = r^2$. The radius of the circle such that the area of the $\Delta$ formed by tangents and chord of contact is maximum is equal to _______.

**37.** Let $f(x)=\begin{cases}(x-1)^2\sin\dfrac{1}{x-1}-|x|, & \text{if } x\neq 1\\ -1, & \text{if } x=1\end{cases}$

be a real valued function. Then the number of points where $f(x)$ is not differentiable is _______.

**38.** Let $a, b, c$ be real numbers, each greater than 1, such that $\frac{2}{3}\log_b a+\frac{3}{5}\log_c b+\frac{5}{2}\log_a c=3$. If the value of $b$ is 9, then find the value of '$a$'.

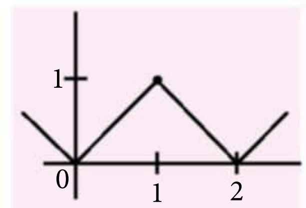
**39.** Suppose $\sin^3 x\sin 3x=\sum_{m=0}^{n} C_m\cos nx$ is an identity in $x$ when $C_0, C_1,..., C_n$ are constants and $C_n \neq 0$, then the value of $n$ = _______.

**40.** Let $p\lambda^4+q\lambda^3+r\lambda^2+s\lambda+t=\begin{vmatrix}\lambda^2+3\lambda & \lambda-1 & \lambda+3\\ \lambda+1 & -2\lambda & \lambda-4\\ \lambda-3 & \lambda+4 & 3\lambda\end{vmatrix}$

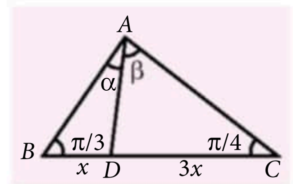
be an identity in $\lambda$ where $p, q, r, s, t$ are constants. Then the value of $t$ is _______.

## SOLUTIONS

**1. (b) :** If we draw the graph of the function $||x - 1| - 1|$ in the interval $[0, 2]$ it will be as follows. Therefore, range of the $f(x)$ will be $[0, 1]$.




**2. (a) :** In $\Delta ABD$, applying sine law, we get




$$\frac{AD}{\sin(\pi/3)}=\frac{x}{\sin\alpha}$$

$$\Rightarrow AD=\frac{\sqrt{3}x}{2\sin\alpha} \quad ...(i)$$

In $\Delta ACD$, applying sine law, we get

$$\frac{AD}{\sin(\pi/4)}=\frac{3x}{\sin\beta}\Rightarrow AD=\frac{3x}{\sqrt{2}\sin\beta} \quad ...(ii)$$

From (i) and (ii), we get

$$\frac{\sqrt{3}x}{2\sin\alpha}=\frac{3x}{\sqrt{2}\sin\beta}\Rightarrow\frac{\sin\alpha}{\sin\beta}=\frac{1}{\sqrt{6}}$$

$$\therefore \frac{\sin\angle BAD}{\sin\angle CAD}=\frac{1}{\sqrt{6}}$$

**3. (b) :** We have, $\frac{1}{2}(A-B+C)=\frac{1}{2}(\pi-B-B)=\frac{\pi}{2}-B$

$$\therefore 2ac\sin\frac{1}{2}(A-B+C)=2ac\sin\left(\frac{\pi}{2}-B\right)$$

$= 2ac\cos B = a^2 + c^2 - b^2$ (By cosine rule)

**4. (c) :** Solving the lines $2x - 3y = 5$ and $3x - 4y = 7$

Centre is $(1, -1)$.

Given, area of the circle is 154 sq. units.

$\Rightarrow \pi r^2 = 154 \Rightarrow r = 7$

Equation of the circle is $(x - 1)^2 + (y + 1)^2 = 7^2$

$\Rightarrow x^2 + y^2 - 2x + 2y - 47 = 0$

**5. (a) :** Consider, $y=\dfrac{1}{ax+b}\Rightarrow x=\dfrac{1}{a}\left(\dfrac{1}{y}-b\right)=\dfrac{1-by}{ay}$

Substituting in the given equation,

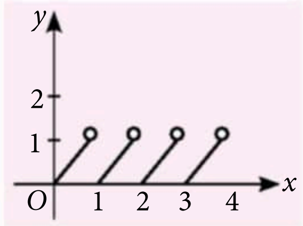
$$\frac{a(1-by)^2}{a^2y^2}+\frac{b(1-by)}{ay}+c=0$$

$\Rightarrow (1 - by)^2 + by(1 - by) + cay^2 = 0$

$\Rightarrow cay^2 - by + 1 = 0$

Hence, the equation is $cax^2 - bx + 1 = 0$.

**6. (a) :** $f(x) = x - [x]$




$f(x+1) = x + 1 - [x+1]$
$= x + 1 - ([x] + 1)$
$= x - [x] = f(x)$

$\therefore$ $f$ is a periodic function with period 1.

Clearly, curve is repeated after 1 unit on $x$-axis, so, it is a periodic function with period 1.

**7. (d) :** We have, $(1 + x + x^2 + x^3)^3 (1 - x)^3 (1 - x)^3$
$= (1 - x^4)^3 (1 - x)^3$
$= (1 - 3x^4 + 3x^8 - x^{12}) (1 - 3x + 3x^2 - x^3)$
$\therefore$ The coefficient of $x^9$ is $3(-3) = -9$

**8. (d) :** We have, $fog(x) = f(g(x)) = \sin(\ln|x|)$
$\therefore$ $R_1 = \{u : -1 \le u \le 1\}$ $(\because -1 \le \sin\theta \le 1, \forall\theta)$
Also, $gof(x) = g(f(x)) = \ln|\sin x|$
$\therefore$ $R_2 = \{v : -\infty < v \le 0\}$

**9. (c) :** By series

$$\lim_{x\to 0}\frac{a\left(x-\frac{x^3}{6}+\frac{x^5}{120}-\ldots\right)-bx+cx^2+x^3}{2x^2\left(x-\frac{x^2}{2}+\frac{x^3}{3}\ldots\right)-2x^3+x^4}$$

$$=\lim_{x\to 0}\frac{(a-b)x+cx^2+\left(1-\frac{a}{6}\right)x^3+\frac{a}{120}x^5+\ldots}{\frac{2}{3}x^5+\ldots}=L$$

$\Rightarrow$ $a = b$, $c = 0$, $1 - \frac{a}{6} = 0$, $a = 6$

$\therefore$ $L = \frac{a}{120}\times\frac{3}{2} = \frac{3}{40}$

**10. (b) :** We have, $3\left[\sin^4\left(\frac{3\pi}{2}-\alpha\right)+\sin^4(3\pi+\alpha)\right]$

$-2\left[\sin^6\left(\frac{\pi}{2}+\alpha\right)+\sin^6(5\pi-\alpha)\right]$

$=3\left(\cos^4\alpha+\sin^4\alpha\right)-2\left(\sin^6\alpha+\cos^6\alpha\right)$

$=3\left[1-2\sin^2\alpha\cos^2\alpha\right]-2\left[1-3\sin^2\alpha\cos^2\alpha\right]=1$

**11. (c) :** $A$ and $B$ can occupy the positions, (1, 6), (2, 7), (3, 8), (4, 9), (5, 10), (6, 11), (7, 12).

$\therefore$ Required probability $=\frac{2\times7\times10!}{12!}=\frac{7}{66}$

**12. (c) :** Length of the side of the square $S_r$ is $a_r$.

Now, $a_1 = 10,\ a_2=\frac{10}{\sqrt{2}},\ a_3=\frac{10}{(\sqrt{2})^2}, \ldots$

$a_7=\frac{10}{(\sqrt{2})^6}=\frac{10}{8}>1,\ a_8=\frac{10}{(\sqrt{2})^7}<1,\ a_9=\frac{10}{(\sqrt{2})^8}<1$

and $a_{10}=\frac{10}{(\sqrt{2})^9}<1$

As, $a_8$, $a_9$, $a_{10}$ are less than 1.

$\therefore$ $S_8=a_7^2<1,\ S_9=a_8^2<1,\ S_{10}=a_9^2<1$

Hence, (c) is the correct answers.

**13. (d) :** Given, $(a^2 - b^2)^{2(x-1)} = (a-b)^{2x}(a+b)^{-2}$
$\Rightarrow$ $(a+b)^{2x-2}.(a-b)^{2x-2} = (a-b)^{2x}(a+b)^{-2}$
$\Rightarrow$ $(a+b)^{2x} = (a-b)^2 \Rightarrow (a+b)^x = |a-b|$
$\Rightarrow$ $x=\frac{\log|a-b|}{\log(a+b)}$

**14. (d)**

**15. (d) :** We have, $f(x)f(y) = f(xy) - f(x) - f(y)$
$\Rightarrow$ $(f(x)+1)(f(y)+1) = f(xy) + 1$
Let $g(x) = f(x) + 1$, then
$g(x)g(y) = g(xy)$ $\Rightarrow$ $g(x) = x^n$, $f(x) = x^n - 1$
Given, $f(2) = 7 \Rightarrow 2^n - 1 = 7 \Rightarrow n = 3$
$\therefore$ $f(x) = x^3 - 1$ and $f(-2) = -9$

**16. (b) :** $f(x)$ is defined if $-1\le\frac{x}{2}-1\le1$ and $\cos x>0$

or $0\le x\le4$ and $-\frac{\pi}{2}<x<\frac{\pi}{2}$

$\therefore$ $0\le x<\frac{\pi}{2}$

**17. (b) :** We have, $A=\begin{bmatrix}1 & \frac{1}{51}\\ 0 & 1\end{bmatrix}$;

$B=\begin{bmatrix}1 & 2\\ -1 & -1\end{bmatrix}A\begin{bmatrix}-1 & -2\\ 1 & 1\end{bmatrix}$

Let $P=\begin{bmatrix}1 & 2\\ -1 & -1\end{bmatrix}$ and $Q=\begin{bmatrix}-1 & -2\\ 1 & 1\end{bmatrix}$

$\because$ $B = PAQ$
$B^2 = (PAQ)(PAQ) = PAQPAQ = PA^2Q$

As, $QP=\begin{bmatrix}-1 & -2\\ 1 & 1\end{bmatrix}\begin{bmatrix}1 & 2\\ -1 & -1\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

$A^2=\begin{bmatrix}1 & \frac{1}{51}\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & \frac{1}{51}\\ 0 & 1\end{bmatrix}=\begin{bmatrix}1 & \frac{2}{51}\\ 0 & 1\end{bmatrix}$

$A^3=\begin{bmatrix}1 & \frac{2}{51}\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & \frac{1}{51}\\ 0 & 1\end{bmatrix}=\begin{bmatrix}1 & \frac{3}{51}\\ 0 & 1\end{bmatrix}$

Similarly, $A^n = \begin{bmatrix} 1 & \frac{n}{51} \\ 0 & 1 \end{bmatrix}$

Now, $B^n = PA^nQ = \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} 1 & \frac{n}{51} \\ 0 & 1 \end{bmatrix}\begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix}$

$$B^n = \begin{bmatrix} 1+\frac{n}{51} & \frac{n}{51} \\ \frac{-n}{51} & 1-\frac{n}{51} \end{bmatrix}$$

$$\sum_{n=1}^{50} B^n = \begin{bmatrix} 50+25 & 25 \\ -25 & 50-25 \end{bmatrix} = \begin{bmatrix} 75 & 25 \\ -25 & 25 \end{bmatrix}$$

Hence, sum of all the elements = 100

**18. (c) :** The expression

$$\left(x+\left(x^3-1\right)^{1/2}\right)^5+\left(x-\left(x^3-1\right)^{1/2}\right)^5$$

$= 2(x^5 + 10x^3(x^3 - 1) + 5x(x^3 - 1)^2)$

$[\because \{(a+b)^n + (a-b)^n\} = 2({}^nC_0a^n + {}^nC_2a^{n-2}b^2 + {}^nC_4a^{n-4}b^4 + \ldots)]$

$\therefore$ The degree of the polynomial is 7.

**19. (d) :** Since, $a = 2$; $b = -4$; $c = 1$ and $d = -2$

Let $A = \begin{bmatrix} 2 & -4 \\ 1 & -2 \end{bmatrix} \Rightarrow A^2 = \begin{bmatrix} 2 & -4 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} 2 & -4 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ = null matrix

Hence, $A$ is nilpotent.

**20. (b) :** For every determinant with value 1 ($\in B$), we can find a determinant $C$ with value $-1$ by interchanging any two rows or columns among themself in $|B|$.

Hence, there are equal number of elements in $B$ and $C$.

**21. (c) :** The tangent at $(x_1, \sin x_1)$ is,

$y - \sin x_1 = \cos x_1(x - x_1)$

It passes through the origin (0, 0). Then,

$\sin x_1 = x_1 \cos x_1 = x_1\sqrt{1-\sin^2 x_1}$

$\Rightarrow \sin^2 x_1 = x_1^2(1 - \sin^2 x_1)$

$\Rightarrow y_1^2 = x_1^2(1 - y_1^2)$ $\qquad (\because y_1^2 = \sin^2 x_1)$

Now, $(x_1, y_1)$ lies on the curve

$y^2 = x^2(1 - y^2)$ or $x^2 - y^2 = x^2y^2$

**22. (d) :** Given expression is

$$(1+i)^{n_1}+(1-i)^{n_1}+(1+i)^{n_2}+(1-i)^{n_2}$$

Using, $1+i=\sqrt{2}(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4})$

and $(1-i)=\sqrt{2}\left(\cos\frac{\pi}{4}-i\sin\frac{\pi}{4}\right)$

$\therefore$ The given expression becomes

$$(\sqrt{2})^{n_1}\left[\cos\frac{n_1\pi}{4}+i\sin\frac{n_1\pi}{4}\right]+(\sqrt{2})^{n_1}\left[\cos\frac{n_1\pi}{4}-i\sin\frac{n_1\pi}{4}\right]+$$

$$(\sqrt{2})^{n_2}\left[\cos\frac{n_2\pi}{4}+i\sin\frac{n_2\pi}{4}\right]+(\sqrt{2})^{n_2}\left[\cos\frac{n_2\pi}{4}-i\sin\frac{n_2\pi}{4}\right]$$

$$=(\sqrt{2})^{n_1}\left[2\cos\frac{n_1\pi}{4}\right]+(\sqrt{2})^{n_2}\left[2\cos\frac{n_2\pi}{4}\right]$$

= Real number irrespective the values of $n_1$ and $n_2$ which is always real $n_1 > 0$, $n_2 > 0$

**23. (d) :** Let $x + y = z \Rightarrow \frac{dz}{dx}-1=\frac{dy}{dx} \Rightarrow \frac{dz}{dx}-1=\frac{a^2}{z^2}$

$$\Rightarrow \frac{dz}{dx}=\frac{a^2+z^2}{z^2} \Rightarrow \frac{z^2}{a^2+z^2}dz=dx$$

$$\Rightarrow x + c = z - a\tan^{-1}\left(\frac{z}{a}\right) \Rightarrow a\tan^{-1}\left(\frac{x+y}{a}\right) = y - c$$

When $x = 0$ and $y = 0 \Rightarrow c = 0$

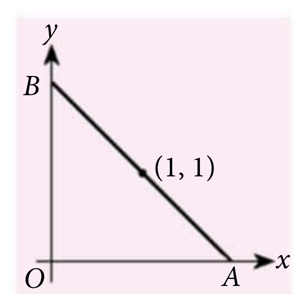
$$\Rightarrow \frac{y}{a}=\tan^{-1}\left(\frac{x+y}{a}\right)$$

Now, $y = a \Rightarrow 1 = \tan^{-1}\left(\frac{x}{a}+1\right) \Rightarrow \frac{x}{a} = \tan 1 - 1$

**24. (c) :** Slope of the tangent at point $(x, y)$ is given by $f'(x) = 2x + b$

slope of tangent at (1,1) is $2 + b$

Equation of tangent at (1,1) is




$y - 1 = (2 + b)(x - 1)$

$\Rightarrow (2 + b)x - y = 1 + b$

or $\frac{x}{(1+b)/(2+b)}-\frac{y}{(1+b)}=1$

This line meets $x$-axis and $y$-axis at

$A\left(\frac{1+b}{2+b},0\right)$ and $B(0,-(1+b))$

$\therefore$ Area of $\Delta OAB = \frac{1}{2}\left(\frac{1+b}{2+b}\right)\{-(1+b)\} = 2$

$\Rightarrow (1 + b)^2 = -4(2 + b) \Rightarrow b^2 + 2b + 1 = -8 - 4b$

$\Rightarrow b^2 + 6b + 9 = 0 \Rightarrow (b + 3)^2 = 0 \Rightarrow b = -3$

**25. (c) :** We know that, ${}^nC_{r-1} + {}^nC_r = {}^{(n+1)}C_r$ ...(i)

Now, ${}^{47}C_4 + \sum_{j=1}^{5} {}^{(52-j)}C_3$

$= {}^{47}C_4 + {}^{47}C_3 + {}^{48}C_3 + {}^{49}C_3 + {}^{50}C_3 + {}^{51}C_3$

$= {}^{52}C_4$ (using (i)).

**26. (0.5) :** We have, $f(x)=\int_1^x \frac{\ln(t)}{1+t}dt;\ f(\frac{1}{x})=\int_1^{1/x}\frac{\ln(t)}{1+t}dt$

Let $t=\frac{1}{y} \Rightarrow dt=\frac{-1}{y^2}dy$

$$f\left(\frac{1}{x}\right)=\int_1^x \frac{\ln 1/y}{1+1/y}\left(\frac{-1}{y^2}\right)dy=\int_1^x \frac{\ln y}{y(1+y)}dy$$

$$f\left(\frac{1}{x}\right)=\int_1^x \frac{\ln(t)}{t(1+t)}dt$$

Now, $f(x)+f\left(\frac{1}{x}\right)=\int_1^x \ln(t)\left[\frac{1}{t+1}+\frac{1}{t(t+1)}\right]dt$

$$=\int_1^x \frac{\ln(t)}{t}dt=\frac{1}{2}[\ln(x)]^2$$

$\therefore\ f(e)+f\left(\frac{1}{e}\right)=\frac{1}{2}$

**27. (75) :** Since, $A+B+C=180°$ and $2B=A+C$

($\because$ $A$, $B$, $C$ are in A.P)

$\therefore$ $3B=180° \Rightarrow B=60°$

Given, $\frac{b}{\sqrt{3}}=\frac{c}{\sqrt{2}}=k \Rightarrow b=k\sqrt{3}$ and $c=k\sqrt{2}$

By the sine rule, $\frac{b}{\sin B}=\frac{c}{\sin C}$

$\Rightarrow \frac{k\sqrt{3}}{\sin 60°}=\frac{k\sqrt{2}}{\sin C}\ \therefore\ \sin C=\frac{1}{\sqrt{2}} \Rightarrow C=45°$

($\because$ $C=135°$ is impossible)

$\therefore$ $A=180°-(B+C)=180°-(60°+45°)=75°$.

**28. (1) :** The given curves are $y=\frac{x^2}{b}$ ...(i)

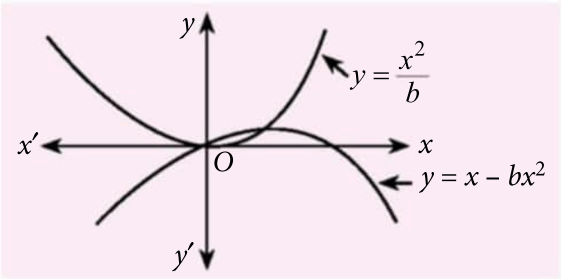
and $y=x-bx^2$ ...(ii)

$\Rightarrow$ $y=-(bx^2-x)$

From (i) and (ii), we get $\frac{x^2}{b}=x-bx^2$

$\Rightarrow$ $x^2-bx+b^2x^2=0 \Rightarrow x^2(b^2+1)=bx$

$\Rightarrow$ $x=0$ and $x=\frac{b}{1+b^2}$




$\because$ Point of intersections are (0,0) and $\left(\frac{b}{1+b^2},\frac{1}{(1+b^2)^2}\right)$.

$\therefore$ Area under the curves *i.e.*, area of the bounded region

$$A=\int_0^{b/1+b^2}(y_2-y_1)dx=\int_0^{b/1+b^2}(x-bx^2-\frac{x^2}{b})dx$$

$$=\left|\frac{x^2}{2}-\frac{bx^3}{3}-\frac{x^3}{3b}\right|_0^{\frac{b}{1+b^2}}=\frac{1}{6}\frac{b^2}{(1+b^2)^2}=\frac{1}{6}\frac{1}{\left(b+\frac{1}{b}\right)^2}$$

Now, the area to be maximum, $\left(b+\frac{1}{b}\right)$ should be minimum.

$\Rightarrow \frac{d}{db}\left[b+\frac{1}{b}\right]=0 \Rightarrow 1-\frac{1}{b^2}=0$

$\Rightarrow$ $b^2=1 \Rightarrow b=\pm 1$

$\because$ $b>0$ $\therefore$ $b=1$ is the required answer.

**29. (0.75) :** For the parabola $y^2=4ax$, the normal to this parabola is $y=mx-2am-am^3$.

Given parabola is $y^2=x$ $\because$ $4a=1$ or $a=\frac{1}{4}$

So equation of normal to the parabola (curve) becomes

$$y=mx-\frac{m}{2}-\frac{m^3}{4}$$

It passes through $(c, 0)$ the given point, so equation of the normal becomes $0=mc-\frac{m}{2}-\frac{m^3}{4}$

$\Rightarrow$ $m=0$ or $m^2=4\left(c-\frac{1}{2}\right)$

Here, $m^2\ge 0 \Rightarrow 4\left(c-\frac{1}{2}\right)\ge 0 \Rightarrow c\ge\frac{1}{2}$

Thus at $c=\frac{1}{2}\Rightarrow m^2=0\Rightarrow m=0$

Now $c$ must be greater than zero for real values of $m$. Thus for $m=0$, $y=0$. It means $x$-axis is the normal.

Now, we have $m^2=4\left(c-\frac{1}{2}\right)$ and we have that $m_1m_2=-1$.

Since other two normals are perpendicular

$\Rightarrow$ $-4\left(c-\frac{1}{2}\right)=-1\Rightarrow c=\frac{3}{4}$

**30. (0) :** Let $f(x)=\sqrt{\frac{x-\sin x}{x+\cos^2 x}}=\frac{\sqrt{x-\sin x}}{\sqrt{x+\cos^2 x}}$

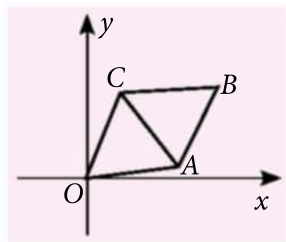
$$\lim_{x\to 0}(x-\sin x)^{1/2}=\lim_{x\to 0}\left[x\left(1-\frac{\sin x}{x}\right)\right]^{1/2}=0\cdot 0=0$$

$$\lim_{x\to 0}(x+\cos^2 x)^{1/2}=(0+1)^{1/2}=1$$

$$\therefore \quad \lim_{x\to 0} f(x)=\frac{0}{1}=0$$

**31. (6) :** Area of the quadrilateral $OABC$

= area of $\Delta OAC$ + area of $\Delta ABC$




$$=\frac{1}{2}|\overrightarrow{OA}\times\overrightarrow{AC}|+\frac{1}{2}|\overrightarrow{AB}\times\overrightarrow{BC}|$$

$$=\frac{1}{2}|\vec{a}\times(\vec{b}-\vec{a})|+\frac{1}{2}|(10\vec{a}+2\vec{b}-\vec{a})\times(\vec{b}-10\vec{a}-2\vec{b})|$$

$$=\frac{1}{2}|\vec{a}\times\vec{b}|+\frac{1}{2}|(9\vec{a}+2\vec{b})\times(-10\vec{a}-\vec{b})|$$

$$=\frac{1}{2}|\vec{a}\times\vec{b}|+\frac{1}{2}\times 11|\vec{a}\times\vec{b}|=6|\vec{a}\times\vec{b}|$$

Area of the required parallelogram $=|\vec{a}\times\vec{b}|$

$\therefore \quad p=6|\vec{a}\times\vec{b}|\ ;\ q=|\vec{a}\times\vec{b}|$

So, $p = kq \Rightarrow 6|\vec{a}\times\vec{b}|=k|\vec{a}\times\vec{b}| \quad \therefore \ k=6$

**32. (1210) :** Roots of $x^2 - 10cx - 11d = 0$ are $a$ and $b$

$\Rightarrow \ a + b = 10c$ and $ab = -11d$

Similarly, $c$ and $d$ are the roots of

$x^2 - 10ax - 11b = 0$

$\Rightarrow \ c + d = 10a$ and $cd = - 11b$

$\Rightarrow \ a + b + c + d = 10(a + c)$ and $abcd = 121\ bd$

$\Rightarrow \ b + d = 9(a + c)$ and $ac = 121$

Also we have, $a^2 - 10ac - 11d = 0$

and $c^2 - 10ac - 11b = 0$

$\Rightarrow \ a^2 + c^2 - 20ac - 11(b + d) = 0$

$\Rightarrow \ (a + c)^2 - 22 \times 121 - 99(a + c) = 0$

$\Rightarrow \ a + c = 121$ or $-22$

For $a + c = -22$,

We get $a = c$ from $ac = 121$.

$\therefore$ Rejecting this value we have $a + c = 121$

$\therefore \ a + b + c + d = 10(a + c) = 1210$.

**33. (3) :** Number of ways in which a student can select at least one and atmost $n$ books out of $(2n + 1)$ books is equal to

$${}^{2n+1}C_1+{}^{2n+1}C_2+\ldots\ldots+{}^{2n+1}C_n$$

$$=\frac{1}{2}\left[2\cdot{}^{2n+1}C_1+2\cdot{}^{2n+1}C_2+\ldots..+2\cdot{}^{2n+1}C_n\right]$$

$$=\frac{1}{2}\left[{}^{2n+1}C_1+{}^{2n+1}C_2+\ldots\ldots..+{}^{2n+1}C_n + {}^{2n+1}C_1+{}^{2n+1}C_2+\ldots\ldots.+{}^{2n+1}C_n\right]$$

Now, ${}^{2n+1}C_1={}^{2n+1}C_{2n}$

${}^{2n+1}C_2={}^{2n+1}C_{2n-1}$

..........................................

..........................................

${}^{2n+1}C_n={}^{2n+1}C_{n+1}$

$$=\frac{1}{2}\left[{}^{2n+1}C_0+{}^{2n+1}C_1+\ldots\ldots+{}^{2n+1}C_n + {}^{2n+1}C_{n+1}+{}^{2n+1}C_n+\ldots\ldots..+{}^{2n+1}C_{2n} + {}^{2n+1}C_{2n+1}-2\right]$$

$$=\frac{1}{2}\left(2^{2n+1}-2\right)=2^{2n}-1=63$$

$\Rightarrow \ 2^{2n} = 64 = 2^6 \ \Rightarrow \ 2n = 6 \ \Rightarrow \ n = 3$

**34. (0.25) :** We have,

$$P(B/(A\cup B^C))=\frac{P(B\cap(A\cup B^C))}{P(A\cup B^C)}$$

$$=\frac{P((B\cap A)\cup(B\cap B^C))}{P(A\cup B^C)}$$

$$=\frac{P(A\cap B)}{P(A\cup B^C)}=\frac{P(A)-P(A\cap B^C)}{P(A\cup B^C)}$$

$$=\frac{(1-0\cdot 3)-0.5}{(1-0.3)+(1-0.4)-0.5}=0.25$$

**35. (18) :** Since it passes through (8, 2).

Let equation of $L$ is $y - 2 = m(x - 8)$

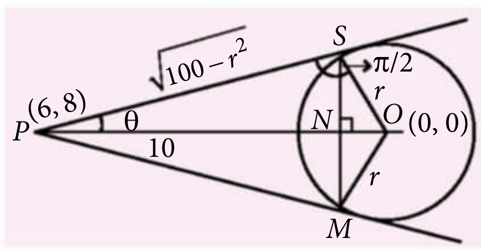
$$OP + OQ = 10 + \frac{2}{-m} + 8(-m) \geq 10 + 2\sqrt{\left(-\frac{2}{m}\right)(-8m)}$$

$\therefore \quad OP + OQ \geq 10 + 8$

Absolute minimum of $OP + OQ = 18$.

**36. (5) :** $\tan\theta = \frac{r}{PS} = \frac{r}{\sqrt{100 - r^2}}$ ; $\sin\theta = \frac{r}{10}$ and

$\cos\theta = \frac{\sqrt{100 - r^2}}{10}$




Area of $\Delta PSM$

$= A = \frac{1}{2} SM \times PN$

$= \frac{1}{2} \cdot 2 \cdot SN \times PN = SN \times PN = SP \sin\theta \times SP \cos\theta$

$$= \left(\sqrt{100 - r^2}\right)^2 \times \sin\theta\ \cos\theta = (100 - r^2)\frac{r}{10}\frac{\sqrt{100 - r^2}}{10}$$

$$= \frac{r(100 - r^2)^{3/2}}{100}$$

$$\frac{dA}{dr} = \frac{1}{100} \cdot \frac{3}{2}(100 - r^2)^{1/2}(-2r)r + \frac{(100 - r^2)^{3/2}}{100} = 0$$

$(100 - r^2)^{1/2}(-3r^2 + 100 - r^2) = 0,$

$r \neq 10$ as $P$ is outside the circle

$\Rightarrow \quad 4r^2 = 100 \Rightarrow r^2 = 25 \Rightarrow r = 5$

Thus for $r = 5$, $A$ is maximum

since $\frac{d^2A}{dr^2} < 0$ for $r = 5$.

**37. (1) :** Since $(x - 1)^2 \sin\left(\frac{1}{x - 1}\right)$

is differentiable at $x = 0$ but $|x|$ is not differentiable at $x = 0$, $f(x)$ is not differentiable at $x = 0$

Now, for differentiability at $x = 1$,

$$f'(1^+) = \lim_{h \to 0}\left[\frac{f(1 + h) - f(1)}{h}\right] = \lim_{h \to 0}\frac{h^2 \sin\frac{1}{h} - (1 + h) + 1}{h}$$

$$= \lim_{h \to 0}\frac{h^2 \sin\left(\frac{1}{h}\right) - h}{h} = \lim_{h \to 0} h \sin\frac{1}{h} - 1$$

$= 0 - 1 = -1$

Similarly $f'(1^-) = -1$ and $f(1) = -1$

$\therefore \quad f$ is differentiable at $x = 1$

$\therefore \quad f$ is differentiable everywhere except at $x = 0$.

**38. (27) :** We have, $\frac{2}{3}\log_b a + \frac{3}{5}\log_c b + \frac{5}{2}\log_a c = 3$

$$\Rightarrow \quad \frac{1}{3}\left(\frac{2}{3}\log_b a + \frac{3}{5}\log_c b + \frac{5}{2}\log_a c\right) = 1$$

$$= \left(\frac{2}{3}\log_b a \times \frac{3}{5}\log_c b \times \frac{5}{2}\log_a c\right)$$

$\Rightarrow \quad$ A.M. = G.M.

$$\Rightarrow \quad \frac{2}{3}\log_b a = \frac{3}{5}\log_c b = \frac{5}{2}\log_a c = 1$$

$$\Rightarrow \quad \frac{2\log a}{3\log 9} = 1 \Rightarrow \log a = \frac{2\log 3}{2} \times 3 \quad \Rightarrow \quad a = 3^3 = 27$$

**39. (6) :** We have,

$$\sin^3 x \sin 3x = \frac{1}{8} \cdot 2\sin 3x(3\sin x - \sin 3x)$$

$$= \frac{1}{8}[2\sin 3x \cdot 3\sin x - 2\sin^2 3x]$$

$$= \frac{1}{8}[3(\cos 2x - \cos 4x) - (1 - \cos 6x)]$$

$$= \frac{3}{8}\cos 2x - \frac{1}{8}\cos 4x - \frac{1}{8} + \frac{1}{8}\cos 6x$$

$$= -\frac{1}{8}\cos 4x + \frac{3}{8}\cos 2x - \frac{1}{8} + \frac{1}{8}\cos 6x = \sum_{m=0}^{n} C_m \cos nx$$

On comparing $n = 6$.

**40. (0) :** Expanding, $p\lambda^4 + q\lambda^3 + r\lambda^2 + s\lambda + t$

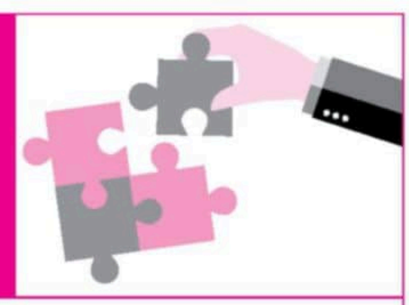
$= (\lambda^2 + 3\lambda)[-6\lambda^2 - (\lambda^2 - 16)] - (\lambda - 1)[(\lambda + 1)(3\lambda) - (\lambda - 3)(\lambda - 4)] + (\lambda + 3)[(\lambda + 1)(\lambda + 4) + 2\lambda(\lambda - 3)]$

Independent term $= t = (1)(-12) + 3(4)$

$= -12 + 12 = 0 \quad \therefore t = 0$

## PUZZLE CORNER

**SOLUTION - AUGUST 2024**



| | | | | | |
|---|---|---|---|---|---|
| 3÷ 2 | 6 | 2– 3 | 1 | 1– 4 | 5 |
| 12× 4 | 1– 2 | 7+ 1 | 6 | 8+ 5 | 3 |
| 1 | 3 | 1– 4 | 5 | 3÷ 6 | 2 |
| 3 | 6+ 1 | 5 | 2÷ 4 | 1– 2 | 5– 6 |
| 1– 5 | 2– 4 | 6 | 2 | 3 | 1 |
| 6 | 10+ 5 | 2 | 3 | 5+ 1 | 4 |

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- NIT Warangal
- IIT Hyderabad
- IISC, Bangalore
- Netaji Subhas Institute of Technology, Delhi
- DTU, Delhi

# JEE WORK CUTS

## SINGLE OPTION CORRECT TYPE

1. If $\text{Re}\left(\frac{z-8i}{z+6}\right)=0$, then $z$ lies on the curve
   (a) $x^2 + y^2 + 6x - 8y = 0$
   (b) $4x - 3y + 24 = 0$
   (c) $x^2 + y^2 - 8 = 0$
   (d) None of these

2. If $\alpha, \beta$ are the distinct roots of $x^2 + bx + c = 0$, then $\lim\limits_{x\to\beta}\frac{e^{2(x^2+bx+c)}-1-2(x^2+bx+c)}{(x-\beta)^2}$ is equal to
   (a) $2(b^2 + 4c)$ (b) $b^2 - 4c$
   (c) $2(b^2 - 4c)$ (d) $b^2 + 4c$

3. Let $C$ be the largest circle centred at (2, 0) and inscribed in the ellipse $\frac{x^2}{36}+\frac{y^2}{16}=1$. If $(1, \alpha)$ lies on $C$, then $10\alpha^2$ is equal to
   (a) 59 (b) 118 (c) 120 (d) 95

4. The integral $\int_0^{\pi}\sqrt{1+4\sin^2\frac{x}{2}-4\sin\frac{x}{2}}\,dx$ equals
   (a) $\frac{2\pi}{3}-4-4\sqrt{3}$ (b) $4\sqrt{3}-4$
   (c) $4\sqrt{3}-4-\frac{\pi}{3}$ (d) $\pi - 4$

5. The domain of the function $f(x)=\sqrt{\log_{0.5} x!}$ is
   (a) {0, 1, 2, 3, ...} (b) {1, 2, 3, ...}
   (c) $(0, \infty)$ (d) {0, 1}

6. Let $f: R - \{2, 6\} \to R$ be real valued function defined as $f(x)=\frac{x^2+2x+1}{x^2-8x+12}$. Then range of $f$ is
   (a) $\left(-\infty, -\frac{21}{4}\right]\cup\left[\frac{21}{4}, \infty\right)$
   (b) $\left(-\infty, -\frac{21}{4}\right]\cup[0, \infty)$
   (c) $\left(-\infty, -\frac{21}{4}\right)\cup(0, \infty)$
   (d) $\left(-\infty, -\frac{21}{4}\right]\cup[1, 2)$

7. $\cos\frac{2\pi}{13}+\cos\frac{4\pi}{13}+\cos\frac{6\pi}{13}+...+\cos\frac{24\pi}{13}=$
   (a) 0 (b) 1
   (c) −1 (d) 1/2

8. If $\frac{3+i\sin\theta}{4-i\cos\theta}, \theta\in[0, 2\pi]$ is a real number, then an argument of $\sin\theta + i\cos\theta$ is
   (a) $\pi-\tan^{-1}\left(\frac{3}{4}\right)$ (b) $\tan^{-1}\left(\frac{4}{3}\right)$
   (c) $-\tan^{-1}\left(\frac{3}{4}\right)$ (d) $\pi-\tan^{-1}\left(\frac{4}{3}\right)$

9. Ten persons, amongst whom are $A$, $B$ and $C$ are speak at a function. The number of ways in which it can be done if $A$ wants to speak before $B$, and $B$ wants to speak before $C$ is
   (a) $\frac{10!}{6!}$ (b) 21870
   (c) $\frac{10!}{3!}$ (d) $\frac{10!}{7!}$

10. The coefficient of $x^{18}$ in the product $(1 + x)(1 - x)^{10}(1 + x + x^2)^9$ is
   (a) 84 (b) −126 (c) 126 (d) −84

**11.** The foci of an ellipse are (–2, 4) and (2, 1). The point $\left(1, \frac{23}{6}\right)$ is an extremity of the minor axis. What is the value of the eccentricity?

(a) $\frac{9}{13}$ (b) $\frac{3}{\sqrt{13}}$ (c) $\frac{2}{\sqrt{13}}$ (d) $\frac{4}{13}$

**12.** The sum of the series $\frac{1}{1\cdot 2}-\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}-\ldots$ upto ∞ is equal to

(a) $\log_e 2 - 1$ (b) $\log_e 2$

(c) $\log_e(4/e)$ (d) $2\log_e 2$

**13.** Find the equations of common tangents to the parabola $y^2 = 16x$ and the circle $x^2 + y^2 = 8$.

(a) $y = x + 8;\ y = -x - 8$

(b) $y = x \pm 8$

(c) $y = x \pm 4$

(d) $y = x + 4;\ y = -x - 4$

**14.** Find the value of

$$\lim_{x\to 0}\frac{\sin x+\log_e(\sqrt{1+\sin^2 x}-\sin x)}{\sin^3 x}.$$

(a) $\frac{1}{6}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{1}{8}$

**15.** The function $f(x) = a \sin |x| - be^{|x|}$ is differentiable at $x = 0$ when

(a) $3a + b = 0$ (b) $3a - b = 0$

(c) $a + b = 0$ (d) $a - b = 0$

## ONE OR MORE THAN ONE OPTION(S) CORRECT TYPE

**16.** If $(a - b)\sin(\theta + \phi) = (a + b)\sin(\theta - \phi)$ and $a\tan\frac{\theta}{2}-b\tan\frac{\phi}{2}=c$, then

(a) $b\tan\phi = a\tan\theta$ (b) $a\tan\phi = b\tan\theta$

(c) $\sin\phi=\frac{2bc}{a^2-b^2-c^2}$ (d) $\sin\theta=\frac{2ac}{a^2-b^2-c^2}$

**17.** Let $a, b, c \in Q^+$ satisfying $a > b > c$. Which of the following statements (s) hold true for the quadratic polynomial $f(x) = (a + b - 2c)x^2 + (b + c - 2a)x + (c + a - 2b)$?

(a) The parabola $y = f(x)$ opens upwards.

(b) Both roots of the equation $f(x) = 0$ are rational.

(c) $x$-coordinate of vertex of the graph is positive

(d) $x$-coordinate of vertex of the graph is negative

**18.** If $\lim_{x\to 0}\frac{3+\alpha\sin x+\beta\cos x+\log_e(1-x)}{3\tan^2 x}=\frac{1}{3}$, then $2\alpha - \beta$ is equal to

(a) 5 (b) 1 (c) 7 (d) 2

**19.** If $f(x)=\cos^{-1}x+\cos^{-1}\left\{\frac{x}{2}+\frac{1}{2}\sqrt{3-3x^2}\right\}$, then

(a) $f\left(\frac{2}{3}\right)=\frac{\pi}{3}$ (b) $f\left(\frac{2}{3}\right)=2\cos^{-1}\frac{2}{3}-\frac{\pi}{3}$

(c) $f\left(\frac{1}{3}\right)=\frac{\pi}{3}$ (d) $f\left(\frac{1}{3}\right)=2\cos^{-1}\frac{1}{3}-\frac{\pi}{3}$

**20.** If $f(x)$, $g(x)$ (where $x > 1$) are non–negative and non–positive functions respectively, such that $f'(x) \le \alpha f(x)$, $g'(x) \ge \beta g(x)$ for some $\alpha, \beta > 0$ and $f(1) = 0$, $g(1) = 0$, then

(a) $\frac{f(e)+f(\pi)}{f^2(e)+f^2(\pi)-4}=0$

(b) $\frac{f(e)+f(\pi)}{f^2(e)+f^2(\pi)-4}=-4$

(c) $\frac{g(\sqrt{e})+g(\sqrt{\pi})}{g(\sqrt[3]{e})+g(\sqrt[3]{\pi})-3}=0$

(d) $\frac{g(\sqrt{e})+g(\sqrt{\pi})}{g(\sqrt[3]{e})+g(\sqrt[3]{\pi})-3}=-3$

**21.** The projection of line $3x - y + 2z - 1 = 0 = x + 2y - z - 2$ on the plane $3x + 2y + z = 0$ is

(a) $\frac{x+1}{11}=\frac{y-1}{-9}=\frac{z-1}{-15}$

(b) $3x - 8y + 7z + 4 = 0 = 3x + 2y + z$

(c) $\frac{x+12}{11}=\frac{y+8}{-9}=\frac{z+14}{15}$

(d) $\frac{x+12}{11}=\frac{y+8}{-9}=\frac{z+14}{-15}$

**22.** Let $f(x)=\sin\left(\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right)\right)$ for all $x \in R$ and $g(x)=\frac{\pi}{2}\sin x$ for all $x \in R$. Let $(fog)(x)$ denote $f(g(x))$ and $(gof)(x)$ denote $g(f(x))$. Then, which of the following is (are) true?

(a) Range of $f$ is $\left[-\frac{1}{2},\frac{1}{2}\right]$

(b) Range of $fog$ is $\left[-\frac{1}{2},\frac{1}{2}\right]$

(c) $\lim_{x\to 0}\frac{f(x)}{g(x)}=\frac{\pi}{6}$

(d) There is an $x \in R$ such that $(gof)(x) = 1$

**23.** Let $P = \begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & \alpha \\ 3 & -5 & 0 \end{bmatrix}$, where $\alpha \in R$.

Suppose $Q = [q_{ij}]$ is a matrix such that $PQ = kI$, where $k \in R$, $k \neq 0$ and $I$ is the identity matrix of order 3. If $q_{23} = -\frac{k}{8}$ and $\det(Q) = \frac{k^2}{2}$, then

(a) $\alpha = 0, k = 8$
(b) $4\alpha - k + 8 = 0$
(c) $\det(P \text{ adj}(Q)) = 2^9$
(d) $\det(Q \text{ adj}(P)) = 2^{13}$

**24.** The function $f(x) = x - |x - x^2|$, $-1 \leq x \leq 1$ is continuous on the interval

(a) $[-1, 1]$ (b) $(-1, 1)$
(c) $[-1, 1]\backslash\{0\}$ (d) $(-1, 1)\backslash\{0\}$

**25.** The greatest and least values of $f(x) = \tan^{-1}x - \frac{1}{2}\ln x$ on $\left[\frac{1}{\sqrt{3}}, \sqrt{3}\right]$ are

(a) $f_{\min} = \sqrt{3} - 1$ (b) $f_{\max} = \frac{\pi}{6} + \frac{1}{4}\ln 3$
(c) $f_{\min} = \frac{\pi}{3} - \frac{1}{4}\ln 3$ (d) $f_{\max} = \frac{\pi}{12} + \ln 5$

**26.** If $\int \frac{d\theta}{\cos^2\theta(\tan 2\theta + \sec 2\theta)} = \lambda\tan\theta + 2\log_e|f(\theta)| + C$, where $C$ is a constant of integration, then the ordered pair $(\lambda, f(\theta))$ is equal to

(a) $(-1, 1 - \tan\theta)$ (b) $(-1, 1 + \tan\theta)$
(c) $(1, 1 + \tan\theta)$ (d) $(1, 1 - \tan\theta)$

**27.** Let $y(x)$ be a solution of $(1 + x^2)\frac{dy}{dx} + 2xy - 4x^2 = 0$ and $y(0) = -1$. Then, $y(1)$ is equal to

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{6}$ (d) $-1$

**28.** In a group of 400 people, 160 are smokers and non- vegetarian; 100 are smokers and vegetarian and the remaining 140 are non-smokers and vegetarian. Their chances of getting a particular chest disorder are 35%, 20% and 10% respectively. A person is chosen from the group at random and is found to be suffering from the chest disorder. The probability that the selected person is a smoker and non-vegetarian is

(a) $\frac{7}{45}$ (b) $\frac{14}{45}$ (c) $\frac{28}{45}$ (d) $\frac{8}{45}$

**29.** There are three bags $B_1$, $B_2$ and $B_3$. The bag $B_1$ contains 5 red and 5 green balls, $B_2$ contains 3 red and 5 green balls and $B_3$ contains 5 red and 3 green balls. Bags $B_1$, $B_2$ and $B_3$ have probabilities 3/10, 3/10 and 4/10 respectively of being chosen. A bag is selected at random and a ball is chosen at random from the bag. Then which of the following options is/are correct?

(a) Probability that the chosen ball is green, given that the selected bag is $B_3$, equals 3/8.
(b) Probability that the selected bag is $B_3$, given that the chosen ball is green, equals 5/13.
(c) Probability that the chosen ball is green equals 39/80.
(d) Probability that the selected bag is $B_3$ and the chosen ball is green equals 3/10.

**30.** In a third order matrix $A$, $a_{ij}$ denotes the element in the $i^{th}$ row and $j^{th}$ column.
If $a_{ij} = 0$ for $i = j = 1$ for $i > j = -1$ for $i < j$
Then, the matrix is

(a) skew symmetric (b) symmetric
(c) not invertible (d) non-singular

## COMPREHENSION TYPE

### Paragraph for Q. No. 31 and 32

Consider the curves $S_1 : y = x^2$, $S_2 : y = -x^2$ and $S_3 : y^2 = 4x - 3$.

**31.** Area bounded by the curves $S_1$, $S_2$ and $S_3$ is

(a) 2/3 sq. unit (b) 1/3 sq. unit
(c) 3 sq. units (d) 4/3 sq. units

**32.** Area bounded by the curves $S_1$, $S_3$ and the line $x = 3$ is

(a) 13 sq. units (b) 3/13 sq. unit
(c) 13/3 sq. units (d) 1/3 sq. unit

### Paragraph for Q. No. 33 to 35

Let $f(x) = f_1(x) - 2f_2(x)$, where

$$f_1(x) = \begin{cases} \min\{x^2, |x|\}, & |x| \leq 1 \\ \max\{x^2, |x|\}, & |x| > 1 \end{cases}$$ and

$$f_2(x) = \begin{cases} \min\{x^2, |x|\}, & |x| > 1 \\ \max\{x^2, |x|\}, & |x| \leq 1 \end{cases}$$ and

let $$g(x) = \begin{cases} \min\{f(t) : -3 \leq t \leq x, & -3 \leq x < 0\} \\ \max\{f(t) : 0 \leq t \leq x, & 0 \leq x \leq 3\} \end{cases}$$

**33.** For $-3 \le x \le -1$, the range of $g(x)$ is
(a) $[-1, 3]$ (b) $[-1, -15]$
(c) $[-1, 9]$ (d) None of these

**34.** For $x \in (-1, 0)$, $f(x) + g(x)$ is
(a) $x^2 - 2x + 1$ (b) $x^2 + 2x - 1$
(c) $x^2 + 2x + 1$ (d) $x^2 - 2x - 1$

**35.** The graph of $y = g(x)$ in its domain is broken at
(a) 1 point (b) 2 points
(c) 3 points (d) None of these

**Paragraph for Q. No. 36 and 37**

Let $f : [0, 1] \to R$ (the set of all real numbers) be a function. Suppose the function $f$ is twice differentiable, $f(0) = f(1) = 0$ and satisfies $f''(x) - 2f'(x) + f(x) \ge e^x$, $x \in [0, 1]$.

**36.** Which of the following is true for $0 < x < 1$?
(a) $0 < f(x) < \infty$ (b) $-\frac{1}{2} < f(x) < \frac{1}{2}$
(c) $-\frac{1}{4} < f(x) < 1$ (d) $-\infty < f(x) < 0$

**37.** If the function $e^{-x} f(x)$ assumes its minimum in the interval $[0, 1]$ at $x = 1/4$, then which of the following is true?
(a) $f'(x) < f(x),\ \frac{1}{4} < x < \frac{3}{4}$
(b) $f'(x) > f(x),\ 0 < x < \frac{1}{4}$
(c) $f'(x) < f(x),\ 0 < x < \frac{1}{4}$
(d) $f'(x) < f(x),\ \frac{3}{4} < x < 1$

## MATRIX MATCH TYPE

**38.** The vertices of a triangle $ABC$ are $A(1, -2)$, $B(-7, 6)$ and $C(11/5, 2/5)$. Match the following columns.

| | Column-I | | Column-II |
|---|---|---|---|
| (A) | Equation of the perpendicular bisector of $AB$ | (P) | $26x + 17y + 8 = 0$ |
| (B) | Equation of the median through $A$ | (Q) | $14x + 23y - 40 = 0$ |
| (C) | Equation of the altitude through $C$ | (R) | $x - y + 5 = 0$ |
| (D) | Equation of $BC$ | (S) | $5x - 5y - 9 = 0$ |

(a) (A) → (P); (B) → (Q); (C) → (R); (D) → (S)
(b) (A) → (Q); (B) → (P); (C) → (S); (D) → (R)
(c) (A) → (R); (B) → (P); (C) → (S); (D) → (Q)
(d) (A) → (S); (B) → (P); (C) → (Q); (D) → (R)

**39.** Match the following columns.

| | Column-I | | Column-II |
|---|---|---|---|
| (A) | $f : [0, \infty) \to [0, \infty)$, $f(x) = \frac{x}{1+x}$ is | (P) | one-one and onto |
| (B) | $f : R - \{0\} \to R$, $f(x) = x - \frac{1}{x}$ is | (Q) | one-one but not onto |
| (C) | $f : R - \{0\} \to R$, $f(x) = x + \frac{1}{x}$ is | (R) | onto but not one-one |
| (D) | $f : R \to R$, $f(x) = 2x + \sin x$ is | (S) | neither one-one nor onto |

(a) (A) → (Q); (B) → (R); (C) → (P); (D) → (S)
(b) (A) → (Q); (B) → (R); (C) → (S); (D) → (P)
(c) (A) → (Q); (B) → (S); (C) → (P); (D) → (R)
(d) (A) → (Q); (B) → (S); (C) → (R); (D) → (P)

**40.** Match the following columns.

| | Column-I | | Column-II |
|---|---|---|---|
| (A) | If $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$, then $x^2 + y^2 + z^2 + 2xyz =$ | (P) | 10 |
| (B) | If $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \frac{3\pi}{2}$ and $f(1) = 1$, $f(p + q) = f(p) . f(q)$ $\forall p, q \in R$, then $x^{f(1)} + y^{f(2)} + x^{f(3)} - \frac{x+y+z}{x^{f(1)} + y^{f(2)} + z^{f(3)}} =$ | (Q) | 20 |
| (C) | If $\sum_{i=1}^{10} \cos^{-1}x_i = 10\pi$, then $\sum_{i=1}^{10} x_i^2 =$ | (R) | 2 |
| (D) | If $\sum_{i=1}^{20} \operatorname{cosec}^{-1}x_i = 10\pi$, then $\sum_{i=1}^{10} 2x_i^2 =$ | (S) | 1 |

(a) (A) → (S); (B) → (R); (C) → (P); (D) → (Q)
(b) (A) → (R); (B) → (S); (C) → (P); (D) → (Q)
(c) (A) → (S); (B) → (P); (C) → (R); (D) → (Q)
(d) (A) → (P); (B) → (R); (C) → (Q); (D) → (S)

## NUMERICAL ANSWER TYPE

**41.** Let tanα, tanβ and tanγ; $\alpha, \beta, \gamma \neq \frac{(2n-1)\pi}{2}$, $n \in N$ be the slopes of three line segments *OA*, *OB* and *OC*, respectively, where *O* is origin. If circumcentre of Δ*ABC* coincides with origin and its orthocentre lies on *y*-axis, then the value of $\left(\frac{\cos 3\alpha + \cos 3\beta + \cos 3\gamma}{\cos\alpha \cos\beta \cos\gamma}\right)^2$ is equal to ______.

**42.** If $A = \{x : x \in N, x \geq 2 \text{ and } x < 3\}$, $B = \{x : -2 \leq x \leq 4, x \in Z\}$ and $C = \{x : x \in Z, -3 < x < 4\}$. Then $n(A) + (n(B) \times n(C))$ is equal to ______.

**43.** The number of ways to give 16 different things to three persons *A*, *B*, *C* so that *B* gets 1 more than *A* and *C* gets 2 more than *B*, is $\frac{16!}{n!\,m!\,r!}$, $(n < m < r)$, then the value of $n + m - r$ is ______.

**44.** The $5^{th}$ and $8^{th}$ terms of a geometric sequence of real numbers are 7! and 8!, respectively. If the sum to first *n* terms of the G.P. is 2205, then *n* equals ______.

**45.** From any point on the hyperbola $\frac{x^2}{4} - y^2 = 1$, tangents are drawn to the hyperbola $\frac{x^2}{4} - y^2 = 2$. The area cut-off by the chord of contact on the asymptotes is equal to______ .

**46.** Let *k* be a positive real number and

$$A = \begin{bmatrix} 2k-1 & 2\sqrt{k} & 2\sqrt{k} \\ 2\sqrt{k} & 1 & -2k \\ -2\sqrt{k} & 2k & -1 \end{bmatrix}, B = \begin{bmatrix} 0 & 2k-1 & \sqrt{k} \\ 1-2k & 0 & 2\sqrt{k} \\ -\sqrt{k} & -2\sqrt{k} & 0 \end{bmatrix}$$

If det (adj *A*) + det (adj *B*) = $10^6$, then [*k*] is equal to ______.

**47.** Let $f : R \to R$ be a differentiable function such that its derivative $f'$ is continuous and $f(\pi) = -6$.

If $F : [0, \pi] \to R$ is defined by $F(x) = \int_0^x f(t)dt$, and if $\int_0^\pi (f'(x) + F(x))\cos x\,dx = 2$, then the value of *f*(0) is ______.

**48.** In a triangle *PQR*, let $\vec{a} = \overrightarrow{QR}$, $\vec{b} = \overrightarrow{RP}$ and $\vec{c} = \overrightarrow{PQ}$.

If $|\vec{a}| = 3$, $|\vec{b}| = 4$ and $\frac{\vec{a}\cdot(\vec{c}-\vec{b})}{\vec{c}\cdot(\vec{a}-\vec{b})} = \frac{|\vec{a}|}{|\vec{a}|+|\vec{b}|}$,

then value of $|\vec{a}\times\vec{b}|^2$ is ______.

**49.** The probability distribution of random variable *X* is given by

| *X* | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| *P*(*X*) | *K* | 2*K* | 2*K* | 3*K* | *K* |

Let $p = P(1 < X < 4 | X < 3)$. If $5p = \lambda K$, then λ is equal to ______.

**50.** Let the function $f : (0, \pi) \to R$ be defined by $f(\theta) = (\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^4$. Suppose the function *f* has a local minimum at θ precisely when $\theta \in \{\lambda_1\pi, \ldots, \lambda_r\pi\}$, where $0 < \lambda_1 < \ldots < \lambda_r < 1$. Then the value of $\lambda_1 + \ldots + \lambda_r$ is ______.

## SOLUTIONS

**1. (a) :** Let $z = x + iy$. Then

$$\frac{z-8i}{z+6} = \frac{x+(y-8)i}{(x+6)+iy} = \frac{(x+(y-8)i)(x+6-iy)}{(x+6)^2+y^2}$$

$$= \frac{(x^2+6x+y^2-8y)+i(xy-8x+6y-48-xy)}{(x+6)^2+y^2}$$

$$\text{Re}\left(\frac{z-8i}{z+6}\right) = 0 \Rightarrow x^2 + y^2 + 6x - 8y = 0$$

**2. (c) :** We have, α and β are the roots of $x^2 + bx + c = 0$

$$\Rightarrow \alpha = \frac{-b+\sqrt{b^2-4c}}{2} \text{ and } \beta = \frac{-b-\sqrt{b^2-4c}}{2}$$

$$\text{Now, } \lim_{x\to\beta} \frac{e^{2(x^2+bx+c)} - 1 - 2(x^2+bx+c)}{(x-\beta)^2} \quad \left(\frac{0}{0}\text{ Form}\right)$$

$$= \lim_{x\to\beta} \frac{e^{2(x^2+bx+c)}\cdot 2(2x+b) - 2(2x+b)}{2(x-\beta)}$$

[By L' Hospital Rule]

$$= \lim_{x\to\beta} \frac{2(2x+b)\cdot[e^{2(x^2+bx+c)}-1]}{2(x-\beta)} \quad \left(\frac{0}{0}\text{ Form}\right)$$

$$= \lim_{x\to\beta} \frac{(2x+b)\cdot e^{2(x^2+bx+c)}\cdot 2(2x+b) + (e^{2(x^2+bx+c)}-1)\cdot 2}{1}$$

[By L' Hospital Rule]

$$= e^0 \cdot 2\,(2\beta+b)^2 = 2\left[2\left(\frac{-b-\sqrt{b^2-4c}}{2}\right)+b\right]^2 = 2(b^2-4c)$$

**3. (b)**

**4. (c) :** Let $I = \int_0^{\pi} \sqrt{1 + 4\sin^2\frac{x}{2} - 4\sin\frac{x}{2}}\, dx$

$= \int_0^{\pi} \sqrt{\left(1 - 2\sin\frac{x}{2}\right)^2}\, dx = \int_0^{\pi} \left|1 - 2\sin\frac{x}{2}\right| dx$

$= \int_0^{\pi/3} \left(1 - 2\sin\frac{x}{2}\right) dx + \int_{\pi/3}^{\pi} \left(2\sin\frac{x}{2} - 1\right) dx$

$= \left[\left(x + 4\cos\frac{x}{2}\right)\right]_0^{\pi/3} + \left[\left(-4\cos\frac{x}{2} - x\right)\right]_{\pi/3}^{\pi}$

$= -\frac{\pi}{3} + 8\cdot\frac{\sqrt{3}}{2} - 4 = 4\sqrt{3} - 4 - \frac{\pi}{3}$

**5. (d):** Since, $f(x)$ is defined if $x! > 0$

$\log_{0.5} x! \ge 0$, $\frac{\log x!}{\log 0.5} \ge 0$, $\log x! \le 0$

$x! \le e^0$, $x! \le 1$

$\therefore$ Domain of the function $f(x)$ is $\{0, 1\}$.

**6. (b):** Let $y = \frac{x^2 + 2x + 1}{x^2 - 8x + 12}$

By cross multiplying, we get

$yx^2 - 8xy + 12y - x^2 - 2x - 1 = 0$

$\Rightarrow x^2(y - 1) - x(8y + 2) + (12y - 1) = 0$

Case I : When $y \ne 1$ ; $D \ge 0$

$\Rightarrow (8y + 2)^2 - 4(y - 1)(12y - 1) \ge 0 \Rightarrow y(4y + 21) \ge 0$

$y \in \left(-\infty, \frac{-21}{4}\right] \cup [0, \infty) - \{1\}$

Case II : When $y = 1$

$x^2 + 2x + 1 = x^2 - 8x + 12 \Rightarrow 10x = 11 \Rightarrow x = \frac{11}{10}$

Hence, $y \in \left(-\infty, \frac{-21}{4}\right] \cup [0, \infty)$

**7. (c)**

**8. (d):** Let $z = \frac{(3 + i\sin\theta)}{(4 - i\cos\theta)} \times \frac{(4 + i\cos\theta)}{(4 + i\cos\theta)}$

$= \frac{12 - \sin\theta\cos\theta + i(3\cos\theta + 4\sin\theta)}{4^2 - (i\cos\theta)^2}$

Since, $z$ is purely real.

$\therefore$ $3\cos\theta + 4\sin\theta = 0 \Rightarrow \tan\theta = -\frac{3}{4}$

$\therefore$ $\arg(\sin\theta + i\cos\theta) = \pi - \tan^{-1}\left|\frac{\cos\theta}{\sin\theta}\right|$

$= \pi - \tan^{-1}\left|-\frac{4}{3}\right| = \pi - \tan^{-1}\left(\frac{4}{3}\right)$

**9. (c) :** $A$, $B$ and $C$ can be chosen in ${}^{10}C_3$ ways is $\frac{10!}{7!3!}$ and the remaining persons can speak in 7! ways.

$\therefore$ Number of ways in which they can speak is

$\frac{10!}{7!3!}(7!) = \frac{10!}{6}$

**10. (a) :** Coefficient of $x^{18}$ in $(1 + x)(1 - x)^{10}(1 + x + x^2)^9$

$=$ Coefficient of $x^{18}$ in $(1 - x^2)[(1 - x)(1 + x + x^2)]^9$

$=$ Coefficient of $x^{18}$ in $(1 - x^2)(1 - x^3)^9$

$=$ Coefficient of $x^{18}$ in

$(1 - x^2)(1 - {}^9C_1x^3 + {}^9C_2x^6 - {}^9C_3x^9 + \ldots + {}^9C_6x^{18} - \ldots$
$- {}^9C_9x^{27}) = {}^9C_6 = 84$

**11. (b):** Let the foci are $A(-2, 4)$ and $B(2, 1)$, then

$AB = \sqrt{16 + 9} = 5 \Rightarrow 2ae = 5$

Mid point of foci is the centre of ellipse

$\therefore$ Centre is $\left(0, \frac{5}{2}\right)$

Let $b$ be the length of semi-minor axis, then

$b = \sqrt{(1 - 0)^2 + \left(\frac{23}{6} - \frac{5}{2}\right)^2} = \frac{5}{3}$

Now, $b^2 = a^2(1 - e^2)$ and $ae = \frac{5}{2}$

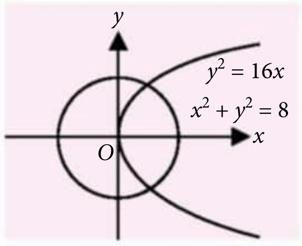
So, we get $a^2 = \frac{325}{36}$

$\because$ $e^2 = \frac{25}{4} \times \frac{36}{325} = \frac{9}{13} \Rightarrow e = \frac{3}{\sqrt{13}}$

**12. (c)**

**13. (d):** Comparing $y^2 = 16x$ with the standard equation of the parabola $y^2 = 4ax$, we get

$4a = 16 \Rightarrow a = 4$




Equation of tangent of parabola is

$y = mx + \frac{a}{m} \Rightarrow y = mx + \frac{4}{m}$

$\Rightarrow mx - y + \frac{4}{m} = 0$ ...(i)

If equation of tangent to the parabola is also tangent to the circle $x^2 + y^2 = 8$, then length of perpendicular from centre (0, 0) to (i) will be radius of the circle, which is $\sqrt{8}$.

$\therefore$ $\frac{0 - 0 + \frac{4}{m}}{\sqrt{m^2 + (-1)^2}} = \sqrt{8} \Rightarrow \frac{\frac{4}{m}}{\sqrt{m^2 + 1}} = \sqrt{8}$

$\Rightarrow \dfrac{16}{m^2} = 8(m^2+1) \Rightarrow 2 = m^2(m^2+1)$

$\Rightarrow (m^2-1)(m^2+2) = 0$

Thus, $m^2 - 1 = 0$ or $m^2 + 2 = 0$, which is impossible.

So, $m = \pm 1$

Hence, equation of common tangents will be

$y = \pm x \pm \dfrac{4}{1} \Rightarrow y = \pm x \pm 4$

Thus, common tangents are $y = x + 4$, $y = -x - 4$.

**14. (a)**

**15. (d):** We have, $f(x) = \begin{cases} a\sin x - be^x, x \ge 0 \\ -a\sin x - be^{-x}, x < 0 \end{cases}$

which is continuous at $x = 0$.

$\Rightarrow f'(x) = \begin{cases} a\cos x - be^x, x \ge 0 \\ -a\cos x + be^{-x}, x < 0 \end{cases}$

If $f(x)$ is differentiable at $x = 0$, then L.H.D. = R.H.D.

$\Rightarrow a - b = -a + b \Rightarrow a - b = 0$

**16. (b, c)**

**17. (a, b, c) :** As, $a > b > c$

$\therefore a - c > 0$ and $b - c > 0 \Rightarrow (a-c) + (b-c) > 0$

$\Rightarrow a + b - 2c > 0$ *i.e.*, coefficients of $x^2 > 0$

$\Rightarrow$ Parabola opens upwards

Now, when $f(x) = 0$, we get $x = 1$, $\dfrac{c+a-2b}{a+b-2c}$ as the roots.

Hence, both roots are rational.

Also, $x$-coordinate of vertex of the graph is positive.

**18. (a) :** Given,

$$\lim_{x\to 0} \frac{3 + \alpha\sin x + \beta\cos x + \log_e(1-x)}{3\tan^2 x} = \frac{1}{3}$$

$$\Rightarrow \lim_{x\to 0} \frac{3 + \alpha\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots\right) + \beta\left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots\right) + \left(-x - \frac{x^2}{2!} - \frac{2x^3}{3!} - \dots\right)}{3\left(\frac{\tan x}{x}\right)^2 \cdot x^2} = \frac{1}{3}$$

$$\Rightarrow \lim_{x\to 0} \frac{3 + \alpha\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots\right) + \beta\left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots\right) + \left(-x - \frac{x^2}{2!} - \frac{2x^3}{3!} - \dots\right)}{3x^2} = \frac{1}{3}$$

Comparing the coefficient of $x^0$, we get

$\beta = -3$

Comparing the coefficient of $x^{-1}$, we get

$\alpha - 1 = 0 \Rightarrow \alpha = 1$

$\therefore 2\alpha - \beta = 2(1) - (-3) = 2 + 3 = 5$

**19. (a, c)**

**20. (a, c) :** Here, $f(x) \ge 0$

and $f'(x) - \alpha f(x) \le 0$ (Given)

$\Rightarrow \dfrac{d}{dx}[e^{-\alpha x} f(x)] \le 0 \;\; \forall x > 1 \Rightarrow f(x) \le 0 \;\forall\; x > 1$

$\because f(1) = 0 \Rightarrow \quad f(x) = 0 \;\; \forall\; x > 1$

Similarly, $g(x) = 0 \;\forall\; x > 1$

Now, $\dfrac{f(e)+f(\pi)}{f^2(e)+f^2(\pi)-4} = 0$ and $\dfrac{g(\sqrt{e})+g(\sqrt{\pi})}{g(\sqrt[3]{e})+g(\sqrt[3]{\pi})-3} = 0$

**21. (a, b) :** Equation of a plane passing through the line $3x - y + 2z - 1 = 0 = x + 2y - z - 2$ is

$3x - y + 2z - 1 + \lambda(x + 2y - z - 2) = 0$

Since, it is perpendicular to the given plane

$\therefore \lambda = -\dfrac{3}{2}$

$\therefore$ Equation of the line of projection is

$3x - 8y + 7z + 4 = 0 = 3x + 2y + z$

Its direction ratios are $< 11, -9, -15 >$ and the point $(-1,1,1)$ lies on the line

$\therefore \dfrac{x+1}{11} = \dfrac{y-1}{-9} = \dfrac{z-1}{-15}$ is also the equation of the line of projection.

**22. (a, b, c)**

**23. (b, c) :** We have, $PQ = kI$

$\Rightarrow Q = P^{-1}kI = \dfrac{k}{\det P}(\text{adj } P)I$

$$= \frac{k}{20+12\alpha}\begin{bmatrix} 5\alpha & 10 & -\alpha \\ 3\alpha & 6 & -(3\alpha+4) \\ -10 & 12 & 2 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

As, $q_{23} = -k/8$

$\Rightarrow \dfrac{k}{20+12\alpha}\cdot(3\alpha+4) = \dfrac{k}{8}$

$\Rightarrow 2(3\alpha + 4) = 5 + 3\alpha$ $\quad [\because k \ne 0]$

$\Rightarrow 6\alpha + 8 = 5 + 3\alpha \Rightarrow \alpha = -1$

Now, $\det Q = \dfrac{k^3}{\det P} \Rightarrow \dfrac{k^2}{2} = \dfrac{k^3}{20+12\alpha}$

$\Rightarrow 2k = 20 + 12\alpha \;\; \therefore k = 4$

$\det(P \text{ adj } Q) = \det(P)(\det \text{adj } Q)$

$$= 8\cdot\left(\frac{k^2}{2}\right)^2 = 2^9$$

**24. (a) :** Given, $f(x) = x - |x - x^2|, -1 \le x \le 1$

$$\Rightarrow \quad f(x) = \begin{cases} 2x - x^2, & -1 \le x \le 0 \\ x^2, & 0 < x \le 1 \end{cases}$$

Now, $\lim_{x\to 0^-} f(x) = \lim_{x\to 0} (2x - x^2) = 0$

and $\lim_{x\to 0^+} f(x) = \lim_{x\to 0} x^2 = 0$

Also, $f(0) = 0$

$\therefore$ $f(x)$ is continuous in the interval $[-1, 1]$.

**25. (b, c) :** We have, $f(x) = \tan^{-1}x - \frac{1}{2}\ln x$

$$\Rightarrow \quad f'(x) = \frac{1}{1+x^2} - \frac{1}{2}\cdot\frac{1}{x} = \frac{2x-1-x^2}{2x(1+x^2)} = \frac{-(x-1)^2}{2x(1+x^2)} < 0$$

$\therefore$ $f(x)$ is a decreasing function in $\left[\frac{1}{\sqrt{3}}, \sqrt{3}\right]$.

$$\therefore \quad f_{max} = f\left(\frac{1}{\sqrt{3}}\right) = \tan^{-1}\frac{1}{\sqrt{3}} - \frac{1}{2}\ln\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6} + \frac{1}{4}\ln 3$$

and $f_{min} = f(\sqrt{3}) = \tan^{-1}(\sqrt{3}) - \frac{1}{2}\ln\sqrt{3} = \frac{\pi}{3} - \frac{1}{4}\ln 3$

**26. (b):** Let $I = \int \frac{\sec^2\theta}{\left(\frac{2\tan\theta}{1-\tan^2\theta}\right) + \left(\frac{1+\tan^2\theta}{1-\tan^2\theta}\right)} d\theta$

$$= \int \frac{\sec^2\theta(1-\tan^2\theta)}{(1+\tan\theta)^2} d\theta = \int \frac{\sec^2\theta(1-\tan\theta)}{1+\tan\theta} d\theta$$

Putting $\tan\theta = t \Rightarrow \sec^2\theta\, d\theta = dt$, we get

$$I = \int\left(\frac{1-t}{1+t}\right) dt = \int\left(-1 + \frac{2}{1+t}\right) dt$$

$= -t + 2\log_e|(1+t)| + C = -\tan\theta + 2\log_e|(1+\tan\theta)| + C$

$\therefore$ $\lambda = -1$ and $f(\theta) = 1 + \tan\theta$

**27. (c) :** We have, $\frac{dy}{dx} + \frac{2x}{1+x^2}\cdot y = \frac{4x^2}{1+x^2}$

It is a linear differential equation.

$$\therefore \quad \text{I.F.} = e^{\int \frac{2x}{1+x^2}dx} = e^{\log(1+x^2)} = (1+x^2)$$

$\therefore$ General solution is

$$y(1+x^2) = \int 4x^2 dx = \frac{4x^3}{3} + c$$

$\because$ $y(0) = -1 \Rightarrow -1(1+0) = 0 + c \Rightarrow c = -1$

$$\therefore \quad y = \frac{4x^3}{3(1+x^2)} - \frac{1}{1+x^2} \quad \therefore \quad y(1) = \frac{4}{3(1+1)} - \frac{1}{1+1} = \frac{1}{6}$$

**28. (c) :** Consider the following events.

$A$ : Person chosen is a smoker and non-vegetarian.

$B$ : Person chosen is a smoker and vegetarian.

$C$ : Person chosen is a non-smoker and vegetarian.

$E$ : Person chosen has a chest disorder.

Now, $P(A) = \frac{160}{400}$, $P(B) = \frac{100}{400}$, $P(C) = \frac{140}{400}$,

and $P\left(\frac{E}{A}\right) = \frac{35}{100}$, $P\left(\frac{E}{B}\right) = \frac{20}{100}$, $P\left(\frac{E}{C}\right) = \frac{10}{100}$

$\therefore$ Required probability $= P\left(\frac{A}{E}\right)$

$$= \frac{P(A)\cdot P\left(\frac{E}{A}\right)}{P(A)\cdot P\left(\frac{E}{A}\right) + P(B)\cdot P\left(\frac{E}{B}\right) + P(C)\cdot P\left(\frac{E}{C}\right)}$$

[Using Bayes' theorem]

$$= \frac{\frac{160}{400}\times\frac{35}{100}}{\frac{160}{400}\times\frac{35}{100} + \frac{100}{400}\times\frac{20}{100} + \frac{140}{400}\times\frac{10}{100}} = \frac{28}{45}$$

**29. (a, c) :** The bag $B_1$ has 5 red and 5 green balls.

Bag $B_2$ has 3 red and 5 green balls

Bag $B_3$ has 5 red and 3 green balls

Clearly, $P\left(\frac{G}{B_3}\right) = \frac{3}{8}$

$$P(G) = P(B_1)\cdot P\left(\frac{G}{B_1}\right) + P(B_2)\cdot P\left(\frac{G}{B_2}\right) + P(B_3)\cdot P\left(\frac{G}{B_3}\right)$$

$$= \frac{3}{10}\cdot\frac{1}{2} + \frac{3}{10}\cdot\frac{5}{8} + \frac{4}{10}\cdot\frac{3}{8} = \frac{3}{20} + \frac{3}{16} + \frac{3}{20} = \frac{12+15+12}{80} = \frac{39}{80}$$

Now, $P(G \cap B_3) = P\left(\frac{G}{B_3}\right)P(B_3) = \frac{3}{8}\times\frac{4}{10} = \frac{3}{20}$

and $P\left(\frac{B_3}{G}\right) = \frac{P(G\cap B_3)}{P(G)} = \frac{3/20}{39/80} = \frac{4}{13}$

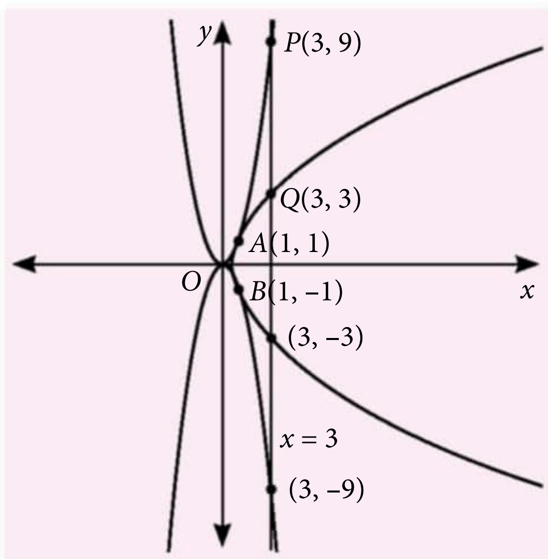
**30. (a, c) :** As defined, $A = \begin{pmatrix} 0 & -1 & -1 \\ 1 & 0 & -1 \\ 1 & 1 & 0 \end{pmatrix}$

Here, $A$ is a skew-symmetric matrix and of odd order.

$\therefore$ $|A| = 0 \Rightarrow$ singular matrix

$\therefore$ $A$ is non-invertible (inverse does not exist).

**31. (b):**




Area of region $OAB = 2\int_0^1 \left(\frac{y^2+3}{4} - \sqrt{y}\right)dy = \frac{1}{3}$ sq. unit

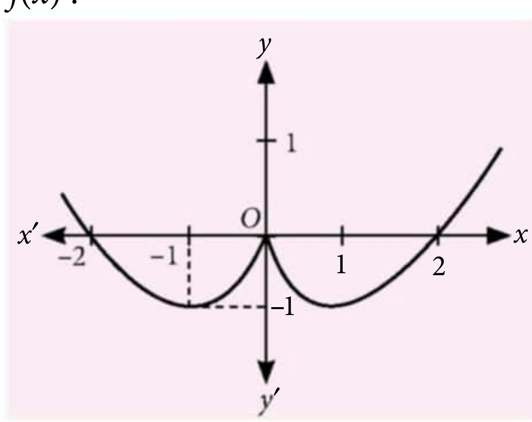
**32. (c) :** Area of region $APQ = \int_1^3 (x^2 - \sqrt{(4x-3)})dx$

$= \frac{13}{3}$ sq. unit

**33. (a) :** $f_1(x) = x^2$ and $f_2(x) = |x|$

or $f(x) = f_1(x) - 2f_2(x) = x^2 - 2|x|$

Graph of $f(x)$ :




The range of $g(x)$ for $[-3, -1]$ is $[-1, 3]$.

**34. (b):** For $x \in (-1, 0)$, $f(x) + g(x) = x^2 + 2x - 1$

**35. (a) :** Obviously, the graph is broken at $x = 0$.

**36. (d):** Observe that

$e^{-x}\{f'' - 2f' + f\} = \frac{d}{dx}\{e^{-x}(f'(x) - f(x))\}$

The given condition is

$f''(x) - 2f'(x) + f(x) \geq e^x \Rightarrow e^{-x}\{f''(x) - 2f'(x) + f(x)\} \geq 1$

$\Rightarrow \frac{d}{dx}\{e^{-x}(f'(x) - f(x))\} \geq 1 \Rightarrow \frac{d^2}{dx^2}\{e^{-x}f(x)\} \geq 1$

Let $g(x) = e^{-x}f(x)$.

$g''(x) > 0$ means that $g$ is concave upward.

$g(0) = g(1) = 0$

$\therefore\ g(x) < 0\ \forall\ x \in (0,1) \Rightarrow e^{-x}f(x) < 0\ \forall\ x \in (0,1)$

$\therefore\ f(x) < 0$

**37. (c) :** We have, $\frac{d}{dx}\{e^{-x}(f'(x) - f(x))\} \geq 1$

Let $A(x) = e^{-x}f(x)$

Then $A'(x) = e^{-x}(f'(x) - f(x))$

$A(x)$ has local minimum at $x = 1/4$

$\therefore\ A'(1/4) = 0$

Let $B(x) = e^{-x}(f'(x) - f(x))$

$B$ is an increasing function.

$B(x) < 0$ in $(0, 1/4) \Rightarrow e^{-x}(f'(x) - f(x)) < 0$ in $(0, 1/4)$

Thus, $f'(x) < f(x)$ in $(0, 1/4)$

**38. (c) :** Mid point of $AB$ is $\left(\frac{1-7}{2}, \frac{-2+6}{2}\right) = (-3, 2)$

and slope of $AB$ is $\frac{6+2}{-7-1} = -1$

(A) Equation of the perpendicular bisector of $AB$ is

$y - 2 = 1 \cdot (x + 3) \Rightarrow x - y + 5 = 0$

(B) Mid point of $BC$ is

$\left(\frac{-7+(11/5)}{2}, \frac{(2/5)+6}{2}\right) = \left(\frac{-12}{5}, \frac{16}{5}\right)$

$\therefore$ Equation of median through $A$ is

$\frac{y+2}{(16/5)+2} = \frac{x-1}{(-12/5)-1} \Rightarrow \frac{y+2}{26} = \frac{x-1}{-17}$

$\Rightarrow 26x + 17y + 8 = 0$

(C) Slope of $AB = -1$

$\therefore$ Equation of the altitude through $C$ is

$y - (2/5) = 1\,(x - (11/5)) \Rightarrow 5x - 5y - 9 = 0$

(D) Equation of $BC$ is

$\frac{y-(2/5)}{6-(2/5)} = \frac{x-(11/5)}{-7-(11/5)} \Rightarrow \frac{5y-2}{28} = \frac{5x-11}{-46}$

$\Rightarrow 14x + 23y - 40 = 0$

**39. (b):** (A) $f'(x) = \frac{1}{(1+x)^2} > 0 \Rightarrow f(x)$ is one-one and

$f(x) \to 1$ as $x \to \infty \Rightarrow f(x)$ not onto

(B) $f'(x) = 1 + \frac{1}{x^2} \Rightarrow f(x)$ is increases both for $x > 0$ and $x < 0$. Also, $f(1) = f(-1) = 0$

$\Rightarrow f(x)$ is onto but not one-one

(C) $f'(x) = 1 - \frac{1}{x^2}$ changes sign $\Rightarrow f(x)$ is not one-one and $f(x)$ is not onto since $f(x) \geq 2$ and $f(x) \leq -2$

(D) $f'(x) = 2 + \cos x > 0 \Rightarrow f(x)$ is one-one and $f(x) \to \pm\infty$ as $x \to \pm\infty \Rightarrow f(x)$ is onto

**40. (a) :** (A) We have, $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$

$\cos(\cos^{-1}x + \cos^{-1}y) = \cos(\pi - \cos^{-1}z) = -\cos(\cos^{-1}z)$

$$\Rightarrow \quad xy - \sqrt{1-x^2}\sqrt{1-y^2} = -z$$

$$\Rightarrow \quad xy + z = \sqrt{1-x^2}\sqrt{1-y^2}$$

Squaring on both sides, we get

$x^2 + y^2 + z^2 + 2xyz = 1$

(B) Since, $-\frac{\pi}{2} \le \sin^{-1}x \le \frac{\pi}{2}$, and

$$\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \frac{3\pi}{2}$$

$$\Rightarrow \quad \sin^{-1}x = \sin^{-1}y = \sin^{-1}z = \frac{\pi}{2} \quad \Rightarrow \quad x = y = z = 1$$

Also, $f(p + q) = f(p) \cdot f(q) \quad \forall p, q \in R$ ...(i)

Given, $f(1) = 1$

From (i), $f(1 + 1) = (f(1))^2$

$\Rightarrow \quad f(2) = 1^2 = 1$

From (i), $f(2 + 1) = f(2) \cdot f(1)$

$\Rightarrow \quad f(3) = 1^2 \cdot 1 = 1$

$$x^{f(1)} + y^{f(2)} + z^{f(3)} - \frac{x+y+z}{x^{f(1)} + y^{f(2)} + z^{f(3)}}$$

$$= (x+y+z) - \frac{(x+y+z)}{(x+y+z)} = 3 - 1 = 2$$

(C) $\sum_{i=1}^{10} \cos^{-1}x_i = 10\pi$

$$\Rightarrow \quad \cos^{-1}x_i = \pi, \forall i \Rightarrow x_i = -1 \forall i \quad \therefore \sum_{i=1}^{10} x_i^2 = 10$$

(D) $\text{cosec}^{-1}x_i \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

Given $\sum_{i=1}^{20} \text{cosec}^{-1}x_i = 10\pi = 20 \cdot \frac{\pi}{2}$

$$\Rightarrow \quad \text{cosec}^{-1}x_i = \frac{\pi}{2} \text{ for } i = 1, 2, ...., 20$$

$$\Rightarrow \quad x_i = \text{cosec}\frac{\pi}{2} = 1, \text{ for } i = 1, 2, ...., 20 \Rightarrow \sum_{i=1}^{10} 2x_i^2 = 20$$

**41. (144) :** Let $A(r\cos\alpha, r\sin\alpha)$, $B(r\cos\beta, r\sin\beta)$ and $C(r\cos\gamma, r\sin\gamma)$ be the vertices of triangle $ABC$.

Since, circumcentre and orthocentre both lies on $y$-axis.

$\therefore$ Centroid also lies on $y$-axis.

$\Rightarrow \cos\alpha + \cos\beta + \cos\gamma = 0$

$\therefore \cos^3\alpha + \cos^3\beta + \cos^3\gamma = 3\cos\alpha \cdot \cos\beta \cdot \cos\gamma$ ...(i)

Now, $\cos3\alpha + \cos3\beta + \cos3\gamma = 4(\cos^3\alpha + \cos^3\beta + \cos^3\gamma)$

$$\Rightarrow \quad \frac{\cos3\alpha + \cos3\beta + \cos3\gamma}{\cos^3\alpha + \cos^3\beta + \cos^3\gamma} = 4 \quad \text{...(ii)}$$

Using (i) and (ii), we get

$$\left(\frac{\cos3\alpha + \cos3\beta + \cos3\gamma}{\cos\alpha \cdot \cos\beta \cdot \cos\gamma}\right)^2 = 12^2 = 144$$

**42. (43) :** We have, $A = \{2\} \Rightarrow n(A) = 1$

$B = \{-2, -1, 0, 1, 2, 3, 4\} \Rightarrow n(B) = 7$

$C = \{-2, -1, 0, 1, 2, 3\} \Rightarrow n(C) = 6$

$\therefore \quad n(A) + (n(B) \times n(C)) = 1 + 7(6) = 1 + 42 = 43$

**43. (2) :** Number of things given to $A$ is 4

Number of things given to $B$ is 5

Number of things given to $C$ is 7

Hence, required number of ways $= \frac{16!}{4!5!7!}$

$\therefore \quad n + m - r = 2$

**44. (3) :** Let $a, ar, ar^2, ar^3, ...$ are in G.P.

Now $ar^4 = 7!$ and $ar^7 = 8!$

On dividing, we get $r^3 = 8 \Rightarrow r = 2$

$$\because \quad a \times 2^4 = 5040 \Rightarrow a = \frac{5040}{16} = 315$$

So, 315, 630, 1260, ...... are in G.P.

$\therefore \quad S_3 = 2205 \Rightarrow n = 3$

**45. (8) :** Let $P(x_1, y_1)$ be a point on the hyperbola $\frac{x^2}{4} - y^2 = 1$, then $\frac{x_1^2}{4} - y_1^2 = 1$

The chord of contact of tangents from $P$ to the hyperbola

$$\frac{x^2}{4} - y^2 = 2 \text{ is } \frac{xx_1}{4} - \frac{yy_1}{1} = 2 \quad \text{...(i)}$$

The equation of the asymptotes are

$$\frac{x}{2} - y = 0 \text{ and } \frac{x}{2} + y = 0$$

The points of intersection of (i) with the two asymptotes are given by

$$x_1' = \frac{4}{\frac{x_1}{2} - y_1}, \; y_1' = \frac{2}{\frac{x_1}{2} - \frac{y_1}{1}}$$

$$x_2' = \frac{4}{\frac{x_1}{2} + y_1}, \; y_2' = \frac{-2}{\frac{x_1}{2} + \frac{y_1}{1}}$$

$$\therefore \quad \text{Area of the triangle} = \left|\frac{1}{2}(x_1'y_2' - x_2'y_1')\right| = 8$$

**46. (4) :** We have,

$$\det A = (2k-1)(4k^2-1) + 2\sqrt{k}\,(4k\sqrt{k} + 2\sqrt{k}) + 2\sqrt{k}\,(4k\sqrt{k} + 2\sqrt{k})$$

$= (2k-1)(4k^2-1) + 8(2k+1)k$

$= (2k+1)((2k-1)^2 + 8k) = (2k+1)^3$

$\therefore$ det (adj $A$) = (det $A)^2 = (2k+1)^6$

Since, $B$ is skew symmetric matrix of order 3.

$\Rightarrow$ det $B = 0$

$\therefore$ det (adj $B$) = (det $B)^2 = 0$

$\therefore$ $(2k+1)^6 = 10^6 \Rightarrow 2k+1 = 10 \Rightarrow k = 4.5$

So, $[k] = [4.5] = 4$

**47. (4) :** We have, $F(x) = \int_0^x f(t)\,dt \Rightarrow F'(x) = f(x)$

Also, $\int_0^\pi \left(f'(x)+F(x)\right)\cos x\,dx = 2$

Let, $I = \int_0^\pi f'(x)\cos x\,dx$

$= \left[\cos x\, f(x)\right]_0^\pi - \int_0^\pi (-\sin x) f(x)\,dx$

$= -f(\pi) - f(0) + \int_0^\pi \sin x\, f(x)\,dx$

$= 6 - f(0) + \int_0^\pi \sin x \cdot F'(x)\,dx$

$= 6 - f(0) + [\sin x\, F(x)]_0^\pi - \int_0^\pi \cos x\, F(x)\,dx$

$= 6 - f(0) + 0 - \int_0^\pi \cos x\, F(x)\,dx$

$\Rightarrow \int_0^\pi \left(f'(x)+F(x)\right)\cos x\,dx = 6 - f(0)$

$\Rightarrow 2 = 6 - f(0) \Rightarrow f(0) = 4$

**48. (108) :** We have, $|\vec{a}| = 3, |\vec{b}| = 4$

Also, $\dfrac{\vec{a}\cdot(\vec{c}-\vec{b})}{\vec{c}\cdot(\vec{a}-\vec{b})} = \dfrac{|\vec{a}|}{|\vec{a}|+|\vec{b}|}$

$\Rightarrow \dfrac{-(\vec{b}+\vec{c})(\vec{c}-\vec{b})}{-(\vec{a}+\vec{b})(\vec{a}-\vec{b})} = \dfrac{3}{7}$ $[\because \vec{a}+\vec{b}+\vec{c} = 0]$

$\Rightarrow \dfrac{|\vec{b}|^2 - |\vec{c}|^2}{|\vec{b}|^2 - |\vec{a}|^2} = \dfrac{3}{7} \Rightarrow \dfrac{16-|\vec{c}|^2}{16-9} = \dfrac{3}{7}$

$\Rightarrow 16 - |\vec{c}|^2 = 3 \Rightarrow |\vec{c}|^2 = 13$

Now, $\vec{a}+\vec{b} = -\vec{c} \Rightarrow |\vec{a}+\vec{b}|^2 = |-\vec{c}|^2$

$\Rightarrow |a|^2 + |b|^2 + 2\vec{a}\cdot\vec{b} = |c|^2 \Rightarrow 3^2 + 4^2 + 2\vec{a}\cdot\vec{b} = 13$

$\Rightarrow 2\vec{a}\cdot\vec{b} = -12 \Rightarrow \vec{a}\cdot\vec{b} = -6$

$\therefore |\vec{a}\times\vec{b}|^2 = |\vec{a}|^2|\vec{b}|^2 - (\vec{a}\cdot\vec{b})^2$

$= 3^2 \cdot 4^2 - 6^2 = 144 - 36 = 108$

**49. (30) :** We know that, $\sum P(X) = 1$

$\Rightarrow K + 2K + 2K + 3K + K = 1 \Rightarrow K = \dfrac{1}{9}$

Now, $p = P(1 < X < 4 | X < 3)$

$= \dfrac{P((1<X<4)\cap(X<3))}{P(X<3)} = \dfrac{P(X=2)}{P(X<3)}$

$= \dfrac{P(X=2)}{P(X=1)+P(X=2)} = \dfrac{\frac{2K}{9K}}{\frac{K}{9K}+\frac{2K}{9K}} = \dfrac{2}{3}$

Also, $5p = \lambda K$ (Given)

$\Rightarrow 5\times\dfrac{2}{3} = \lambda\times\dfrac{1}{9} \Rightarrow \lambda = 30$

**50. (0.5) :** We have, $f(\theta) = (\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^4$

$= 1 + \sin 2\theta + (1 - \sin 2\theta)^2$

Let $1 - \sin 2\theta = t$. As $\theta \in (0, \pi)$, $\sin 2\theta \in [-1, 1]$

$\therefore t \in [0, 2]$

$\therefore g(t) = 1 + 1 - t + t^2 = t^2 - t + 2$

Now, $g(t)$ has local minima at $t = 1/2$

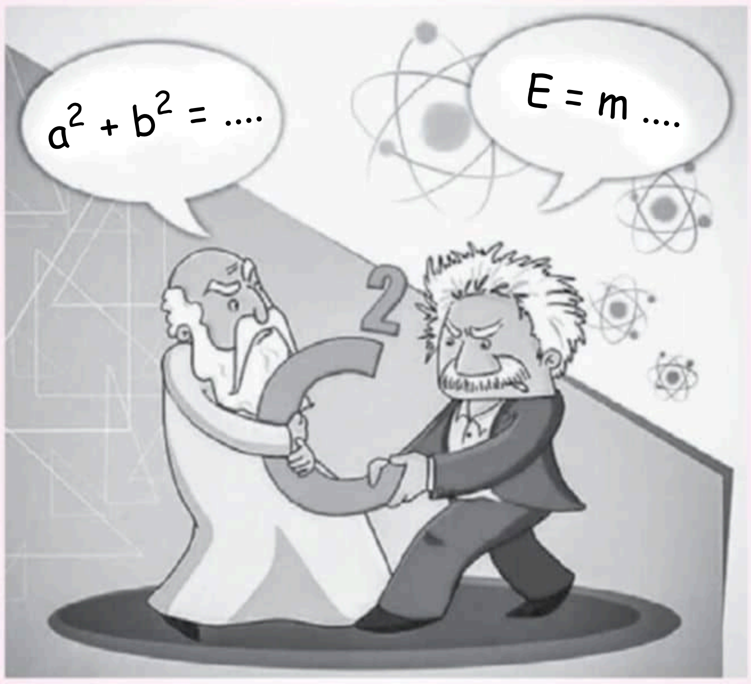
$\therefore 1 - \sin 2\theta = \dfrac{1}{2} \Rightarrow \sin 2\theta = \dfrac{1}{2}$

$\Rightarrow 2\theta = \dfrac{\pi}{6}, \dfrac{5\pi}{6}$. $(\because 2\theta \in (0, 2\pi))$

$\therefore \theta = \dfrac{\pi}{12}, \dfrac{5\pi}{12}$. So, $\lambda_1 = \dfrac{1}{12}$ and $\lambda_2 = \dfrac{5}{12}$

Thus, $\lambda_1 + \lambda_2 = \dfrac{1}{12} + \dfrac{5}{12} = \dfrac{6}{12} = 0.5$

## COMIC CAPSULE



# YOU ASK WE ANSWER

**Do you have a question that you just can't get answered?**
Use the vast expertise of our MTG team to get to the bottom of the question. From the serious to the silly, the controversial to the trivial, the team will tackle the questions, easy and tough.
The best questions and their solutions will be printed in this column each month.

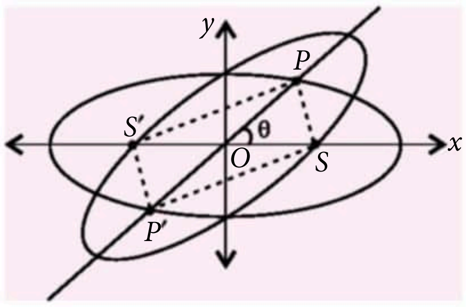
**1.** If $e_1$ and $e_2$ be the eccentricities of two concentric ellipses such that foci of one lie on the other and they have same length of major axes. If $e_1^2 + e_2^2 = p$, then find the least value of $p$. ***(Prithvik, Agra)***

**Ans.** Let θ be the angle of inclination of major axes and equation of one of the ellipses be

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \quad \text{... (i)}$$

whose foci are $S$ and $S'$ and eccentricity = $e_1$

$\therefore \quad b^2 = a^2(1 - e_1^2)$ ... (ii)




Let $P$ and $P'$ be foci of other ellipse whose eccentricity = $e_2$

Let $PP' = 2R \quad \Rightarrow \quad OP = R = ae_2$

$\because$ Diagonals $PP'$ and $SS'$ bisect each other.

$\therefore$ $PS'P'S$ is a parallelogram.

Here, co-ordinates of $P$ are ($R$cosθ, $R$sinθ).

$$\therefore \text{ From (i), } \frac{R^2\cos^2\theta}{a^2}+\frac{R^2\sin^2\theta}{b^2}=1$$

$$\Rightarrow \frac{a^2e_2^2(1-\sin^2\theta)}{a^2}+\frac{a^2e_2^2\sin^2\theta}{a^2(1-e_1^2)}=1 \quad \text{[Using (ii)]}$$

$$\Rightarrow \sin^2\theta\left(\frac{e_2^2}{1-e_1^2}-e_2^2\right)=1-e_2^2$$

$$\Rightarrow \sin^2\theta\left(\frac{e_1^2\,e_2^2}{1-e_1^2}\right)=1-e_2^2$$

$$\Rightarrow \frac{(1-e_1^2)(1-e_2^2)}{e_1^2\,e_2^2}=\sin^2\theta\le 1$$

$\Rightarrow \quad 1 - e_1^2 - e_2^2 + e_1^2e_2^2 \le e_1^2e_2^2$

$\Rightarrow \quad e_1^2 + e_2^2 \ge 1 \quad \Rightarrow \quad p \ge 1$

So, the least value of $p$ is 1.

**2.** Let $\left[\sqrt{n^2+1}\right]=\left[\sqrt{n^2+\lambda}\right]$, where $n, \lambda \in N$ and $[\cdot]$ represents greatest integer function. Show that λ can have $2n$ different values. ***(Tejaswin Jha, Bihar)***

**Ans.** We have,

$n^2 + 1 = (n + 1)^2 - 2n < (n + 1)^2 \ \forall \ n \in N$

$\Rightarrow \quad \sqrt{n^2+1} < n+1$

Thus, we have, $n < \sqrt{n^2+1} < n+1$

$\therefore \quad \left[\sqrt{n^2+1}\right]=n$

Now, the given equation reduces to $\left[\sqrt{n^2+\lambda}\right]=n$

$\Rightarrow \quad n<\sqrt{n^2+\lambda}<(n+1) \ \Rightarrow \ n^2<n^2+\lambda<(n+1)^2$

$\Rightarrow \quad 0 < \lambda < 2n + 1.$

Hence, λ can have $2n$ different values.

**3.** Find the general solution of the equation $\sin^4 2x + \cos^4 2x = \sin 2x \cos 2x$. ***(Ritu, West Bengal)***

**Ans.** We have, $(\sin^2 2x + \cos^2 2x)^2 - 2\sin^2 2x \cos^2 2x = \sin 2x \cos 2x$

$\Rightarrow \quad 2\sin^2 2x\cos^2 2x + \sin 2x\cos 2x - 1 = 0$

Put $\sin 2x \cos 2x = t$.

Therefore, we have $2t^2 + t - 1 = 0$

$\Rightarrow \quad (2t - 1)(t + 1) = 0 \Rightarrow t = \frac{1}{2}, -1$

**Case 1 :** When $t = -1$, we have $\sin 2x \cos 2x = -1$

$\Rightarrow \quad \sin 4x = -2$, which is impossible.

Therefore $t = -1$ is to be rejected.

**Case 2 :** When $t = 1/2$, we have $\sin 4x = 1$

$\Rightarrow \quad 4x = 2n\pi + (\pi/2)$

$$\Rightarrow \quad x=\frac{n\pi}{2}+\frac{\pi}{8}=(4n+1)\frac{\pi}{8}$$

## Recipe for Success

"Climbing to the top demands strength, whether it is to the top of Mount Everest or to the top of your career."

**–APJ Abdul Kalam**

# beat the TIME TRAP

**Duration : 30 minutes**

## SECTION-I

### Single Option Correct Type

**1.** If a set $A$ has 5 elements, then the number of ways of selecting two subsets $P$ and $Q$ from $A$ such that $P$ and $Q$ are mutually disjoint, is

(a) 64 (b) 128 (c) 243 (d) 729

**2.** Let $\frac{\sin(\theta-\alpha)}{\sin(\theta-\beta)}=\frac{a}{b}$ and $\frac{\cos(\theta-\alpha)}{\cos(\theta-\beta)}=\frac{c}{d}$.

Then, the value of $\cos(\alpha - \beta)$ equals

(a) $\frac{ac-bd}{ad+bc}$ (b) $\frac{ac+bd}{ad+bc}$

(c) $\frac{ac+bd}{ab+cd}$ (d) $\frac{ac-bd}{ab+cd}$

**3.** Let $f: N \to R$ be such that $f(1) = 1$ and $f(1) + 2f(2) + 3f(3) + .... + nf(n) = n(n + 1) f(n)$, for all $n \in N$, $n \geq 2$, where $N$ is the set of natural numbers and $R$ is the set of real numbers. Then the value of $f(500)$ is

(a) 1000 (b) 500 (c) 1/500 (d) 1/1000

**4.** Two matrices, $A$ and $B$ are both $5 \times 5$ square matrices such that $A = kB$, where $k$ is a non-zero constant. If $|B| = z$, then what will be the value of $|4(A + B)|$?

(a) $5z$ (b) $(20k + 20)z$

(c) $3125z$ (d) $(4k + 4)^5 z$

**5.** Let $X$ be the set $\left\{1, \pi, \left\{42, \sqrt{2}\right\}, \{1, 3\}\right\}$. Which of the following statement(s) is/are true?

$P : \pi \in X$ $Q : \{1, 3\} \subseteq X$ $R : \{1, \pi\} \subseteq X$

(a) $P$ only (b) $Q$ only

(c) $R$ only (d) $P$ and $R$ only

**6.** The value of $\cot\left[\sum_{n=1}^{19}\cot^{-1}\left(1+\sum_{p=1}^{n}2p\right)\right]$ is

(a) $\frac{22}{23}$ (b) $\frac{23}{22}$

(c) $\frac{21}{19}$ (d) $\frac{19}{21}$

**7.** Let $[x]$ denote the greatest integer less than or equal to $x$. Then the value of $\alpha$ for which the function

$$f(x)=\begin{cases}\frac{\sin[-x^2]}{[-x^2]}, & x \neq 0\\ \alpha, & x=0\end{cases}$$ is continuous at $x = 0$ is

(a) $\alpha = 0$ (b) $\alpha = \sin(-1)$

(c) $\alpha = \sin(1)$ (d) $\alpha = 1$

**8.** Let the $n^{\text{th}}$ term of a sequence be

$$t_n=\frac{1}{2}\{(1+\sqrt{3})^n+(1-\sqrt{3})^n\}, \text{ for } n = 3, 4, 5, .....$$

Then, for $m = 100$

(a) $\frac{1}{4}t_m$ is the arithmetic mean of $t_{m-1}$ and $t_{m-2}$

(b) $\frac{1}{4}t_{m-1}$ is the arithmetic mean of $t_m$ and $t_{m-2}$

(c) $\frac{1}{4}t_m$ is the geometric mean of $t_{m-1}$ and $t_{m-2}$

(d) $\frac{1}{4}t_{m-1}$ is the geometric mean of $t_m$ and $t_{m-2}$

**9.** If $\alpha$ is a real number, then the value of

$$\int\frac{dx}{\cos x\sqrt{\sin(2x+\alpha)+\sin\alpha}}$$ is

(a) $\sqrt{(\tan x+\tan\alpha)}+C$; where $C$ is an arbitrary constant

(b) $\sqrt{2(\tan x+\tan\alpha)}+C$; where $C$ is an arbitrary constant

(c) $\sqrt{\dfrac{2(\tan x+\tan\alpha)}{\sin\alpha}}+C$; where $C$ is an arbitrary constant

(d) $\sqrt{\dfrac{2(\tan x+\tan\alpha)}{\cos\alpha}}+C$; where $C$ is an arbitrary constant

**10.** If the straight lines $2x + 3y - 1 = 0$, $x + 2y - 1 = 0$ and $ax + by - 1 = 0$ form a triangle with origin as orthocentre, then $(a, b) =$

(a) (6, 4) (b) (–3, 3)
(c) (–8, 8) (d) (0, 7)

## SECTION-II

### Numerical Answer Type

**11.** Let $\alpha_1, \alpha_2, ...., \alpha_7$ be the roots of the equation $x^7 + 3x^5 - 13x^3 - 15x = 0$ and $|\alpha_1| \ge |\alpha_2| \ge .... \ge |\alpha_7|$. Then $\alpha_1\alpha_2 - \alpha_3\alpha_4 + \alpha_5\alpha_6$ is equal to _______.

**12.** If $\sum_{k=1}^{n} f(a+k)=16(2^n-1)$, where the function '$f$' satisfies the relation $f(x + y) = f(x) f(y)$ for all natural numbers $x, y$ and further $f(1) = 2$, then value of $a$ is _______.

**13.** Let $p(x)$ be a real polynomial of least degree which has a local maximum at $x = 1$ and a local minimum at $x = 3$. If $p(1) = 6$ and $p(3) = 2$, then $p'(0)$ is _______.

**14.** The area (in sq. units) of the region bounded by the curves $y = |x - 1|$ and $y = 3 - |x|$ is _______.

**15.** A pair of dice is rolled together till a sum of either 5 or 7 is obtained. Then the probability that 5 comes before 7 is _______.

## SOLUTIONS

**1. (c) :** $A$ has 5 elements and $P \cap Q = \phi$

$\therefore\ {}^5C_0\,[{}^5C_0 + {}^5C_1 + {}^5C_2 + {}^5C_3 + {}^5C_4 + {}^5C_5] + {}^5C_1\,[{}^4C_4 + {}^4C_3 + {}^4C_2 + {}^4C_1 + {}^4C_0] + {}^5C_2\,[{}^3C_3 + {}^3C_2 + {}^3C_1 + {}^3C_0] + {}^5C_3\,[{}^2C_2 + {}^2C_1 + {}^2C_0] + {}^5C_4\,[{}^1C_1 + {}^1C_0] + {}^5C_5$

$= 32 + 80 + 80 + 40 + 10 + 1 = 243$

**2. (b):** Given, $\dfrac{\sin(\theta-\alpha)}{\sin(\theta-\beta)}=\dfrac{a}{b}$

Applying componendo & dividendo, we get

$$\frac{\sin(\theta-\alpha)+\sin(\theta-\beta)}{\sin(\theta-\alpha)-\sin(\theta-\beta)}=\frac{a+b}{a-b}$$

$$\Rightarrow \frac{2\sin\left(\frac{2\theta-\alpha-\beta}{2}\right)\cos\left(\frac{\beta-\alpha}{2}\right)}{2\sin\left(\frac{\beta-\alpha}{2}\right)\cos\left(\frac{2\theta-\alpha-\beta}{2}\right)}=\frac{a+b}{a-b} \quad ...(i)$$

Also, $\dfrac{\cos(\theta-\alpha)}{\cos(\theta-\beta)}=\dfrac{c}{d}$

Applying componendo & dividendo, we get

$$\frac{\cos(\theta-\alpha)+\cos(\theta-\beta)}{\cos(\theta-\alpha)-\cos(\theta-\beta)}=\frac{c+d}{c-d}$$

$$\Rightarrow \frac{2\cos\left(\frac{2\theta-\alpha-\beta}{2}\right)\cos\left(\frac{\beta-\alpha}{2}\right)}{-2\sin\left(\frac{2\theta-\alpha-\beta}{2}\right)\sin\left(\frac{\beta-\alpha}{2}\right)}=\frac{c+d}{c-d} \quad ...(ii)$$

On multiplying (i) & (ii), we get

$$\frac{\cos^2\left(\frac{\beta-\alpha}{2}\right)}{\sin^2\left(\frac{\beta-\alpha}{2}\right)}=\frac{-ac-ad-bc-bd}{ac-ad-bc+bd}$$

Again, applying componendo & dividendo

$$\frac{\cos^2\left(\frac{\beta-\alpha}{2}\right)+\sin^2\left(\frac{\beta-\alpha}{2}\right)}{\cos^2\left(\frac{\beta-\alpha}{2}\right)-\sin^2\left(\frac{\beta-\alpha}{2}\right)}=\frac{-2ad-2bc}{-2ac-2bd}$$

$$\Rightarrow \frac{1}{\cos(\beta-\alpha)}=\frac{ad+bc}{ac+bd}$$

$$\Rightarrow \frac{1}{\cos(\alpha-\beta)}=\frac{ad+bc}{ac+bd} \Rightarrow \cos(\alpha-\beta)=\frac{ac+bd}{ad+bc}$$

**3. (d):** We have, $f(1) + 2f(2) + 3f(3) + ... + nf(n)$
$= n(n + 1) f(n)$ for $n \ge 2$ ...(i)

Putting $n = n + 1$, we get

$f(1) + 2f(2) + 3f(3) + ... + nf(n) + (n + 1) f(n + 1)$
$= (n + 1)(n + 2) f(n + 1)$ ...(ii)

(ii) – (i), we get

$(n + 1) f(n + 1) = (n + 1)(n + 2) f(n + 1) - n(n + 1) f(n)$

$\Rightarrow f(n + 1) = (n + 2) f(n + 1) - n f(n)$

$\Rightarrow (n + 1) f(n + 1) = n f(n)$

$\Rightarrow 2f(2) = 3f(3) = 4 f(4) ..... = nf(n)$

From (i),

$$f(1)+\underbrace{nf(n)+nf(n)+nf(n)+....+nf(n)}_{(n-1)\text{ times}}=n(n+1)f(n)$$

$\Rightarrow 1 + (n - 1)\, n f(n) = n(n + 1) f(n) \Rightarrow 2n f(n) = 1$

$\Rightarrow f(n)=\dfrac{1}{2n} \quad \therefore f(500)=\dfrac{1}{1000}$

**4. (d):** We are given that,

$A = kB$ ...(i) and $|B| = z$ ...(ii)

Now, $|4(A + B)| = |4(kB + B)|$ (Using (i))

$= |4(k + 1)B| = (4k + 4)^5|B| = (4k + 4)^5 z$ (Using (ii))

**5. (d):** We have, $1 \in X$, $\pi \in X$, $\{42, \sqrt{2}\} \in X$ and $\{1, 3\} \in X$

So, only $P$ and $R$ are true.

**6. (c):** Given, $\cot\left[\sum_{n=1}^{19} \cot^{-1}\left(1 + \sum_{p=1}^{n} 2p\right)\right]$

$$= \cot\left[\sum_{n=1}^{19} \cot^{-1}\left(1 + 2 \times \frac{n(n+1)}{2}\right)\right]$$

$$= \cot\left[\sum_{n=1}^{19} \cot^{-1}(1 + n(n+1))\right]$$

$$= \cot\left[\sum_{n=1}^{19} \tan^{-1}\left(\frac{n+1-n}{1+n(n+1)}\right)\right]$$

$$= \cot\left[\sum_{n=1}^{19} (\tan^{-1}(n+1) - \tan^{-1} n)\right]$$

$$= \cot[\tan^{-1}2 - \tan^{-1}1 + \tan^{-1}3 - \tan^{-1}2 + \ldots + \tan^{-1}20 - \tan^{-1}19]$$

$$= \cot[\tan^{-1}20 - \tan^{-1}1] = \cot\left[\tan^{-1}\left(\frac{20-1}{1+20}\right)\right]$$

$$= \cot\left(\tan^{-1}\frac{19}{21}\right) = \cot\left(\cot^{-1}\frac{21}{19}\right) = \frac{21}{19}$$

**7. (c):** $\lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{\sin[-x^2]}{[-x^2]} = \lim_{x \to 0} \frac{\sin(-1)}{(-1)} = \sin 1$

and $f(0) = \alpha$

$\therefore$ $f(x)$ will be continuous at $x = 0$ if $\alpha = \sin 1$.

**8. (a):** $t_{m-1} = t_{99} = \frac{1}{2}\left\{(1+\sqrt{3})^{99} + (1-\sqrt{3})^{99}\right\}$

$$t_{m-2} = t_{98} = \frac{1}{2}\left\{(1+\sqrt{3})^{98} + (1-\sqrt{3})^{98}\right\}$$

Now,

$$\frac{t_{m-1} + t_{m-2}}{2} = \frac{1}{2} \times \frac{1}{2}\left[(1+\sqrt{3})^{99} + (1-\sqrt{3})^{99} + (1+\sqrt{3})^{98} + (1-\sqrt{3})^{98}\right]$$

$$= \frac{1}{4}\left\{(1+\sqrt{3})^{98}(2+\sqrt{3}) + (1-\sqrt{3})^{98}(2-\sqrt{3})\right\}$$

$$= \frac{1}{4}\left\{\frac{(1+\sqrt{3})^{98}}{2-\sqrt{3}} + \frac{(1-\sqrt{3})^{98}}{2+\sqrt{3}}\right\}$$

$$= \frac{1}{4}\left\{\frac{(1+\sqrt{3})^{100}}{(2-\sqrt{3})(1+\sqrt{3})^2} + \frac{(1-\sqrt{3})^{100}}{(2+\sqrt{3})(1-\sqrt{3})^2}\right\}$$

$$= \frac{1}{4}\left\{\frac{(1+\sqrt{3})^{100}}{2} + \frac{(1-\sqrt{3})^{100}}{2}\right\}$$

$$= \frac{1}{4} \times \frac{1}{2}\left\{(1+\sqrt{3})^{100} + (1-\sqrt{3})^{100}\right\} = \frac{1}{4}t_{100} = \frac{1}{4}t_m$$

Hence, $\frac{1}{4}t_m$ is the arithmetic mean of $t_{m-1}$ and $t_{m-2}$.

**9. (d):** Let $I = \int \frac{dx}{\cos x\sqrt{\sin(2x+\alpha) + \sin\alpha}}$

$$= \int \frac{\sec x\, dx}{\sqrt{\sin(2x+\alpha) + \sin\alpha}}$$

$$= \int \frac{\sec x\, dx}{\sqrt{2\sin(x+\alpha)\cos x}} = \frac{1}{\sqrt{2}}\int \frac{\left(\frac{\sec x}{\cos x}\right)dx}{\frac{\sqrt{\sin(x+\alpha)\cos x}}{\cos x}}$$

$$= \frac{1}{\sqrt{2}}\int \frac{\sec^2 x\, dx}{\sqrt{\frac{\sin x\cos\alpha + \cos x\sin\alpha}{\cos x}}}$$

## SAMURAI SUDOKU

### ANSWER - AUGUST 2024

| | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 5 | 3 | 1 | 7 | 9 | 4 | 6 | 8 | | | | 9 | 8 | 1 | 2 | 3 | 6 | 7 | 4 | 5 |
| 8 | 9 | 7 | 6 | 5 | 4 | 1 | 2 | 3 | | | | 4 | 2 | 3 | 7 | 8 | 5 | 6 | 1 | 9 |
| 6 | 1 | 4 | 2 | 3 | 8 | 5 | 7 | 9 | | | | 5 | 6 | 7 | 4 | 1 | 9 | 8 | 2 | 3 |
| 4 | 2 | 5 | 8 | 1 | 3 | 7 | 9 | 6 | | | | 6 | 9 | 4 | 1 | 7 | 2 | 3 | 5 | 8 |
| 3 | 6 | 1 | 9 | 2 | 7 | 8 | 4 | 5 | | | | 1 | 5 | 2 | 3 | 6 | 8 | 4 | 9 | 7 |
| 7 | 8 | 9 | 4 | 6 | 5 | 3 | 1 | 2 | | | | 3 | 7 | 8 | 5 | 9 | 4 | 1 | 6 | 2 |
| 5 | 3 | 2 | 7 | 9 | 1 | 6 | 8 | 4 | 3 | 9 | 2 | 7 | 1 | 5 | 9 | 4 | 3 | 2 | 8 | 6 |
| 9 | 7 | 8 | 5 | 4 | 6 | 2 | 3 | 1 | 5 | 6 | 7 | 8 | 4 | 9 | 6 | 2 | 7 | 5 | 3 | 1 |
| 1 | 4 | 6 | 3 | 8 | 2 | 9 | 5 | 7 | 8 | 1 | 4 | 2 | 3 | 6 | 8 | 5 | 1 | 9 | 7 | 4 |
| | | | | | | 8 | 9 | 3 | 4 | 5 | 1 | 6 | 2 | 7 | | | | | | |
| | | | | | | 4 | 6 | 2 | 7 | 3 | 8 | 5 | 9 | 1 | | | | | | |
| | | | | | | 1 | 7 | 5 | 6 | 2 | 9 | 3 | 8 | 4 | | | | | | |
| 4 | 3 | 6 | 8 | 1 | 5 | 7 | 2 | 9 | 1 | 8 | 5 | 4 | 6 | 3 | 5 | 9 | 2 | 8 | 1 | 7 |
| 2 | 1 | 5 | 7 | 6 | 9 | 3 | 4 | 8 | 9 | 7 | 6 | 1 | 5 | 2 | 3 | 8 | 7 | 6 | 4 | 9 |
| 7 | 8 | 9 | 4 | 2 | 3 | 5 | 1 | 6 | 2 | 4 | 3 | 9 | 7 | 8 | 1 | 4 | 6 | 3 | 2 | 5 |
| 5 | 7 | 8 | 9 | 3 | 2 | 1 | 6 | 4 | | | | 6 | 8 | 4 | 2 | 1 | 5 | 7 | 9 | 3 |
| 6 | 9 | 3 | 1 | 4 | 7 | 2 | 8 | 5 | | | | 2 | 9 | 7 | 4 | 3 | 8 | 1 | 5 | 6 |
| 1 | 2 | 4 | 5 | 8 | 6 | 9 | 3 | 7 | | | | 3 | 1 | 5 | 6 | 7 | 9 | 2 | 8 | 4 |
| 8 | 5 | 1 | 2 | 7 | 4 | 6 | 9 | 3 | | | | 5 | 3 | 6 | 8 | 2 | 4 | 9 | 7 | 1 |
| 3 | 4 | 7 | 6 | 9 | 1 | 8 | 5 | 2 | | | | 7 | 2 | 1 | 9 | 5 | 3 | 4 | 6 | 8 |
| 9 | 6 | 2 | 3 | 5 | 8 | 4 | 7 | 1 | | | | 8 | 4 | 9 | 7 | 6 | 1 | 5 | 3 | 2 |

***Winners:*** *Bidisha Halder, Hema Chandran*

$$= \frac{1}{\sqrt{2}} \int \frac{\sec^2 x \, dx}{\sqrt{\tan x \cos\alpha + \sin\alpha}}$$

Put $\tan x \cos\alpha + \sin\alpha = t \Rightarrow (\sec^2 x \cos\alpha)\, dx = dt$

$$\therefore \; I = \frac{1}{\sqrt{2}\cos\alpha} \int \frac{dt}{\sqrt{t}} = \frac{1}{\sqrt{2}\cos\alpha} 2\sqrt{t} + C$$

$$= \frac{\sqrt{2}}{\cos\alpha} \sqrt{\tan x \cos\alpha + \sin\alpha} + C$$

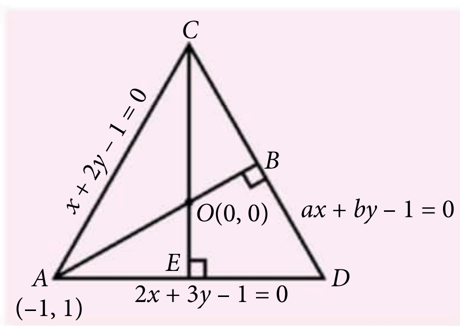
$$= \sqrt{\frac{2(\tan x \cos\alpha + \sin\alpha)}{\cos^2\alpha}} + C = \sqrt{\frac{2(\tan x + \tan\alpha)}{\cos\alpha}} + C$$

**10. (c) :** We have the lines

$2x + 3y - 1 = 0$ ...(i)

$x + 2y - 1 = 0$ ...(ii)

and $ax + by - 1 = 0$ ...(iii)




From (i) and (ii), the point of intersection is $A(-1, 1)$.

Since, line $AB$ is perpendicular to line (iii).

So, we get $a = -b$

Also, the point of intersection of line (ii) and (iii) is

$$C\left(\frac{b-2}{b-2a}, \frac{1-a}{b-2a}\right)$$

Equation of line passing through $O(0, 0)$ and $C$ is,

$(a - 1)x + (b - 2)y = 0$ ...(iv)

Since, line (iv) is perpendicular to line (i)

$\Rightarrow \; a = -8$ and $b = 8$

$\therefore \; (a, b) \equiv (-8, 8)$

**11. (9):** Given equation is, $x^7 + 3x^5 - 13x^3 - 15x = 0$

$\Rightarrow \; x(x^6 + 3x^4 - 13x^2 - 15) = 0$

Here, $x = 0$ is one of the root. Replacing $x^2 = t$,

$\because \; t^3 + 3t^2 - 13t - 15 = 0 \Rightarrow (t - 3)(t^2 + 6t + 5) = 0$

$\Rightarrow \; t = 3, t = -1$ and $t = -5$

Now, $x^2 = 3, x^2 = -1$ and $x^2 = -5$

$\Rightarrow \; x = \pm\sqrt{3}, x = \pm i, x = \pm\sqrt{5}i$

From the given condition $|\alpha_1| \geq |\alpha_2| \geq .... \geq |\alpha_7|$

we can say that $|\alpha_7| = 0$ and $|\alpha_1| = \sqrt{5} = |\alpha_2|$

and $|\alpha_4| = \sqrt{3} = |\alpha_3|$ and $|\alpha_5| = 1 = |\alpha_6|$

So, we have,

$\alpha_5 = i, \alpha_6 = -i, \alpha_3 = \sqrt{3}, \alpha_4 = -\sqrt{3}, \alpha_2 = \sqrt{5}i$

and $\alpha_1 = -\sqrt{5}i$

So, $\alpha_1\alpha_2 - \alpha_3\alpha_4 + \alpha_5\alpha_6 = 5 - (-3) + 1 = 9$

**12. (3):** Given, $f(x + y) = f(x) f(y)$ for all natural numbers and $f(1) = 2$

$\Rightarrow \; f(1 + 1) = f(1) \cdot f(1) = 2^2$

We can prove by induction that $f(n) = 2^n$ for all $n$

Now, by hypothesis

$$\sum_{k=1}^{n} f(a+k) = 16(2^n - 1) \Rightarrow \; f(a)\sum_{k=1}^{n} f(k) = 16(2^n - 1)$$

$$\Rightarrow \; f(a)[2 + 2^2 + ........ + 2^n] = 16(2^n - 1)$$

$$\Rightarrow \; f(a).\frac{2(2^n - 1)}{2 - 1} = 16(2^n - 1)$$

$\Rightarrow \; f(a) = 8 = 2^3 \;\Rightarrow\;$ Conclusion is $a = 3$.

**13. (9):** Let $p'(x) = \lambda(x - 1)(x - 3)$

$$\Rightarrow p(x) = \lambda\left(\frac{x^3}{3} - 2x^2 + 3x\right) + k$$

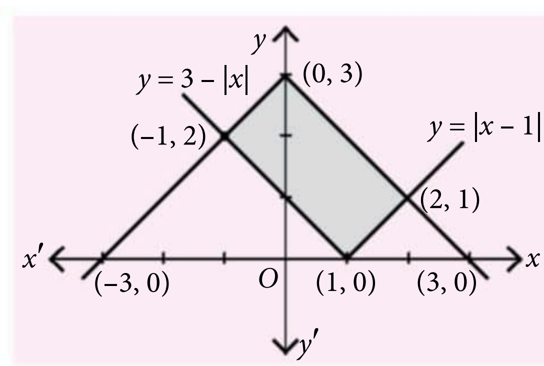
$p(1) = 6 \;\Rightarrow\; \frac{4}{3}\lambda + k = 6$

$p(3) = 2 \;\Rightarrow\; k = 2$. Thus, $\lambda = 3$

$\therefore \; p'(0) = 3\lambda = 9$

**14. (4):** Required area

$$= \int_{-1}^{0} [(3 + x) - (-x + 1)]\, dx + \int_{0}^{1} [(3 - x) - (-x + 1)]\, dx + \int_{1}^{2} [(3 - x) - (x - 1)]\, dx$$




$$\Rightarrow \; \text{Area} = \int_{-1}^{0} 2(1 + x)\, dx + \int_{0}^{1} 2\, dx + \int_{1}^{2} (4 - 2x)\, dx$$

= 4 sq. units

**15. (0.4) :** When a pair of dice is thrown, the chances of getting sum 5 is (1, 4), (2, 3), (3, 2), (4, 1) and sum 7 is (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1).

Probability of getting sum 5 when a pair of dice is rolled

$$= \frac{4}{36} = \frac{1}{9}$$

Probability of not getting sum 5 or 7 $= 1 - \frac{10}{36} = \frac{26}{36} = \frac{13}{18}$

Required probability $= \frac{1}{9} + \frac{13}{18} \cdot \frac{1}{9} + \left(\frac{13}{18}\right)^2 \cdot \frac{1}{9} + .......$

$$= \frac{1/9}{1 - \frac{13}{18}} = \frac{2}{5}$$

# Are you ready for Olympiads?

**LEVEL 1 Exam on**

**22nd Oct., 19th Nov. & 12th Dec., 2024**

SOF SCIENCE OLYMPIAD FOUNDATION | IMO SOF INTERNATIONAL MATHEMATICS OLYMPIAD

**CLASS XI**

## SYLLABUS*

Following the protocol of NEP (2020), NCF (2023), NCERT and CBSE guidelines, National and various State Boards for the convenience of schools and students, any change/reduction in the syllabi will be reflected in actual question papers.

**Section – 1 :** Verbal and Non-Verbal Reasoning.

**Section – 2 :** Sets, Relations and Functions, Logarithms, Complex Numbers & Quadratic Equations, Linear Inequalities, Sequences and Series, Trigonometry, Straight Lines, Conic Sections, Permutations and Combinations, Binomial Theorem, Statistics, Limits and Derivatives, Probability, Introduction to 3-D Geometry.

**OR**

**Section – 2 :** Numbers, Quantification, Numerical Applications, Sets, Relations and Functions, Sequences and Series, Permutations and Combinations, Mathematical Reasoning, Limits, Continuity and Differentiability, Probability, Descriptive Statistics, Basics of Financial Mathematics, Straight Lines, Circles.

**Section – 3 :** The syllabus of this section will be based on the syllabus of Quantitative Aptitude.

**Section – 4 :** Sets, Relations and Functions, Sequences and Series, Permutations and Combinations, Limits and Derivatives, Straight Lines, Circles, Parabola, Probability.

Total Questions : 50 | Time : 1 hr.

**PATTERN & MARKING SCHEME**

| Section | (1) Logical Reasoning | (2) Mathematical Reasoning or Applied Mathematics | (3) Everyday Mathematics | (4) Achievers Section |
|---|---|---|---|---|
| No. of Questions | 15 | 20 | 10 | 5 |
| Marks per Ques. | 1 | 1 | 1 | 3 |

## Practice Questions

**1.** If $X = \{-2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8\}$ and $A = \{x : |x - 2| \leq 3, x \text{ is an integer}\}$, then $X \cap A =$

A. $\{-2, 6, 7, 8\}$ B. $\{-2, -1, 1, 2, 3, 4, 5, 6\}$
C. $\{-1, 0, 1, 2, 3, 4, 5\}$ D. $\{-2, -1, 2, 3, 6, 7, 8\}$

**2.** If $f(x) = 2x^2 + bx + c$ and $f(0) = 3$ and $f(2) = 1$, then $f(1)$ is equal to

A. 1 B. 2 C. 0 D. 1/2

**3.** If $f(x) = [x]^2 - 5[x] + 6 = 0$, where $[\cdot]$ denote the greatest integer function, then

A. $x \in [3, 4]$ B. $x \in (2, 3]$
C. $x \in [2, 3]$ D. $x \in [2, 4)$

**4.** The value of

$\sin^2\frac{\pi}{10} + \sin^2\frac{2\pi}{5} + \sin^2\frac{3\pi}{5} + \sin^2\frac{9\pi}{10}$ is equal to

A. 1 B. –4 C. –2 D. 2

**5.** The value of $\sin 50° - \sin 70° + \sin 10°$ is equal to

A. 1 B. 0 C. 1/2 D. 2

**6.** A real value of $x$ satisfies the equation

$\left(\frac{3 - 4ix}{3 + 4ix}\right) = \alpha - i\beta \, (\alpha, \beta \in R)$, if $\alpha^2 + \beta^2 =$

A. 1 B. –1 C. 2 D. –2

**7.** If $-8 \leq 5x - 3 < 7$, then $x \in$

A. $(-1, 2)$ B. $[-1, 2)$ C. $[-2, \infty)$ D. $[-2, 0)$

**8.** Number of ways in which the letters of word MEDICAL can be arranged if A and E are put together but all the vowels never come together, is

A. 240 B. 720 C. 960 D. 1240

**9.** If $(1 + x - 2x^2)^6 = 1 + C_1x + C_2x^2 + C_3x^3 + ... + C_{12}x^{12}$, then the value of $C_2 + C_4 + C_6 + ... + C_{12}$, is

A. 30 B. 32
C. 31 D. None of these

**10.** If $6^{th}$ term of a G.P. is 2, then the product of first 11 terms of the G.P. is equal to

A. 512 B. 1024 C. 2048 D. 256

**11.** Find the distance of the point (3, –5) from the line $3x - 4y - 26 = 0$.

A. $\frac{2}{5}$ unit B. 4 units C. $\frac{3}{5}$ unit D. 6 units

**12.** What is the eccentricity of hyperbola whose vertices and foci are $(\pm 2, 0)$ and $(\pm 3, 0)$ respectively?

A. $\frac{4}{5}$ B. $\frac{5}{4}$ C. $\frac{3}{2}$ D. $\frac{2}{3}$

**13.** The image of the point (7, 2, –1) in the $ZX$-plane is

A. (7, –2, 1) B. (–7, –2, –1)
C. (7, –2, –1) D. (7, 2, –1)

*For latest updates visit www.sofworld.org

**14.** In an experiment with 15 observations on $x$, the following results were available

$\sum x^2 = 2935, \sum x = 185$

One observation that was 20, we found to be wrong and was replaced by the correct value 30. Then, the corrected variance is

A. 60.00 B. 188.66 C. 177.33 D. 8.33

**15.** If the mean of the numbers $a$, $b$, 8, 5, 10 is 6 and their variance is 6.8, then $ab$ is equal to

A. 6 B. 7 C. 12 D. 14

## ACHIEVERS SECTION

**16.** Match the following :

| | Column I | | Column II |
|---|---|---|---|
| (P) | A side of an equilateral triangle is 20 cm long. A second equilateral triangle is inscribed in it by joining the mid-points of the sides of the first triangle. The process is continued, the perimeter (in cm) of the sixth inscribed equilateral triangle is | (1) | 2480 |
| (Q) | In a potato race 20 potatoes are placed in a line at intervals of 4 metres with the first potato 24 metres from the starting point. A contestant is required to bring the potatoes back to the starting place one at a time, then the distance (in m) he would run in bringing back all the potatoes is | (2) | 725 |
| (R) | In a cricket tournament 16 school teams participated. A sum of ₹ 8000 is to be awarded among themselves as prize money. If the last placed team is awarded ₹ 275 in prize money and the award increases by the same amount for successive finishing places, then the amount received by first place team is | (3) | $\frac{15}{8}$ |

A. (P) → (1), (Q) → (2), (R) → (3)
B. (P) → (2), (Q) → (1), (R) → (3)
C. (P) → (3), (Q) → (1), (R) → (2)
D. (P) → (1), (Q) → (3), (R) → (2)

**17.** Which is the following is correct?

A. There are 12 points in a plane of which 5 points are collinear, then the number of lines obtained by joining these points in pairs is ${}^{12}C_2 - {}^{5}C_2$.

B. In the permutations of $n$ things, taken $r$ at a time, the number of permutations in which $m$ particular things occur together is ${}^{n-m}P_{r-m} \times {}^{r}P_{m}$.

C. In a steamer there are stalls for 12 animals, and there are horses, cows and calves (not less than 12 each) ready to be shipped. They can be loaded in $3^{12}$ ways.

D. None of these

**18.** **Statement-I :** Domain of $f(x) = \log_a x$ $(x, a > 0)$ and $a \neq 1$ is $(0, \infty)$ and range of $f(x) = R$.

**Statement-II :** Range of $f(x) = \sqrt{x}$ $\forall\, x \geq 0$ is $[0, \infty)$.

A. Both Statement-I and Statement-II are true.
B. Statement-I is true but Statement-II is false.
C. Statement-I is false but Statement-II is true.
D. Both Statement-I and Statement-II are false.

**19.** The letters of the word ASSASSINATION are arranged at random.

(i) The probability that four S's come consecutively in the word is $\frac{2}{143}$.

(ii) The probability that two I's and two N's come together is $\frac{1}{143}$.

(iii) The probability that all A's are not coming together is $\frac{1}{26}$.

(iv) The probability that no two A's are coming together is $\frac{15}{26}$.

Then, which of the following is true?

A. (i), (ii) B. (i), (iii), (iv)
C. (i), (iv) D. (i), (ii) and (iii)

**20.** The value of $\lim_{x \to 3} \frac{x^5 - 3^5}{x^8 - 3^8}$ is equal to

A. $\frac{5}{8}$ B. $\frac{5}{64}$
C. $\frac{5}{216}$ D. $\frac{1}{27}$

Darken your choice with HB Pencil

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1. | Ⓐ Ⓑ Ⓒ Ⓓ | 5. | Ⓐ Ⓑ Ⓒ Ⓓ | 9. | Ⓐ Ⓑ Ⓒ Ⓓ | 13. | Ⓐ Ⓑ Ⓒ Ⓓ | 17. | Ⓐ Ⓑ Ⓒ Ⓓ |
| 2. | Ⓐ Ⓑ Ⓒ Ⓓ | 6. | Ⓐ Ⓑ Ⓒ Ⓓ | 10. | Ⓐ Ⓑ Ⓒ Ⓓ | 14. | Ⓐ Ⓑ Ⓒ Ⓓ | 18. | Ⓐ Ⓑ Ⓒ Ⓓ |
| 3. | Ⓐ Ⓑ Ⓒ Ⓓ | 7. | Ⓐ Ⓑ Ⓒ Ⓓ | 11. | Ⓐ Ⓑ Ⓒ Ⓓ | 15. | Ⓐ Ⓑ Ⓒ Ⓓ | 19. | Ⓐ Ⓑ Ⓒ Ⓓ |
| 4. | Ⓐ Ⓑ Ⓒ Ⓓ | 8. | Ⓐ Ⓑ Ⓒ Ⓓ | 12. | Ⓐ Ⓑ Ⓒ Ⓓ | 16. | Ⓐ Ⓑ Ⓒ Ⓓ | 20. | Ⓐ Ⓑ Ⓒ Ⓓ |

## SOLUTIONS

**1. (C) :** We have, $X = \{-2, -1, 0, 1, ...., 8\}$ and
$A = \{x : |x - 2| \le 3, x \text{ is an integer}\}$
$\therefore \quad A = \{-1, 0, 1, 2, 3, 4, 5\}$
Now, $X \cap A = \{-1, 0, 1, 2, 3, 4, 5\}$

**2. (C) :** We have, $f(x) = 2x^2 + bx + c$,
$f(0) = 3 \Rightarrow 3 = c$ and $f(2) = 1$
$\Rightarrow 1 = 8 + 2b + c \Rightarrow 2b + 3 = -7 \Rightarrow b = -5$
$\therefore f(1) = 2 \times 1^2 + (-5) \times 1 + 3 = 2 - 5 + 3 = 0$

**3. (D) :** Given, $f(x) = [x]^2 - 5[x] + 6 = 0$
$\Rightarrow [x]^2 - 3[x] - 2[x] + 6 = 0 \Rightarrow ([x] - 3)([x] - 2) = 0$
$\Rightarrow [x] = 3 \text{ or } [x] = 2 \Rightarrow 3 \le x < 3 + 1 \text{ or } 2 \le x < 2 + 1$
$\Rightarrow x \in [3, 4) \text{ or } x \in [2, 3) \Rightarrow x \in [3, 4) \cup [2, 3)$
$\therefore \quad x \in [2, 4)$

**4. (D) :** We know, $\sin\frac{2\pi}{5} = \sin\left[\frac{\pi}{2} - \frac{\pi}{10}\right] = \cos\frac{\pi}{10}$

$$\sin\frac{3\pi}{5} = \sin\left[\frac{\pi}{2} + \frac{\pi}{10}\right] = \cos\frac{\pi}{10}$$

$$\sin\frac{9\pi}{10} = \sin\left[\pi - \frac{\pi}{10}\right] = \sin\frac{\pi}{10}$$

$$\therefore \quad \sin^2\frac{\pi}{10} + \sin^2\frac{2\pi}{5} + \sin^2\frac{3\pi}{5} + \sin^2\frac{9\pi}{10}$$

$$= \left[\sin^2\frac{\pi}{10} + \cos^2\frac{\pi}{10}\right] + \left[\cos^2\frac{\pi}{10} + \sin^2\frac{\pi}{10}\right] = 1 + 1 = 2$$

**5. (B) :** We have, $\sin 50° - \sin 70° + \sin 10°$

$$= 2\cos\left(\frac{50° + 70°}{2}\right) \cdot \sin\left(\frac{50° - 70°}{2}\right) + \sin 10°$$

$$\left[\because \sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right) \cdot \sin\left(\frac{A-B}{2}\right)\right]$$

$= -2\cos 60° \sin 10° + \sin 10°$

$$= -2 \cdot \frac{1}{2} \sin 10° + \sin 10° = 0$$

**6. (A) :** We have, $\alpha - i\beta = \dfrac{3 - 4ix}{3 + 4ix}$ ...(i)

Taking conjugate of (i) on both sides, we get

$$\alpha + i\beta = \overline{\left(\frac{3 - 4ix}{3 + 4ix}\right)} = \frac{\overline{(3 - 4ix)}}{\overline{(3 + 4ix)}} = \frac{3 + 4ix}{3 - 4ix} \quad \text{...(ii)}$$

Multiplying (i) and (ii), we get
$(\alpha - i\beta)(\alpha + i\beta) = 1 \Rightarrow \alpha^2 + \beta^2 = 1$

**7. (B) :** We have, $-8 \le 5x - 3 < 7$
or $-5 \le 5x < 10$ or $-1 \le x < 2 \therefore x \in [-1, 2)$

**8. (C) :** Letters M, D, C, L can be arranged in 4! ways. From 5 gaps created, 2 gaps for (AE) and I can be selected in ${}^5C_2$ ways.
In these two gaps, (AE) and I can be arranged in 2! ways.
Also, AE can be arranged in 2! ways.
$\therefore$ Total number of ways $= 4! \times {}^5C_2 \times 2! \times 2! = 960$

**9. (C) :** We have,
$(1 + x - 2x^2)^6 = 1 + C_1x + C_2x^2 + C_3x^3 + ... + C_{12}x^{12}$
Put $x = 1$, we get
$1 + C_1 + C_2 + C_3 + ... + C_{12} = (1 + 1 - 2)^6 = 0$ ...(i)
Put $x = -1$, we get
$1 - C_1 + C_2 - C_3 + ... + C_{12} = 2^6 = 64$ ...(ii)
Adding (i) and (ii), we get $2 + 2(C_2 + C_4 + ... + C_{12}) = 64$

$$\Rightarrow C_2 + C_4 + ...... + C_{12} = \frac{64 - 2}{2} = 31$$

**10. (C) :** Consider the 11 terms of the G.P. as,

$$\frac{a}{r^5}, \frac{a}{r^4}, \frac{a}{r^3}, \frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2, ar^3, ar^4, ar^5$$

Now, $T_6 = a = 2$

$\therefore$ Product of 11 terms $= a^{11} = 2^{11} = 2048$

**11. (C) :** Given line is $3x - 4y - 26 = 0$.

Here, $A = 3$, $B = -4$ and $C = -26$.

Given point is $(x_1, y_1) = (3, -5)$. The distance of the given point from given line is

$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}} = \frac{|3 \cdot 3 + (-4)(-5) - 26|}{\sqrt{3^2 + (-4)^2}} = \frac{3}{5} \text{ unit}$$

**12. (C) :** Vertices are $(\pm 2, 0)$, foci are $(\pm 3, 0)$

$\Rightarrow a = 2, c = 3 \quad \therefore \quad e = \frac{c}{a} = \frac{3}{2}\ (> 1)$

**13. (C) :** The image of the point $(7, 2, -1)$ in the $ZX$-plane is $(7, -2, -1)$.

**14. (A) :** Given : $N = 15, \Sigma x^2 = 2935, \Sigma x = 185$

One observation 20 was replaced by 30, then

$\Sigma x^2 = 2935 - 400 + 900 = 3435$,

$\Sigma x = 185 - 20 + 30 = 195$

$\therefore$ Correct variance,

$$\sigma^2 = \frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2 = \frac{3435}{15} - \left(\frac{195}{15}\right)^2 = 229 - 169 = 60$$

**15. (C) :** According to question,

Mean $= 6 \Rightarrow \frac{a + b + 8 + 5 + 10}{5} = 6$

$\Rightarrow a + b + 8 + 5 + 10 = 30$

$\Rightarrow a + b = 7$ ...(i)

Also, variance = 6.8

$\Rightarrow (a - 6)^2 + (b - 6)^2 + 2^2 + (-1)^2 + 4^2 = (6.8) \times 5$

$\Rightarrow a^2 - 12a + 36 + b^2 - 12b + 36 + 4 + 1 + 16 = 34$

$\Rightarrow a^2 + b^2 - 84 + 59 = 0$

$\Rightarrow a^2 + b^2 = 25$ ...(ii)

On solving (i) and (ii), we get $ab = 12$

**16. (C) :** (P) Side of equilateral $\Delta ABC$ = 20 cm. By joining the mid-points of this triangle, we get another equilateral triangle of side equal to half of the length of side of $\Delta ABC$.

Continuing in this way, we get a set of equilateral triangles with side equal to half of the side of the previous triangle.

$\therefore$ Perimeter of first triangle $= 20 \times 3 = 60$ cm

[$\because$ Perimeter = sum of all sides of $\Delta$]

Perimeter of second triangle $= 10 \times 3 = 30$ cm

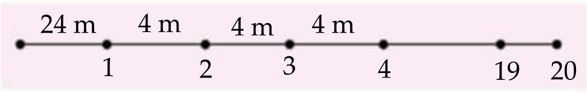
So, the series will be 60, 30, 15, .... which forms a G.P.

Here, first term $(a) = 60$ and common ratio $(r) = \frac{30}{60} = \frac{1}{2}$

Since perimeter of sixth inscribed triangle is the sixth term of the series.

$\therefore \quad a_6 = ar^{6-1} = 60 \times \left(\frac{1}{2}\right)^5 = \frac{60}{32} = \frac{15}{8}$ cm

(Q) According to the given statement, we have the following diagram.




Distance travelled to bring first potato

$= 24 + 24 = 2 \times 24 = 48$ m

Distance travelled to bring second potato

$= 2(24 + 4) = 2 \times 28 = 56$ m

Distance travelled to bring third potato

$= 2(24 + 4 + 4) = 2 \times 32 = 64$ m

Hence, the series of distances is 48, 56, 64, ..... , where $a = 48$, $d = 56 - 48 = 8$ and $n = 20$

$\therefore \quad S_{20} = \frac{20}{2}\{2 \times 48 + 19 \times 8\} = 10 \times 248 = 2480$

$\therefore$ Total distance covered in bringing back all the potatoes = 2480 m.

(R) Let the first place team got ₹ $a$.

Since, award money increases by the same amount for successive finishing places.

$\therefore$ Series formed is in A.P.

Let the constant amount be ₹ $d$.

Now, $l = 275$, $n = 16$ and $S_{16} = 8000$

$\therefore \quad l = a + (n - 1)d \Rightarrow 275 = a + 15d$ ...(i)

and $S_{16} = \frac{16}{2}[2a + (16 - 1)(d)]$

$\Rightarrow 8000 = 8[2a + 15d] \Rightarrow 1000 = 2a + 15d$ ...(ii)

Solving (i) and (ii), we get $a = 725$

Hence, ₹ 725 is received by first place team.

**17. (C) :** A. Since 5 points are collinear

$\therefore$ They will make a line.

$\therefore$ Required number of lines $= {}^{12}C_2 - {}^{5}C_2 + 1$

B. Here, we have to arrange $n$ things, taken $r$ at a time in which $m$ particular things occur together.

Consider the $m$ things as 1 group. Then,

Number of things excluding $m$ particular things $= (n - m)$

Number of ways of selecting $(r - m)$ things from $(n - m)$ things $= {}^{n-m}C_{r-m}$

Now, consider $m$ particular things temporarily as a single thing and mix it with $(r - m)$ selected things. Now, these $(r - m + 1)$ things can be arranged in $(r - m + 1)!$ and $m$ particular things can be put together in $m!$ ways.

$\therefore$ Required number of arrangements

$= {}^{n-m}C_{r-m}(r-m+1)! \times m!$

C. Since, there are three types of animals and each stall is available for 12 animals.

$\therefore$ Number of ways of loading = $3^{12}$

**18. (A) : Statement-I :** $f(x) = \log_a x$; $x, a > 0$; $a \neq 1$

or $f(x) = \dfrac{\log x}{\log a}$

Domain of $f(x)$ is $(0, \infty)$.

Range of $f(x)$ is $(-\infty, \infty)$ or $R$.

**Statement-II :** $f(x) = \sqrt{x}$, $x \geq 0$

Range of $f(x)$ is $[0, \infty)$.

**19. (C) :** Total number of letters in the word ASSASSINATION are 13.

Out of which 3A's, 4S's, 2I's, 2N's, 1 T and 1 O.

(i) If four S's come consecutively, then we consider these 4S's as 1 group.

| S | S | S | S | A | A | A | I | I | N | N | T | O |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | | | 9 | | | | | | | | |

$\therefore$ Number of words when all S's are together $= \dfrac{10!}{3!2!2!}$

Also, total number of words using letter of the word Assassination $= \dfrac{13!}{3!4!2!2!}$

$\therefore$ Required probability $= \dfrac{10! \times 3! \times 4! \times 2! \times 2!}{3!2!2! \times 13!}$

$= \dfrac{10! \times 4!}{13!} = \dfrac{4!}{13 \times 12 \times 11} = \dfrac{24}{1716} = \dfrac{2}{143}$

(ii) If 2I's and 2N's come together, then there are 10 alphabets.

$\therefore$ Number of words when 2I's and 2N's come together

$= \dfrac{10!}{3!4!} \times \dfrac{4!}{2!2!}$

$\therefore$ Required probability $= \dfrac{\dfrac{10!4!}{3!4!2!2!}}{\dfrac{13!}{3!4!2!2!}} = \dfrac{4!10!}{13!} = \dfrac{2}{143}$

(iii) Number of words when all A's come together

$= \dfrac{11!}{4!2!2!}$

$\therefore$ Probability when all A's come together

$= \dfrac{\dfrac{11!}{4!2!2!}}{\dfrac{13!}{4!3!2!2!}} = \dfrac{11! \times 3!}{13!} = \dfrac{6}{13 \times 12} = \dfrac{1}{26}$

$\therefore$ Required probability when all A's does not come together $= 1 - \dfrac{1}{26} = \dfrac{25}{26}$

(iv) For no two A's are coming together, let us first arrange the letters except A's

| | S | | S | | S | | S | | I | | N | | T | | I | | O | | N | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|

All the letters except A's are arranged in $\dfrac{10!}{4!2!2!}$ ways

There are 11 vacant places between these letters.

Therefore, 3A's can be placed in 11 places in ${}^{11}C_3 = \dfrac{11!}{3!8!}$ ways

$\therefore$ Total number of words when no two A's are together $= \dfrac{11!}{3!8!} \times \dfrac{10!}{4!2!2!}$

$\therefore$ Required probability

$= \dfrac{11! \times 10!}{3!8!4!2!2!} \times \dfrac{4!3!2!2!}{13!} = \dfrac{10!}{8! \times 13 \times 12} = \dfrac{10 \times 9}{13 \times 12} = \dfrac{15}{26}$

**20. (C) :** $\lim\limits_{x \to 3} \dfrac{x^5 - 3^5}{x^8 - 3^8} = \lim\limits_{x \to 3} \dfrac{x^5 - 3^5}{(x-3)} \cdot \left(\dfrac{x-3}{x^8 - 3^8}\right)$

$= \dfrac{5 \cdot 3^4}{8 \cdot 3^7} = \dfrac{5}{8 \cdot 3^3} = \dfrac{5}{216}$

*For other sections/subjects please refer to Physics For You, Chemistry Today and Biology Today*

◆◆

# Are you ready for Olympiads?

**LEVEL 1 Exam on**

**22nd Oct., 19th Nov. & 12th Dec., 2024**

SOF — SCIENCE OLYMPIAD FOUNDATION | IMO — SOF INTERNATIONAL MATHEMATICS OLYMPIAD

**CLASS XII**

## SYLLABUS*

Following the protocol of NEP (2020), NCF (2023), NCERT and CBSE guidelines, National and various State Boards for the convenience of schools and students, any change/reduction in the syllabus will be reflected in actual question papers.

**Section – 1 :** Verbal and Non-Verbal Reasoning.

**Section – 2 :** Relations and Functions, Inverse Trigonometric Functions, Matrices and Determinants, Continuity and Differentiability, Application of Derivatives, Integrals, Application of Integrals, Differential Equations, Vector Algebra, Three Dimensional Geometry, Probability, Linear Programming.

**OR**

**Section – 2 :** Numbers, Quantification, Numerical Applications, Solutions of Simultaneous Linear Equations, Matrices, Determinants, Application of Derivatives, Integration, Application of Integrations, Differential Equations, Probability, Inferential Statistics, Index numbers, Time-based data, Financial Mathematics, Linear Programming.

**Section – 3 :** The syllabus of this section will be based on the syllabus of Quantitative Aptitude.

**Section – 4 :** Matrices, Determinants, Application of Derivatives, Integration, Application of Integrations, Differential Equations, Linear Programming, Probability.

Total Questions : 50 | Time : 1 hr.

**PATTERN & MARKING SCHEME**

| Section | (1) Logical Reasoning | (2) Mathematical Reasoning or Applied Mathematics | (3) Everyday Mathematics | (4) Achievers Section |
|---|---|---|---|---|
| No. of Questions | 15 | 20 | 10 | 5 |
| Marks per Ques. | 1 | 1 | 1 | 3 |

## Practice Questions

**1.** Let $f: N \to N$ defined by $f(n) = \begin{cases} \frac{n+1}{2}, & \text{if } n \text{ is odd} \\ \frac{n}{2}, & \text{if } n \text{ is even} \end{cases}$ then $f$ is

A. onto but not one-one
B. one-one and onto
C. neither one-one nor onto
D. one-one but not onto

**2.** Let $S$ be the set of all real numbers. Then the relation $R = \{(a, b) : 1 + ab > 0\}$ on $S$ is

A. reflexive and symmetric but not transitive
B. reflexive and transitive but not symmetric
C. symmetric and transitive but not reflexive
D. an equivalence relation

**3.** The domain of the function $f(x) = \sin^{-1}\left(\frac{x+5}{2}\right)$ is

A. $[-1, 1]$ B. $[2, 3]$ C. $[3, 7]$ D. $[-7, -3]$

**4.** If matrix $A = \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$, where $a$, $b$ and $c$ are real positive numbers, $abc = 1$ and $A^T A = I$, then the value of $a^3 + b^3 + c^3$ is

A. 1 B. 2 C. 3 D. 4

**5.** Find the value of $\lambda$, if the points $(1, -5)$, $(-4, 5)$, $(\lambda, 7)$ are collinear.

A. $-4$ B. $-5$ C. $-10$ D. 15

**6.** If $f(x) = \begin{cases} mx^3 & \text{if } x \le 2 \\ 4 & \text{if } x > 2 \end{cases}$ is continuous at $x = 2$, then the value of $m$ is

A. 1/3 B. 1/2 C. 2 D. 4

**7.** The function $f(x) = a \sin|x| + be^{|x|}$ is differentiable at $x = 0$ when

A. $3a + b = 0$ B. $3a - b = 0$
C. $a + b = 0$ D. $a - b = 0$

**8.** The function $f(x) = 1 - x^3 - x^5$ is decreasing for

A. $1 \le x \le 5$ B. $x \le 1$
C. $x \ge 1$ D. all values of $x$

**9.** Find the coordinates of the point on the parabola $y^2 = 8x$ which is at minimum distance from the circle $x^2 + (y + 6)^2 = 1$.

A. $(2, -4)$ B. $(18, -12)$
C. $(2, 4)$ D. None of these

**10.** $\int \frac{(x+3)e^x}{(x+4)^2} dx$ is equal to



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A. $\frac{e^x}{(x+4)}+C$  B. $\frac{e^x}{(x+4)^2}+C$

C. $\frac{e^x}{(x+3)}+C$  D. $\frac{1}{(x+4)^2}+C$

**11.** $\int \frac{2x-1}{(x-1)(x+2)(x-3)}dx = A\log|x-1| + B\log|x+2| + C\log|x-3| + K$, then $A, B, C$ are respectively

A. $\frac{-1}{6}, \frac{1}{3}, \frac{-1}{2}$  B. $\frac{1}{6}, \frac{-1}{3}, \frac{1}{3}$

C. $\frac{1}{6}, \frac{1}{3}, \frac{1}{5}$  D. $\frac{-1}{6}, \frac{-1}{3}, \frac{1}{2}$

**12.** Find the area bounded by the $X$-axis, $y = \sin x$ and the ordinates $x = \frac{\pi}{4}$ and $x = \frac{\pi}{2}$.

A. 1 sq. unit  B. $\frac{1}{\sqrt{2}}$ sq. unit

C. $\frac{1}{\sqrt{3}}$ sq. unit  D. None of these

**13.** The particular solution of $\log(dy/dx) = 3x + 4y$, $y(0) = 0$ is

A. $e^{3x} + 3e^{-4y} = 4$  B. $4e^{3x} - 3^{-4y} = 3$

C. $3e^{3x} + 3e^{-4y} = 4$  D. $4e^{3x} + 3e^{-4y} = 7$

**14.** Find the coordinates of a point on the line $\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2}$ at a distance of $\frac{6}{\sqrt{2}}$ from the point (1, 2, 3).

A. (56, 43, 111)  B. $\left(\frac{56}{17}, \frac{43}{17}, \frac{111}{17}\right)$

C. (2, 1, 3)  D. (–2, –1, –3)

**15.** The angle between the vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$, where $\vec{a} = \hat{i}+\hat{j}+4\hat{k}$ and $\vec{b} = \hat{i}-\hat{j}+4\hat{k}$ is

A. 90°  B. 45°  C. 30°  D. 15°

## ACHIEVERS SECTION

**16.** Read the given statements carefully and state 'T' for true and 'F' for false.

(i) If $A$ is any square matrix, then $\frac{A+A'}{2}$ is always skew-symmetric.

(ii) If $A$ and $B$ are invertible matrices of the same order, then $A + B$ is also invertible.

(iii) If $A$, $B$, $C$ are three matrices such that both $AB$ and $AC$ are defined and are equal, then it implies that $B$ and $C$ are equal matrices.

(iv) For any matrix $A$, $AA'$ is always defined and is a square matrix.

| | (i) | (ii) | (iii) | (iv) |
|---|---|---|---|---|
| A. | T | F | F | T |
| B. | F | F | F | T |
| C. | F | T | T | F |
| D. | T | F | T | F |

**17.** Fill in the blanks and select the correct option.

(i) $f(x) = \sin x + \sqrt{3}\cos x$ has maximum value at __P__.

(ii) The values of $a$ for which the function $f(x) = \sin x - ax + b$ increases on $R$ are __Q__.

(iii) The least value of the function $f(x) = ax + \frac{b}{x}$ $(a > 0, b > 0, x > 0)$ is __R__.

(iv) The function $f(x) = x^2 - 2x$ is strictly decreasing in the interval __S__.

| | (P) | (Q) | (R) | (S) |
|---|---|---|---|---|
| A. | $\frac{\pi}{2}$ | $(-\infty, 1)$ | $\sqrt{ab}$ | $(1, \infty)$ |
| B. | $\frac{\pi}{6}$ | $(-\infty, 1)$ | $2\sqrt{ab}$ | $(-\infty, 1)$ |
| C. | $\pi$ | $(-\infty, -1)$ | $\sqrt{ab}$ | $(-\infty, \infty)$ |
| D. | 1 | $(-\infty, 1)$ | $2\sqrt{ab}$ | $(0, \infty)$ |

**18.** **Statement-I :** Consider the experiment of drawing a card from a deck of 52 playing cards, in which the elementary events are assumed to be equally likely. If $E$ and $F$ denote the events the card drawn is a spade and the card drawn is an ace respectively, then $P(E|F) = \frac{1}{4}$ and $P(F|E) = \frac{1}{13}$.

**Statement-II :** $E$ and $F$ are two events such that the probability of occurrence of one of them is not affected by occurrence of the other. Such events are called independent events.

A. Both Statement-I and Statement-II are true.
B. Both Statement-I and Statement-II are false.
C. Statement-I is true but Statement-II is false.
D. Statement-I is false but Statement-II is true.

**19.** Match the following.

| | Column I | | Column II |
|---|---|---|---|
| (P) | The solution of $\frac{dy}{dx} = 2^{y-x}$ is | (1) | $x^2(y+3)^3 = e^{y+2}$ |
| (Q) | The solution of $\frac{dy}{dx} = 1 + x + y^2 + xy^2$, when $y = 0$, $x = 0$ is | (2) | $2^{-x} - 2^{-y} = k$ |

| | | | |
|---|---|---|---|
| (R) | Solution of $x^2\frac{dy}{dx}=x^2+xy+y^2$ is | (3) | $y=\tan\left(x+\frac{x^2}{2}\right)$ |
| (S) | The solution of $2(y+3)-xy\frac{dy}{dx}=0$, given that $y(1) = -2$ is | (4) | $\tan^{-1}\left(\frac{y}{x}\right)$ $=\log\lvert x\rvert+C$ |

A. P → 1; Q → 2; R → 3; S → 4
B. P → 1; Q → 3; R → 4; S → 2
C. P → 2; Q → 1; R → 3; S → 4
D. P → 2; Q → 3; R → 4; S → 1

**20.** The maximum value of the objective function $z = 4x + 5y$ subject to constraints $2x + 3y \le 12$, $2x+ y \le 8$ and $x \ge 0, y \ge 0$ is

A. 24 B. 23
C. 22 D. 21

Darken your choice with HB Pencil

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1. | Ⓐ Ⓑ Ⓒ Ⓓ | 5. | Ⓐ Ⓑ Ⓒ Ⓓ | 9. | Ⓐ Ⓑ Ⓒ Ⓓ | 13. | Ⓐ Ⓑ Ⓒ Ⓓ | 17. | Ⓐ Ⓑ Ⓒ Ⓓ |
| 2. | Ⓐ Ⓑ Ⓒ Ⓓ | 6. | Ⓐ Ⓑ Ⓒ Ⓓ | 10. | Ⓐ Ⓑ Ⓒ Ⓓ | 14. | Ⓐ Ⓑ Ⓒ Ⓓ | 18. | Ⓐ Ⓑ Ⓒ Ⓓ |
| 3. | Ⓐ Ⓑ Ⓒ Ⓓ | 7. | Ⓐ Ⓑ Ⓒ Ⓓ | 11. | Ⓐ Ⓑ Ⓒ Ⓓ | 15. | Ⓐ Ⓑ Ⓒ Ⓓ | 19. | Ⓐ Ⓑ Ⓒ Ⓓ |
| 4. | Ⓐ Ⓑ Ⓒ Ⓓ | 8. | Ⓐ Ⓑ Ⓒ Ⓓ | 12. | Ⓐ Ⓑ Ⓒ Ⓓ | 16. | Ⓐ Ⓑ Ⓒ Ⓓ | 20. | Ⓐ Ⓑ Ⓒ Ⓓ |

## SOLUTIONS

**1. (A) :** We have, $f(1)=\frac{(1+1)}{2}=\frac{2}{2}=1$ and $f(2)=\frac{2}{2}=1$

Thus, $f(1) = f(2)$ but $1 \ne 2$

$\therefore$ $f$ is not one-one.

In order to find that $f$ is onto or not, consider an arbitrary element $n \in N$.

If $n$ is odd, then $(2n - 1)$ is also odd and

$$f(2n-1)=\frac{(2n-1+1)}{2}=\frac{2n}{2}=n$$

If $n$ is even, then $2n$ is also even and $f(2n)=\frac{2n}{2}=n$

Thus, for each $n \in N$, there exists its pre-image in $N$.

$\therefore$ $f$ is onto.

**2. (A) :** We have, $R = \{(a, b) : 1 + ab > 0\}$

Consider, $(a, a)$ then $1 + a \times a = 1 + a^2 > 0$

[$\because$ $a^2$ is always positive]

$\Rightarrow$ $(a, a) \in R$

So, $R$ is reflexive.

Now, if $(a, b) \in R$, then $1 + ab > 0 \Rightarrow 1 + ba > 0$

$\Rightarrow$ $(b, a) \in R$

So, $R$ is symmetric.

Now, let $(a, b)$ and $(b, c) \in R$ then it is not necessary that $1 + ac > 0$

[e.g., $a = -0.5$, $b = -0.1$, $c = 3$ then $1 + ab > 0$, $1 + bc > 0$ but $1 + ac < 0$].

$\therefore$ $R$ is not transitive.

**3. (D) :** We have, $f(x)=\sin^{-1}\left(\frac{x+5}{2}\right)$

$\Rightarrow -1\le\frac{x+5}{2}\le1 \Rightarrow -2-5\le x\le 2-5$

$\Rightarrow -7 \le x \le -3 \quad \therefore D(f(x)) = [-7, -3]$

**4. (D) :** Since, $A^TA = I$

$$\Rightarrow \begin{bmatrix} a & b & c\\ b & c & a\\ c & a & b\end{bmatrix}\begin{bmatrix} a & b & c\\ b & c & a\\ c & a & b\end{bmatrix}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a^2+b^2+c^2 & ab+bc+ca & ab+bc+ca\\ ab+bc+ca & a^2+b^2+c^2 & ab+bc+ca\\ ab+bc+ca & ab+bc+ca & a^2+b^2+c^2\end{bmatrix}$$

$$=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix} \Rightarrow a^2+b^2+c^2=1 \text{ and } ab+bc+ca=0$$

Now, $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

$= 1 + 2\cdot0 = 1 \Rightarrow a + b + c = 1$ ...(i)

Now, $a^3 + b^3 + c^3$

$= (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) + 3abc$

$\Rightarrow a^3 + b^3 + c^3 = 1 + 3 = 4$ [Using (i)]

**5. (B) :** The given points are collinear iff

$$\begin{vmatrix}1 & -5 & 1\\ -4 & 5 & 1\\ \lambda & 7 & 1\end{vmatrix}=0$$

$\Rightarrow 1(-2) + 5(-4 - \lambda) + 1(-28 - 5\lambda) = 0$

$\Rightarrow -50 - 10\lambda = 0 \quad \Rightarrow \lambda = -5$

**6. (B) :** Given, $f(x)$ is continuous at $x = 2$.

$$\Rightarrow \lim_{x\to2^-}f(x)=\lim_{x\to2^+}f(x)=f(2)$$

$$\Rightarrow \lim_{x\to2}(mx^3)=4 \Rightarrow 8m=4 \Rightarrow m=\frac{1}{2}$$

**7. (C)** : We have, $f(x)=\begin{cases} a\sin x+be^x, x\ge 0 \\ -a\sin x+be^{-x}, x<0\end{cases}$

$\Rightarrow \quad f'(x)=\begin{cases} a\cos x+be^x, x>0 \\ -a\cos x-be^{-x}, x<0\end{cases}$

If $f(x)$ is differentiable at $x = 0$, then L.H.D. = R.H.D.

$\Rightarrow \quad a + b = -a - b \quad \Rightarrow a + b = 0$

**8. (D)** : Given, $f(x) = 1 - x^3 - x^5$

Differentiating w.r.t. $x$, we get

$f'(x) = -3x^2 - 5x^4 \Rightarrow f'(x) = -(3x^2 + 5x^4)$

$\Rightarrow f'(x) < 0$ for all values of $x$.

**9. (A)** : A point on the parabola is at a minimum distance from the circle if and only if it is at a minimum distance from the centre of the circle. A point on the parabola $y^2 = 8x$ is of the type $P(2t^2, 4t)$. Centre $C$ of circle $x^2 + (y + 6)^2 = 1$ is $(0, -6)$.

$\therefore \quad (CP)^2 = 4t^4 + (4t + 6)^2 = 4(t^4 + 4t^2 + 12t + 9)$

$\Rightarrow \quad \frac{d}{dt}(CP)^2 = 4(4t^3+8t+12) = 16(t + 1)(t^2 - t + 3)$

Also $\frac{d^2}{dt^2}(CP)^2 = 48t^2+32$

Now, $\frac{d}{dt}(CP)^2 = 0 \Rightarrow t = -1$ (real value)

and $\left[\frac{d^2}{dt^2}(CP)^2\right]_{t=-1} = 80 > 0$ *i.e.*, minima

$\therefore$ Required point is $(2, -4)$.

**10. (A)** : Let $I = \int \frac{(x+3)e^x}{(x+4)^2}dx$

$= \int \frac{(x+4-1)e^x}{(x+4)^2}dx = \int\left[\frac{1}{x+4}-\frac{1}{(x+4)^2}\right]e^x dx = \frac{e^x}{x+4}+C$

$\left[\because \int e^x(f(x)+f'(x))dx = e^x f(x)+C\right]$

**11. (D)** : Let $I = \int \frac{2x-1}{(x-1)(x+2)(x-3)}dx$

Let $\frac{2x-1}{(x-1)(x+2)(x-3)} = \frac{P}{x-1}+\frac{Q}{x+2}+\frac{R}{x-3}$

$\Rightarrow \quad 2x - 1 = P(x + 2)(x - 3) + Q(x - 1)(x - 3) + R(x - 1)(x + 2)$

Putting $x = 1$, we get $1 = P(3)(-2) \Rightarrow P = \frac{-1}{6}$

Putting $x = -2$, we get $-5 = Q(-3)(-5) \Rightarrow Q = \frac{-1}{3}$

Putting $x = 3$, we get $5 = R(2)(5) \Rightarrow R = \frac{1}{2}$

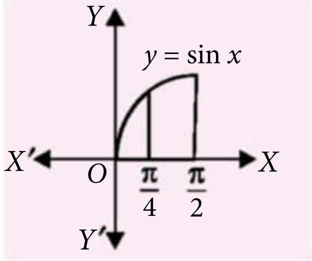
$\therefore \quad I = \frac{-1}{6}\int\frac{dx}{x-1}-\frac{1}{3}\int\frac{dx}{x+2}+\frac{1}{2}\int\frac{dx}{x-3}$

$= \frac{-1}{6}\log|x-1| - \frac{1}{3}\log|x+2| + \frac{1}{2}\log|x-3| + K$

$\therefore \quad A = \frac{-1}{6}, \; B = \frac{-1}{3}, \; C = \frac{1}{2}$

**12. (B)** : We have, $y = \sin x$, $x$-axis and $x = \frac{\pi}{4}, x = \frac{\pi}{2}$

Required area = Area of shaded region




$= \int_{\pi/4}^{\pi/2} \sin x\, dx = -[\cos x]_{\pi/4}^{\pi/2}$

$= -\left[0-\frac{1}{\sqrt{2}}\right] = \frac{1}{\sqrt{2}}$ sq. unit

**13. (D)** : We have, $dy/dx = e^{3x + 4y} = e^{3x} e^{4y}$

$\Rightarrow \quad e^{-4y}\, dy = e^{3x}\, dx$

Integrating both sides, we get

$\frac{e^{-4y}}{-4} = \frac{e^{3x}}{3}+C$

Since, $y(0) = 0 \Rightarrow -1/4 - 1/3 = C \Rightarrow C = -7/12$

Hence, $\frac{e^{-4y}}{-4} = \frac{e^{3x}}{3} - \frac{7}{12} \quad \Rightarrow 7 = 4e^{3x} + 3e^{-4y}$

**14. (B)** : Given equation of line is

$\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2} = \lambda$ (say)

$\Rightarrow \quad x = 3\lambda - 2, y = 2\lambda - 1, z = 2\lambda + 3$

$\therefore$ Coordinates of any point on the line are

$(3\lambda-2, 2\lambda-1, 2\lambda+3)$ ...(i)

Let the distance between this point and $(1, 2, 3)$ is $\frac{6}{\sqrt{2}}$.

$\therefore \quad \sqrt{(3\lambda-2-1)^2+(2\lambda-1-2)^2+(2\lambda+3-3)^2} = \frac{6}{\sqrt{2}}$

$\Rightarrow \quad (3\lambda-3)^2+(2\lambda-3)^2+(2\lambda)^2 = \frac{36}{2}$

$\Rightarrow \quad 9\lambda^2 + 9 - 18\lambda + 4\lambda^2 + 9 - 12\lambda + 4\lambda^2 = 18$

$\Rightarrow \quad 17\lambda^2 - 30\lambda = 0$

$\Rightarrow \quad \lambda(17\lambda-30) = 0 \Rightarrow \lambda = 0, \frac{30}{17}$

Substituting the values of $\lambda$ in (i), we get the required point as $(-2, -1, 3)$ or $\left(\frac{56}{17}, \frac{43}{17}, \frac{111}{17}\right)$.

**15. (A)** : Given that, $\vec{a}=\hat{i}+\hat{j}+4\hat{k}$ and $\vec{b}=\hat{i}-\hat{j}+4\hat{k}$

$\therefore\quad \vec{a}+\vec{b}=2\hat{i}+8\hat{k}$ and $\vec{a}-\vec{b}=2\hat{j}$

Let θ be the angle between $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$, then

$$\cos\theta=\frac{(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b})}{|\vec{a}+\vec{b}||\vec{a}-\vec{b}|}$$

$$=\frac{(2\hat{i}+0\hat{j}+8\hat{k})\cdot(0\hat{i}+2\hat{j}+0\hat{k})}{\sqrt{2^2+0^2+8^2}\sqrt{0^2+2^2+0^2}}=\frac{0+0+0}{\sqrt{4+64}\sqrt{4}}=0$$

$\Rightarrow\quad \cos\theta=0 \Rightarrow \theta=\frac{\pi}{2}=90°$

**16. (B)**

**17. (B)** : (i) We have,

$$f(x)=\sin x+\sqrt{3}\cos x=2\left\{\frac{1}{2}\sin x+\frac{\sqrt{3}}{2}\cos x\right\}, x\in R$$

$$=2\left\{\sin x\cos\frac{\pi}{3}+\cos x\sin\frac{\pi}{3}\right\}=2\left\{\sin\left(x+\frac{\pi}{3}\right)\right\},\ x\in R$$

Since maximum value of $\sin\left(x+\frac{\pi}{3}\right)$ is 1, therefore, maximum value of $\sin x+\sqrt{3}\cos x$ is 2 and it occurs when $\sin\left(x+\frac{\pi}{3}\right)=1$,

*i.e.*, when $x+\frac{\pi}{3}=\frac{\pi}{2}\quad\Rightarrow\quad x=\frac{\pi}{2}-\frac{\pi}{3}\quad\Rightarrow\quad x=\frac{\pi}{6}$

$\therefore$ Given function assumes maximum value at $\frac{\pi}{6}$.

(ii) We have, $f(x)=\sin x-ax+b$

$\Rightarrow\ f'(x)=\cos x-a$

For increasing function, $f'(x)>0$

$\Rightarrow\ \cos x>a$

Since, $\cos x\in[-1,1]$

$\Rightarrow\ a<1\ \Rightarrow\ a\in(-\infty,1)$

(iii) We have, $f(x)=ax+\frac{b}{x}$

$\Rightarrow\ f'(x)=a-\frac{b}{x^2}$

Put $f'(x)=0\Rightarrow\quad a=\frac{b}{x^2}\ \Rightarrow\ x=\pm\sqrt{\frac{b}{a}}$

Now, $f''(x)=-b\cdot\frac{(-2)}{x^3}=\frac{2b}{x^3}$

At $x=\sqrt{\frac{b}{a}}$, $f''(x)=\frac{2b}{\left(\frac{b}{a}\right)^{3/2}}=\frac{2b\cdot a^{3/2}}{b^{3/2}}>0\quad[\because a, b>0]$

$\therefore$ Least value of $f(x)$ *i.e.*, $f\left(\sqrt{\frac{b}{a}}\right)=a\cdot\sqrt{\frac{b}{a}}+\frac{b}{\sqrt{\frac{b}{a}}}$

$=a\cdot a^{-1/2}\cdot b^{1/2}+b\cdot b^{-1/2}\cdot a^{1/2}=\sqrt{ab}+\sqrt{ab}=2\sqrt{ab}$

(iv) We have, $f(x)=x^2-2x$

$\Rightarrow\ f'(x)=2x-2=2(x-1)<0$ if $x<1$

*i.e.*, $x\in(-\infty,1)$. Hence, $f$ is strictly decreasing in $(-\infty,1)$.

**18. (A)** : We have, $P(E)=\frac{13}{52}=\frac{1}{4}$ and $P(F)=\frac{4}{52}=\frac{1}{13}$

Also, $E\cap F$ denote the event that the card drawn is the ace of spades.

$\therefore\ P(E\cap F)=\frac{1}{52}$

Hence, $P(E|F)=\frac{P(E\cap F)}{P(F)}=\frac{\frac{1}{52}}{\frac{1}{13}}=\frac{1}{4}$

Since, $P(E)=\frac{1}{4}=P(E|F)$, we can say that the occurrence of event $F$ has not affected the probability of occurrence of the event $E$. We also have,

$$P(F|E)=\frac{P(E\cap F)}{P(E)}=\frac{\frac{1}{52}}{\frac{1}{4}}=\frac{1}{13}=P(F)$$

Again, $P(F)=\frac{1}{13}=P(F|E)$ shows that occurrence of event $E$ has not affected the probability of occurrence of the event $F$. Thus, $E$ and $F$ are two events such that the

probability of occurrence of one of them is not affected by occurrence of the other.
Such events are called independent events.

**19. (D) :** (P) We have, $\frac{dy}{dx}=2^{y-x} \Rightarrow \frac{dy}{2^y}=\frac{dx}{2^x}$

Integrating both sides, we get $\frac{-2^{-y}}{\log 2}=\frac{-2^{-x}}{\log 2}+C$

$\Rightarrow -2^{-y}+2^{-x}=C\log 2=k(\text{say}) \Rightarrow 2^{-x}-2^{-y}=k$

(Q) We have, $\frac{dy}{dx}=1+x+y^2+xy^2$

$\Rightarrow \frac{dy}{dx}=(1+y^2)(1+x) \Rightarrow \frac{dy}{1+y^2}=(1+x)dx$

Integrating both sides, we get

$\tan^{-1} y = x+\frac{x^2}{2}+c$ ...(i)

When, $y = 0, x = 0 \Rightarrow \tan^{-1}(0) = 0 + 0 + c \quad \therefore \quad c = 0$

Now, from (i), we get $\tan^{-1} y = x+\frac{x^2}{2}$

$\Rightarrow y=\tan\left(x+\frac{x^2}{2}\right)$

(R) We have, $x^2\frac{dy}{dx}=x^2+xy+y^2$

$\Rightarrow \frac{dy}{dx}=1+\frac{y}{x}+\left(\frac{y^2}{x^2}\right)$ ...(i)

Put $y=vx \Rightarrow \frac{dy}{dx}=v+x\frac{dv}{dx}$

From (i), we get

$v+x\frac{dv}{dx}=1+v+v^2$

$\Rightarrow x\frac{dv}{dx}=1+v^2 \Rightarrow \frac{dv}{1+v^2}=\frac{dx}{x}$

Integrating both sides, we get

$\tan^{-1}v = \log|x| + C \Rightarrow \tan^{-1}\left(\frac{y}{x}\right)=\log|x|+C$

(S) We have, $2(y+3)-xy\frac{dy}{dx}=0$

$\Rightarrow 2(y+3)=xy\frac{dy}{dx} \Rightarrow 2\frac{dx}{x}=\left(\frac{y}{y+3}\right)dy$

$\Rightarrow 2\cdot\frac{dx}{x}=\left(1-\frac{3}{y+3}\right)dy$

Integrating both sides, we get

$2\log x = y - 3\log(y+3) + C$ ...(i)

When $x = 1, y = -2$

From (i), $2\log 1 = -2 - 3\log(-2+3) + C$

$\Rightarrow 2\cdot 0 = -2 - 3\cdot 0 + C \Rightarrow C = 2$

On substituting the value of $C$ in (i), we get

$2\log x = y - 3\log(y+3) + 2$

$\Rightarrow \log x^2 + \log(y+3)^3 = (y+2) \Rightarrow x^2(y+3)^3 = e^{y+2}$

**20. (C) :** Given inequalities are

$2x + 3y \le 12$, $2x + y \le 8$ and $x \ge 0, y \ge 0$

Corresponding equations are

$2x + 3y = 12$ and $2x + y = 8$

Now, plotting the graph of $2x + 3y = 12$ using the table

| | | |
|---|---|---|
| $x$ | 0 | 6 |
| $y$ | 4 | 0 |

and $2x + y = 8$ using the table

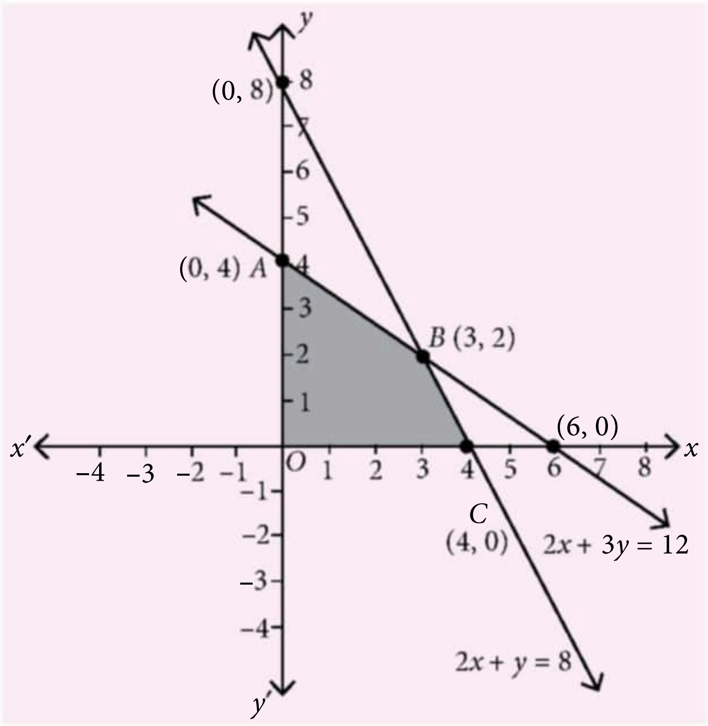
| | | |
|---|---|---|
| $x$ | 0 | 4 |
| $y$ | 8 | 0 |

Intersection point of both lines is (3, 2).

For inequality $2x + 3y \le 12$ by substituting $(x, y) = (0, 0)$ we get $0 + 0 \le 12$, which is true, therefore, shaded region will be towards the origin.

Now, for $2x + y \le 8$, $0 + 0 \le 8$, which is true.

$\therefore$ Shaded region will be towards the origin.




$\therefore$ Required feasible region is $ABCO$ having points $A(0, 4)$, $B(3, 2)$, $C(4, 0)$ and $O(0, 0)$.

The values of $z = 4x + 5y$ at corner points are

$z(A) = 4\cdot 0 + 5\cdot 4 = 20$

$z(B) = 4\cdot 3 + 5\cdot 2 = 22$

$z(C) = 4\cdot 4 + 5\cdot 0 = 16$

$z(O) = 0$

$\therefore$ Maximum value of $z$ is 22.

Class 11 & 12

# Maths in FOCUS

TOPIC

Permutations and Combinations, Determinants

*Detailed theory with High Definition images of the given topic is covered under this heading.*

# PERMUTATIONS AND COMBINATIONS

## Fundamental Principle of Counting

### Multiplication Principle

If one operation can be performed in $m$ different ways and if corresponding to each way of performing this operation, there are $n$ different ways of performing second operation, the two operations together in the given order can be performed in $m \times n$ ways.

### Addition Principle

If there are two operations such that one can be performed in $m$ ways and another operation, which is independent of first, can be performed in $n$ ways then either of the two operations can be performed in $(m + n)$ ways.

## Factorial Notation

The continued product of first $n$ natural numbers *i.e.*, $1 \times 2 \times 3 \times .... \times (n - 1) \times n$ is called '$n$ factorial' and is denoted by $n!$ or $\lfloor n$. Also, we define $0! = 1$.

**Note :** If $n$ is negative or a fraction, $n!$ is not defined.

### Exponent of Prime '$p$' in $n$!

Let $p$ be a prime number and $n$ is a positive integer (*i.e.*, natural number). If $E_p(n)$ denote the exponent of the prime $p$ in the positive integer $n$, then exponent of prime $p$ in $n!$ is denoted by $E_p(n!)$ and defined by

$$E_p(n!) = E_p(1 \cdot 2 \cdot 3 \ldots (n-1) \cdot n) = \left[\frac{n}{p}\right] + \left[\frac{n}{p^2}\right] + \ldots.. + \left[\frac{n}{p^k}\right]$$

where $k$ is the largest positive integer satisfying $p^k \le n < p^{k+1}$

## Permutations

A permutation is an arrangement in a definite order of a number of objects taken some or all at a time.

## Important Theorems

The number of permutations of $n$ different objects taken $r$ at a time, where $0 \le r \le n$ and the objects do not repeat is $n(n-1)(n-2) \ldots (n-r+1)$, which is denoted by ${}^nP_r$.

**Note :** ${}^nP_r = \dfrac{n!}{(n-r)!}, \quad 0 \le r \le n$

The number of permutations of $n$ different objects taken $r$ at a time, where repetition is allowed, is $n^r$.

The number of permutations of $n$ objects, where $p$ objects are of same kind and rest all are different is $\dfrac{n!}{p!}$.

The number of permutations of $n$ objects, where $p_1$ objects are of one kind, $p_2$ are of second kind, ..., $p_k$ are of $k^{th}$ kind and the rest, if any, are of different kind is $\dfrac{n!}{p_1!\, p_2! \ldots p_k!}$.

## Restricted Permutations

The number of ways in which $r$ objects can be arranged from $n$ dissimilar objects if $k$ particular objects are

- always included (or never excluded) $= {}^rP_k \, {}^{n-k}P_{r-k}$
- always excluded (never included) $= {}^{n-k}P_r$

## De-arrangements

Any change in the existing order of things is called De-arrangement. If $m$ things are arranged in a row, the number of ways in which they can be dearranged so that none of them occupies its original place (no one of them occupies the place assigned to it)

$$= m!\left[1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\ldots+(-1)^m\frac{1}{m!}\right] = m!\sum_{r=0}^{m}(-1)^r\frac{1}{r!} = {}^mP_m - {}^mP_{m-1} + {}^mP_{m-2} - \ldots + (-1)^m\,{}^mP_0$$

## Circular Arrangement

- Number of circular arrangement of $n$ objects/things $= \dfrac{n!}{n} = (n-1)!$
- The number of circular permutations of $n$ different things, when clockwise and anticlockwise circular permutations are considered as same is $\dfrac{(n-1)!}{2}$.
- Number of circular permutations of $n$ dissimilar things taken $r$ at a time

  $\dfrac{{}^nP_r}{r}$, if clockwise and anticlockwise orders are considered as different.

  $\dfrac{{}^nP_r}{2r}$, if clockwise and anticlockwise order is considered as same.

# Combinations

Each of the different groups or selections which can be formed by taking some or all of a number of objects, irrespective of their arrangements, is called a combination.

The number of combinations of $n$ distinct objects taken $r$ at a time, is denoted by $C(n, r)$ or ${}^nC_r$ or $\binom{n}{r}$ and given by ${}^nC_r = \dfrac{n!}{r!(n-r)!}, \; 0 \le r \le n$.

## Properties

Let $n$ and $r$ are integers such that $0 \le r \le n$, then

(i) ${}^nC_{n-r} = {}^nC_r$

(ii) ${}^nC_x = {}^nC_y$, then $x = y$ or $x + y = n$

(iii) ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

(iv) $n\,{}^{n-1}C_{r-1} = (n - r + 1)\,{}^nC_{r-1}$

(v) If $n$ is even, then the greatest value of ${}^nC_r$ is ${}^nC_m$, where $m = n/2$. If $n$ is odd, then the greatest value of ${}^nC_r$ is ${}^nC_m$, where $m = (n - 1)/2$ or $(n + 1)/2$.

(vi) If $n = 2m + 1$, then
${}^nC_0 < {}^nC_1 < {}^nC_2 \ldots < {}^nC_m = {}^nC_{m+1}$
${}^nC_{m+1} > {}^nC_{m+2} > {}^nC_{m+3} > \ldots > {}^nC_n$

(vii) If $n = 2m$, then
${}^nC_0 < {}^nC_1 < {}^nC_2 \ldots < {}^nC_m$
${}^nC_m > {}^nC_{m+1} > {}^nC_{m+2} \ldots > {}^nC_n$

(viii) ${}^nC_r = \frac{n}{r} \cdot {}^{n-1}C_{r-1}$, $1 \le r \le n$

(ix) ${}^nC_0 + {}^nC_1 + {}^nC_2 + \ldots + {}^nC_n = 2^n$

(x) ${}^nC_0 + {}^nC_2 + \ldots = {}^nC_1 + {}^nC_3 + \ldots = 2^{n-1}$

(xi) ${}^{2n+1}C_0 + {}^{2n+1}C_1 + \ldots + {}^{2n+1}C_n = {}^{2n+1}C_{n+1} + {}^{2n+1}C_{n+2} + \ldots + {}^{2n+1}C_{2n+1} = 2^{2n}$

## Relation between ${}^nP_r$ and ${}^nC_r$

${}^nP_r = r!\,{}^nC_r = {}^{n-1}P_r + r\,{}^{n-1}P_{r-1}$

## Restricted Combinations

The number of ways in which $r$ objects can be selected from $n$ dissimilar objects if $k$ particular objects are

- always included $= {}^{n-k}C_{r-k} = {}^{n-k}C_{n-r}$
- never included (always excluded) $= {}^{n-k}C_r$

## Division into Groups

- The number of ways in which $m + n$ different things can be divided into two groups which contain $m$ and $n$ $(m \ne n)$ things respectively is ${}^{m+n}C_n \times {}^mC_m = \frac{(m+n)!}{m!\,n!}$, if order of the groups is not to be taken into account and $\frac{(m+n)!}{m!\,n!} \times 2!$, if order of the groups is to be taken into account.
- The number of ways in which $2n$ different things can be divided into two equal groups, is $\frac{{}^{2n}C_n \times {}^nC_n}{2!} = \frac{(2n)!}{2!(n!)^2}$, if order of the groups is not to be taken into account and $\frac{(2n)!}{(n!)^2}$, if order of the groups is to be taken into account.
- The number of ways in which $m + n + p$ different things can be divided into three groups which contain $m$, $n$ and $p$ $(m \ne n \ne p)$ things respectively is ${}^{m+n+p}C_m \times {}^{n+p}C_n \times {}^pC_p = \frac{(m+n+p)!}{m!\,n!\,p!}$, if order of the groups is not to be taken into account and $\frac{(m+n+p)!}{m!\,n!\,p!} \times 3!$, if order of the groups is to be taken into account.

CONCEPT MAP

# PERMUTATIONS AND COMBINATIONS

Class XI

## FUNDAMENTAL PRINCIPLES OF COUNTING

If a certain work $A$ can be done in $m$ different ways and another work $B$ can be done in $n$ different ways, then

- The number of ways of doing the work $A$ or $B$ is $m + n$.
- The number of ways of doing both the work is $mn$.

## PERMUTATIONS

Arranging $r$ objects out of $n$ different things

- When repetition is not allowed $= {}^nP_r = \frac{n!}{(n-r)!}$, where $0 \le r \le n$
- When repetition is allowed $= n^r$

### Properties

- ${}^nP_n = n! = n(n-1)\cdot(n-2) \ldots 3\cdot2\cdot1 = {}^nP_{n-1}$
- ${}^nP_1 = n$
- ${}^nP_0 = \frac{n!}{(n-0)!} = 1$
- ${}^nP_r = n \cdot {}^{n-1}P_{r-1} = n(n-1)\cdot{}^{n-2}P_{r-2} = n(n-1)(n-2)\cdot{}^{n-3}P_{r-3}$ and so on
- ${}^{n-1}P_r + r \cdot {}^{n-1}P_{r-1} = {}^nP_r$
- $\frac{{}^nP_r}{{}^nP_{r-1}} = n - r + 1$

### Important Results on Permutation

- The number of permutations of $n$ different things taken all at a time is ${}^nP_n = n!$.
- The number of permutations of $n$ things taken all at a time, in which $p$ are alike of one kind, $q$ are alike of second kind and $r$ are alike of third kind and rest are different is $\frac{n!}{p!q!r!}$.
- Number of permutations of $n$ different things taken all at a time, when $m$ specified things always come together is $m!\,(n - m + 1)!$
- Number of permutation of $n$ different things taken $r$ at a time :
  (a) when a particular thing is to be included in each arrangements is $r\,{}^{n-1}P_{r-1}$.
  (b) when a particular thing is always excluded, then number of arrangements is ${}^{n-1}P_r$.

### Circular Permutations

- Arrangement of $n$ different things taken all at a time in form of circle is $(n-1)!$, if clockwise and anticlockwise orders are taken as different.
- Number of circular permutations of $n$ different things taken all at a time $= \frac{1}{2}(n-1)!$, if clockwise or anticlockwise orders are not different.
- Number of circular permutations of $n$ dissimilar things, taken $r$ at a time $= \frac{{}^nP_r}{r}$, if clockwise and anticlockwise orders are considered as different.
- Number of circular permutations of $n$ dissimilar things, taken $r$ at a time $= \frac{{}^nP_r}{2r}$, if clockwise and anticlockwise orders are considered as same.

## FACTORIAL NOTATION

Product of first $n$ natural numbers is denoted by $n!$

*i.e.*, $n! = n(n-1)(n-2)\ldots 3\cdot2\cdot1$

### Properties

For any positive integers $m$ and $n$,

- $n! = n \times (n-1)!$
- $(m + n)!$ is divisible by $m!$ as well as $n!$
- $(m \times n)! \ne m! \times n!$
- $(m + n)! \ne m! + n!$
- $(m \div n)! \ne m! \div n!$
- $m > n$, $(m - n)! \ne m! - n!$ but $m!$ is divisible by $n!$

## COMBINATIONS

Ways of selecting $r$ objects out of $n$ different things $= {}^nC_r = \frac{n!}{r!(n-r)!}, 0 \le r \le n$

### Properties

- ${}^nP_r = {}^nC_r\, r!,\ 0 \le r \le n$
- ${}^nC_r = {}^nC_{n-r}$
- ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$
- ${}^nC_a = {}^nC_b$, then either $a = b$ or $n = a + b$
- The largest value of ${}^nC_r$ is ${}^nC_{n/2}$, if $n$ is even and ${}^nC_{\frac{n+1}{2}}$, if $n$ is odd
- ${}^nC_r \div {}^nC_{r-1} = \frac{n-r+1}{r}$

### Important Results on Combination

- If there are $n$ points in the plane, no three of which are collinear, then number of line segments that can be drawn $= {}^nC_2 = \frac{n(n-1)}{2}$
- If $n$ be number of points, no three points are collinear, then number of triangles so formed $= {}^nC_3$
- If $n$ be number of points out of which $m$ are collinear, then number of triangles $= {}^nC_3 - {}^mC_3$

  Number of diagonals in a regular polygon having $n$ sides $= {}^nC_2 - n = \frac{n(n-3)}{2}$
- If $l, m, n$ points are on three parallel lines $L_1, L_2, L_3$ all of which lie in one plane, then number of maximum triangles which can be formed with vertices at these points are ${}^{l+m+n}C_3 - {}^lC_3 - {}^mC_3 - {}^nC_3$
- Total number of selections of any number of objects from $n$ distinct objects $= 2^n$
- Number of selections from $n$ distinct objects, taking at least one $= 2^n - 1$
- Total number of selection of any number of objects from $n$ identical objects $= (n + 1)$

# DETERMINANTS

Class XII

CONCEPT MAP

## DETERMINANT

Corresponding to every square matrix $A$, there exists a number called the determinant of $A$ and denoted by $|A|$.

Let $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$.

Then, $|A| = a_{11}(a_{22}a_{33} - a_{32}a_{23}) - a_{12}(a_{21}a_{33} - a_{23}a_{31}) + a_{13}(a_{21}a_{32} - a_{22}a_{31})$

### Properties of Determinants

- The value of a determinant remains unaltered if its rows and columns are interchanged.
- If two rows (or columns) of a determinant are interchanged, the value of the determinant is multiplied by –1.
- If any two rows (or columns) of a determinant are identical, then the value of determinant is zero.
- If the elements of a row (or column) of a determinant are multiplied by any scalar, then the value of the new determinant is equal to same scalar times the value of the original determinant.
- If each element of any row (or column) of a determinant is the sum of two numbers, then the determinant is expressible as the sum of two determinants of the same order.

### Minors and Cofactors

- For any matrix $A = [a_{ij}]_{n \times n}$, if we leave the $i^{th}$ row and the $j^{th}$ column of the element $a_{ij}$, then the value of determinant thus obtained is called the minor of $a_{ij}$ and it is denoted by $M_{ij}$.
- The minor $M_{ij}$ multiplied by $(-1)^{i+j}$ is called the cofactor of the element $a_{ij}$ and denoted by $A_{ij}$ *i.e.*, $A_{ij} = (-1)^{i+j} M_{ij}$.

### Adjoint of a Matrix

Let $B = [A_{ij}]$ be the matrix of cofactors of matrix $A = [a_{ij}]$. Then the transpose of $B$ is called the adjoint of matrix $A$ and denoted by adj $A$.

### Properties

If $A$ is non-singular matrix of order $n$, then

- $A(\text{adj } A) = (\text{adj } A)\, A = |A|\, I_n$
- $|A \text{ adj } A| = |A|^n$
- $\text{adj }(AB) = (\text{adj } B) \cdot (\text{adj } A)$
- $|\text{adj } A| = |A|^{n-1}$
- $\text{adj }(\text{adj } A) = |A|^{n-2} A$
- $|\text{adj }(\text{adj } A)| = |A|^{(n-1)^2}$

### Area of a Triangle

Let $ABC$ be a triangle with vertices $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$,

then area of $\Delta ABC$ is $\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$.

**Note :**

- We always take absolute value of the determinant as area of triangle is always positive.
- If area is given, we should take both negative and positive values of determinant for calculation.

### Singular and Non-Singular Matrices

A square matrix $A$ of order $n$ is said to be

- Singular if $|A| = 0$.
- Non-singular if $|A| \neq 0$.

**Note :** If $A$ and $B$ are non-singular matrices of the same order, then $AB$ and $BA$ are also non-singular matrices of the same order.

### Inverse of a Matrix

Let $A$ be a non-zero square matrix of order $n$, then a square matrix $B$, such that $AB = BA = I$, is called inverse of $A$, denoted by $A^{-1}$.

*i.e.* $A^{-1} = \frac{1}{|A|}(\text{adj } A)$, $|A| \neq 0$

### Properties

Let $A$ and $B$ be two square matrices of same order $n$. Then,

- $(A^{-1})^{-1} = A$
- $(kA)^{-1} = \frac{1}{k} A^{-1}$ ; where $k$ is any constant.
- $(AB)^{-1} = B^{-1}A^{-1}$
- $(A^T)^{-1} = (A^{-1})^T$
- $AA^{-1} = A^{-1}A = I$

### Solution of System of Linear Equations

Let $AX = B$ be the given system of equations.

- If $|A| \neq 0$, then the system is consistent and has a unique solution.
- If $|A| = 0$ and $(\text{adj } A)B \neq O$, then the system is inconsistent and hence it has no solution.
- If $|A| = 0$ and $(\text{adj } A)B = O$, then the system may be either consistent or inconsistent according as the system have either infinitely many solutions or no solution.

- The number of ways in which $3n$ different things can be divided into three equal groups, is $\frac{(3n)!}{3!(n!)^3}$, if order of the groups is not to be taken into account and $\frac{(3n)!}{(n!)^3}$, if order of the groups is to be taken into account.

## Number of Integral Solutions of Linear Equation

- Number of non-negative integral solutions of the equation $x_1 + x_2 + ... + x_r = n$ is ${}^{n+r-1}C_{r-1}$.
- Number of positive integral solutions of the equation $x_1 + x_2 + .... + x_r = n$ is ${}^{n-1}C_{r-1}$.

## Applications of Combination in Geometry

(i) If there are $n$ points in the plane, no three of which are collinear, then number of line segments can be drawn $= {}^nC_2 = \frac{n(n-1)}{2}$

(ii) If $n$ be number of points out of which $m$ are collinear, then number of line segments drawn
$= {}^nC_2 - {}^mC_2 + 1 = \frac{n(n-1)}{2} - \frac{m(m-1)}{2} + 1 = \frac{1}{2}[n(n-1) - m(m-1) + 2] = \frac{1}{2}(n-m)(n+m-1) + 1$

(iii) If $n$ be number of points no three of which are collinear, then number of triangles so formed $= {}^nC_3$

(iv) If $n$ be number of points out of which $m$ $(\geq 3)$ are collinear, then number of triangles $= {}^nC_3 - {}^mC_3$

(v) Number of rectangles of any size in a rectangle of size $m \times n$ $(n \leq m)$
$= {}^{m+1}C_2 \cdot {}^{n+1}C_2 = \frac{mn}{4}(m+1)(n+1)$

(vi) Number of diagonals in a regular polygon having $n$ sides $= {}^nC_2 - n = \frac{n(n-3)}{2}$

(vii) If $m$ points are on one straight line, are joined to $n$ points on the another straight line, then number of points of intersections of the line segment thus obtained
$= {}^mC_2 \times {}^nC_2 = \frac{mn(m-1)(n-1)}{4}$

(viii) Number of rectangles formed on a chess board $= {}^9C_2 \times {}^9C_2$ ($\because$ Chess board consists 9 horizontal and 9 vertical lines)

(ix) Number of parallelograms when a parallelogram is cut by two sets of $m$ lines parallel to its sides
$= {}^{m+2}C_2 \times {}^{m+2}C_2 = ({}^{m+2}C_2)^2$

(x) If $l$, $m$, $n$ points are on three parallel lines $L_1$, $L_2$, $L_3$ all of which lies in one plane, then number of maximum triangles which can be formed with vertices at these points
$= {}^{l+m+n}C_3 - {}^lC_3 - {}^mC_3 - {}^nC_3$

(xi) Any $R$ points are taken on each of three coplanar parallel lines, then maximum number of $\Delta$'s with vertices at these points $= R^2(4R - 3)$

(xii) Number of non-congruent rectangles that can be formed on a normal chess board
$= {}^8C_2 + 8 = 36$

# DETERMINANTS

## Determinant

Corresponding to every square matrix $A$, there exists a number called the determinant of $A$ and denoted by $|A|$.

Let $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$. Then, $|A| = a_{11}(a_{22}a_{33} - a_{32}a_{23}) - a_{12}(a_{21}a_{33} - a_{23}a_{31}) + a_{13}(a_{21}a_{32} - a_{22}a_{31})$

### Summation of Determinants

Let $\Delta(r) = \begin{vmatrix} f(r) & g(r) & h(r) \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$

where $a_1, a_2, a_3, b_1, b_2, b_3$ are constants or independent of $r$, then $\sum_{r=1}^{n} \Delta(r) = \begin{vmatrix} \sum_{r=1}^{n} f(r) & \sum_{r=1}^{n} g(r) & \sum_{r=1}^{n} h(r) \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$

### Area of a Triangle

- Let $ABC$ be a triangle with vertices $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$, then area of $\Delta ABC$ is

$$\Delta = \frac{1}{2}\left\| \begin{matrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{matrix} \right\|$$

- The given points $A$, $B$ and $C$ will be collinear if and only if area of $\Delta ABC = 0$.

### Minors

- For any matrix $A = [a_{ij}]_{n \times n}$, if we leave the $i^{th}$ row and the $j^{th}$ column of the element $a_{ij}$, then the determinant thus obtained is called the minor of $a_{ij}$ and it is denoted by $M_{ij}$.

Let $\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$. Thus $M_{11} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}$

### Cofactors

Cofactor of an element $a_{ij}$, denoted by $c_{ij}$ is defined by $c_{ij} = (-1)^{i+j} M_{ij}$, where $M_{ij}$ is minor of $a_{ij}$.

### Adjoint of a Matrix

Adjoint of $A$, denoted by adj $A$, is defined as the transpose of the cofactor matrix.

## Properties of Adjoint of a Matrix

- $A(\text{adj } A) = (\text{adj } A)\, A = |A|\, I_n$
- $\text{adj }(AB) = (\text{adj } B) \cdot (\text{adj } A)$
- $|\text{adj } A| = |A|^{n-1}$, where $n$ is the order of $A$.
- $\text{adj }(\text{adj } A) = |A|^{n-2} A$, where $A$ is a non-singular matrix.
- $|\text{adj }(\text{adj } A)| = |A|^{(n-1)^2}$, where $A$ is a non-singular matrix.
- $\text{adj }(A') = (\text{adj } A)'$, where $A$ is a non-singular matrix.
- $\text{adj }(kA) = k^{n-1}(\text{adj } A)$, for any non-zero scalar $k$.

## Singular and Non-singular Matrix

A square matrix $A = [a_{ij}]_{n \times n}$ is called non-singular if $|A| \neq 0$ and singular if $|A| = 0$.

## Inverse of a Matrix

If $A$ be a non-singular square matrix, then inverse of $A$ is defined as

$$A^{-1} = \frac{1}{|A|}(\text{adj}A), \quad |A| \neq 0$$

# APPLICATIONS OF DETERMINANTS AND MATRICES

## Consistent and inconsistent system

A system of equations is said to be consistent if its solution (one or more) exists and said to be inconsistent if its solution does not exist.

# SOLVING A SYSTEM OF LINEAR EQUATIONS USING MATRIX METHOD (INVERSE OF A MATRIX)

Consider non-homogeneous system of equations in three variables as

$a_1x + b_1y + c_1z = d_1$;

$a_2x + b_2y + c_2z = d_2$;

$a_3x + b_3y + c_3z = d_3$

These equations can be written as

$$\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} \text{ i.e., } AX = B, \text{ where}$$

$$A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}, \; X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

**Case I :** If $A$ is a non-singular matrix, $|A| \neq 0$ *i.e.*, $A^{-1}$ exists.

As inverses of a matrix is unique. So, the system of equation is consistent and this matrix equation gives unique solution for the given system of equations. This method of solving system of equations is known as matrix method.

**Case II :** If $A$ is a singular matrix, $|A| = 0$ *i.e.*, $A^{-1}$ does not exist, then two sub cases arise :

**(i)** If $(\text{adj } A)B \neq O$, then the given system of equations is inconsistent and has no solution.

**(ii)** If $(\text{adj } A)\, B = O$, then the system of equations may be either consistent or inconsistent, according as system has either infinitely many solutions or no solution.

# Brain Teaser

# LOGICAL REASONING
## For Various Competitive Exams

**1.** Find the wrong term in the given series.
157.5, 45, 15, 6, 3, 2, 1
(a) 1 (b) 2 (c) 6 (d) 157.5

**2.** Find the next term in given series.
12, 36, 80, 150, 252, 392, 576, ?
(a) 676 (b) 976 (c) 750 (d) 810

***Direction Q.(3 and 4) : Three of the following four are alike in a certain way and so form a group. Which is the one that does not belong to that group?***

**3.** (a) JN (b) DH
(c) WZ (d) LP

**4.** (a) Break (b) Change
(c) Split (d) Divide

***Direction Q.(5 and 6) : Study the following information to answer the given questions :***
In a certain code language,
I. 'she likes apples' is written as 'pic sip dip';
II. 'parrot likes apples lots' is written as 'dip pic tif nit';
III. 'she likes parrot' is written as 'tif sip dip'.

**5.** How is 'parrot' written in that code language?
(a) pic (b) dip
(c) tif (d) None of these

**6.** Which of the following statements is not necessary in order to answer the above question?
(a) (I) (b) (II)
(c) (III) (d) All are required

**7.** Read the following information carefully and answer the question which follows :
If 'A @ B' means 'A is father of B'.
If 'A + B' means 'A is son of B'.
If 'A $ B' means 'A is daughter of B'.
If 'A % B' means 'A is mother of B'.
If 'A & B' means 'A is husband of B'.
What should come in place of the question mark (?) to establish that P is the mother-in-law of T in the below given expression ?
P % Q + R @ S ? T
(a) @ (b) Either & or %
(c) $ (d) &

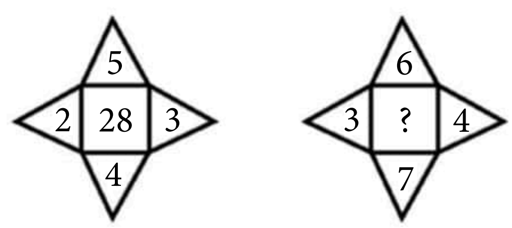
**8.** Pointing to a lady, a man said, "The son of her only brother is the brother of my wife". How is the lady related to that man?
(a) Mother's sister (b) Grandmother
(c) Sister of father-in-law (d) Mother-in-law

***Direction Q.(9 and 10) : In each of the following questions, all the equations except one have been solved according to a certain rule. You are required to solve the unsolved equation following the same rule and to choose the correct answer out of the given options.***

**9.** If 16 @ 3 = 55 and 30 @ 5 = 157, then 9 @ 25 = ?
(a) 335 (b) 125 (c) 265 (d) 232

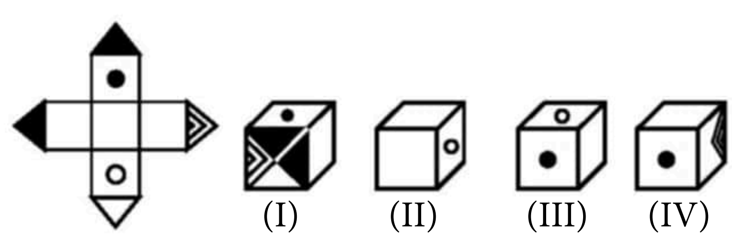
**10.** If 4 × 4 = 6, 8 × 8 = 54 and 9 × 9 = 71, then 2 × 3 = ?
(a) 4 (b) – 6 (c) – 4 (d) 5

**11.** In the following figures, find the number/letter which replaces the sign of '?'.




(a) 66 (b) 48 (c) 30 (d) 60

**12.** In the question given below an unfolded dice is given in the left side while in the right side, four answer choices are given in the form of complete dices. You are required to select the correct answer choice(s) which is/are formed by folding the unfolded dice.

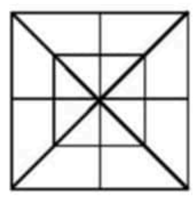



(a) I and II (b) II and III
(c) III and IV (d) II and IV

**13.** 6 equidistant vertical lines are drawn on a board. 6 equidistant horizontal lines are also drawn on the board cutting the 6 vertical lines and the distance between any two consecutive horizontal lines is equal to that between any two consecutive vertical lines. What is the maximum number of squares thus formed?

(a) 37 (b) 55
(c) 126 (d) 225

**14.** Count the number of triangles and squares in the following figure.



(a) 28 triangles, 10 squares
(b) 28 triangles, 8 squares
(c) 32 triangles, 10 squares
(d) 32 triangles, 8 squares

**15. Statements:**
All arrows are bows.
All bows are swords.
Some swords are daggers.
All daggers are knives.
**Conclusions:**
I. All knives are bows.
II. Some swords are knives.
III. All bows are arrows.
IV. All arrows are swords.
(a) Only (II) follows
(b) Only (II) and (IV) follow
(c) Only (III) and (IV) follow
(d) Only (I) and (III) follow

***Direction Q.(16 to 20) : Study the following information carefully and answer the questions given below :***

P, Q, R, S, T, V, W and Z are travelling to three destinations Delhi, Chennai and Hyderabad in three different vehicles – Honda City, Swift Dzire and Ford Ikon. There are three females among them one in each car. There are at least two persons in each car.

R is not travelling with Q and W. T is a male is travelling with only Z and they are not travelling to Chennai. P is travelling in Honda City to Hyderabad. S is sister of P and travels by Ford Ikon. V and R travel together. W does not travel to Chennai.

**16.** Members in which car are travelling to Chennai ?
(a) Honda City (b) Swift Dzire
(c) Ford Ikon
(d) Either Swift Dzire or Ford Ikon

**17.** In which car are four members travelling?
(a) Ford Ikon (b) Honda City
(c) Swift Dzire (d) None of these

**18.** Which of the following combinations represents the three female members?
(a) QSZ (b) WSZ
(c) PSZ (d) Cannot be determined

**19.** Who is travelling with W ?
(a) Only Q (b) Only P
(c) Both P and Q (d) Cannot be determined

**20.** Members in which of the following combinations are travelling in Honda City?
(a) PRS (b) PQW
(c) PWS (d) Data inadequate

## SOLUTIONS

**1. (a) :** $157.5 \xrightarrow{\div 3.5} 45 \xrightarrow{\div 3} 15 \xrightarrow{\div 2.5}$
$6 \xrightarrow{\div 2} 3 \xrightarrow{\div 1.5} 2 \xrightarrow{\div 1} \boxed{1}$
Wrong term is 1.

**2. (d) :** $2^3 + 2^2 = 12$; $6^3 + 6^2 = 252$;
$3^3 + 3^2 = 36$; $7^3 + 7^2 = 392$;
$4^3 + 4^2 = 80$; $8^3 + 8^2 = 576$;
$5^3 + 5^2 = 150$;
So, $9^3 + 9^2 = \boxed{810}$

**3. (c) :** $J \xrightarrow{+4} N$;
$D \xrightarrow{+4} H$;
$L \xrightarrow{+4} P$;
$P \xrightarrow{+4} T$
But, $W \xrightarrow{+3} Z$

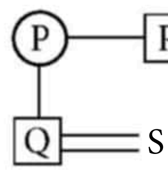
**4. (b) :** 'Change' is different from the other words. Except the word 'Change' all other words show fragmentation.

**5. (c) :** The code for 'parrot' is 'tif'.

she (likes) apples → pic sip (dip)
[parrot] (likes) apples lots → (dip) pic [tif] nit
she (likes) [parrot] → [tif] sip (dip)

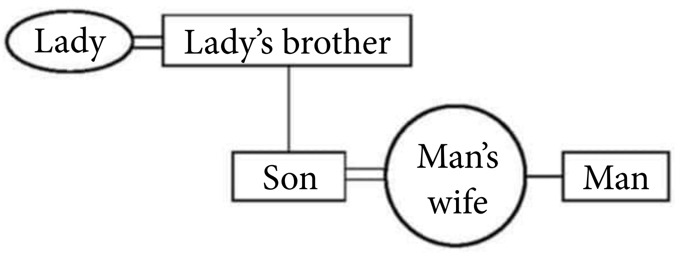
**6. (b) :**
I. she [likes apples] → [pic] sip [dip]
II. parrot [likes apples] lots → [dip pic] tif nit
The code for ''parrot' may be 'tif' or 'nit'.
I. [she likes] apples → pic [sip dip]
II. [she likes] parrots → tif [sip dip]
The code for 'parrot' is 'tif'.
Thus, statement (II) is not necessary.

**7. (d) :** Drawing the family tree from given information:




Now, for P to be mother-in-law of T, T should be husband/wife of S. Hence, it should be S & T *i.e.*, S is husband of T.

**8. (c) :**



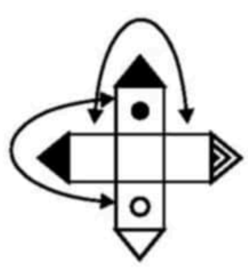

Lady is sister of father-in-law of that man.

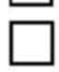
**9. (d) :** $(16 \times 3) + 7 = 55$ and $(30 \times 5) + 7 = 157$,
Similarly, $(9 \times 25) + 7 = 232$

**10. (c) :** $4 \times 4 = 16 - 10 = 6$;
$8 \times 8 = 64 - 10 = 54$ and
$9 \times 9 = 81 - 10 = 71$
Similarly, $2 \times 3 = 6 - 10 = -4$

**11. (d) :** Figure 1 : $(5^2 + 4^2) - (3^2 + 2^2) = 28$
Figure 2 : $? = (6^2 + 7^2) - (3^2 + 4^2) = 60$

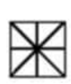
**12. (a) :** According to the unfolded dice,



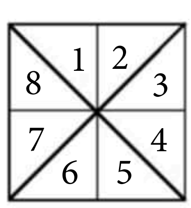
So,  is opposite to 
 is opposite to 
and  is opposite to 

Clearly, cube figures I and II can be formed.

**13. (b) :** A $5 \times 5$ matrix will be formed and total number of squares in a $5 \times 5$ matrix are $5^2 + 4^2 + 3^2 + 2^2 + 1^2 = 55$
Hence, total 55 squares will be formed.

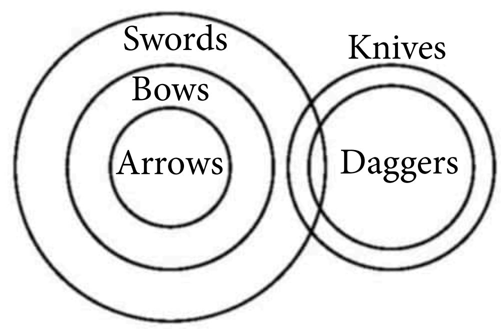
**14. (c) :** The figure consists of 2 similar figures *i.e.*, 

In this figure we have 5 squares, while there are 16 triangles as listed below:




Squares = 18, 23, 45, 67 and 1234578
∴ Number of squares = 5
Triangles = 1, 2, 3, 4, 5, 6, 7, 8, 1234, 5678, 7812, 3456, 12, 34, 56, 78
∴ Number of triangles = 16
Now, there are 2 similar figures.
Hence, total squares = 5 × 2 = 10 and
total triangles = 16 × 2 = 32.

**15. (b) :** All arrows are bows and all bows are swords mean that arrows is a subset of bows and bows further is a subset of swords. Now, some swords are daggers will mean swords and daggers will intersect, but daggers intersecting with arrows and bows is not a necessary condition. All daggers are knives will mean that daggers is a subset of knives. We get the following diagram :




We do not know anything about knives and bows, thus, conclusion (I) is invalid. Swords and knives do intersect, thus conclusion (II) is valid. Arrows may or may not overlap the bows, thus conclusion (III) is invalid. Arrows is a subset of swords, thus conclusion (IV) is valid.
Hence, only conclusions (II) and (IV) follow.

**(16 - 20) :**

| Person | Sex | Vehicle | Destination |
|---|---|---|---|
| P | M/F | Honda City | Hyderabad |
| Q | M/F | Honda City | Hyderabad |
| R | Male | Ford Ikon | Chennai |
| S | Female | Ford Ikon | Chennai |
| T | Male | Swift Dzire | Delhi |
| V | Male | Ford Ikon | Chennai |
| W | M/F | Honda City | Hyderabad |
| Z | Female | Swift Dzire | Delhi |

**16. (c) :** R, S and V are travelling to Chennai in car Ford Ikon.

**17. (d) :** Four members are not travelling in any car.

**18. (d) :** S and Z are female members. The third female member is either P, Q or W.

**19. (c) :** P and Q are travelling with W.

**20. (b) :** P, Q and W are travelling in Honda City.

◇◇

## MONTHLY TEST DRIVE CLASS XI ANSWER KEY

| | | | | |
|---|---|---|---|---|
| 1. (d) | 2. (a) | 3. (d) | 4. (c) | 5. (b) |
| 6. (d) | 7. (a,b,c) | 8. (a,b,c) | 9. (c,d) | 10. (a,c) |
| 11. (c,d) | 12. (a,c) | 13. (a,b) | 14. (b) | 15. (c) |
| 16. (a) | 17. (4) | 18. (2) | 19. (3) | 20. (6) |

# QUANTITATIVE APTITUDE

For Various Competitive Exams

**1.** Akash scored 73 marks in subject *A*. He scored 56% marks in subject *B* and $x$ marks in subject *C*. Maximum marks in each subject were 150. The overall percentage marks obtained by Akash in all the three subjects together were 54%. How many marks did he score in subject *C*?

(a) 84 (b) 86 (c) 79 (d) 73

**2.** A man divided his share to his sons *A* and *B* in such a way that the interest received by *A* at 15% per annum for 3 years is double the interest received by *B* at 12% per annum for 5 years. At what ratio was his share divided?

(a) 2/3 (b) 8/3 (c) 3/8 (d) 3/2

**3.** A person bought 76 cows and sold 20 cows at 15% profit, 40 cows at 19% profit and remaining 16 cows at 25% profit and got a profit of ₹ 6570 as a whole. The cost price of each cow is

(a) ₹ 450 (b) ₹ 425 (c) ₹ 420 (d) ₹ 400

**4.** A shopkeeper marks up his goods by 20% and then gives a discount of 20%. Besides he cheats both his supplier and customer by 100 g *i.e.*, he takes 1100 g from his supplier and sells only 900 g to his customer. What is his net profit percentage?

(a) 25% (b) 24.5% (c) 17.33% (d) 32.5%

**5.** In a mixture of 120 litres, the ratio of milk and water is 2 : 1. If the ratio of milk and water is 1 : 2, then the amount of water (in litres) is required to be added is

(a) 20 (b) 40 (c) 80 (d) 120

**6.** The arithmetic mean of the scores of a group of students in a test was 52. The brightest 20% of them secured a mean score of 80 and the dullest 25%, a mean score of 31. The mean score of remaining 55% is

(a) 50 (b) 51.4 approx.
(c) 54.6 approx. (d) 45

**7.** The value of

$$\frac{1}{1+x^{b-a}+x^{c-a}}+\frac{1}{1+x^{a-b}+x^{c-b}}+\frac{1}{1+x^{b-c}+x^{a-c}} \text{ equals}$$

(a) 3 (b) 2
(c) 1 (d) None of these

**8.** Two candles of the same length are lighted at the same time. The first is consumed in 6 hours and the second in 4 hours. Assuming that each candle burns at a constant rate, in how many hours after being lighted was the first candle twice the length of the second?

(a) 1 hour (b) 2 hours (c) 3 hours (d) 4 hours

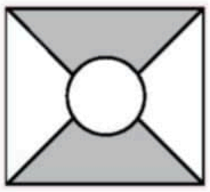
**9.** A man works twice as fast as a woman. A woman works twice as fast as a child. If 32 men can complete a job in 12 days, how many days would be required for 64 women and 128 children together to complete the same job?

(a) 2 days (b) 3 days (c) 4 days (d) 6 days

**10.** Excluding stoppages, the speed of a bus is 54 km/hr and including stoppages, it is 45 km/hr. For how many minutes does the bus stop per hour?

(a) 9 min. (b) 10 min. (c) 12 min. (d) 20 min.

**11.** A square lawn has a circular pond in the centre. Find the shaded region's area given that the diameter of circle is 1/3 times the diagonal of square with sides 3 cm.



(a) $\frac{9-\pi}{2}\text{ cm}^2$ (b) $\frac{(18-\pi)}{4}\text{ cm}^2$

(c) $\frac{(18-\pi)}{18}\text{ cm}^2$ (d) $\frac{(2+\pi)}{8}\text{ cm}^2$

**12.** In how many different ways, 5 boys and 5 girls can sit on a circular table so that the boys and girls are alternate?

(a) 2480 (b) 2800 (c) 2680 (d) 2880

**13.** A bag contains 13 white and 7 black balls. Two balls are drawn at random. What is the probability that they are of the same colour?

(a) $\frac{41}{190}$ (b) $\frac{21}{190}$ (c) $\frac{59}{190}$ (d) $\frac{99}{190}$

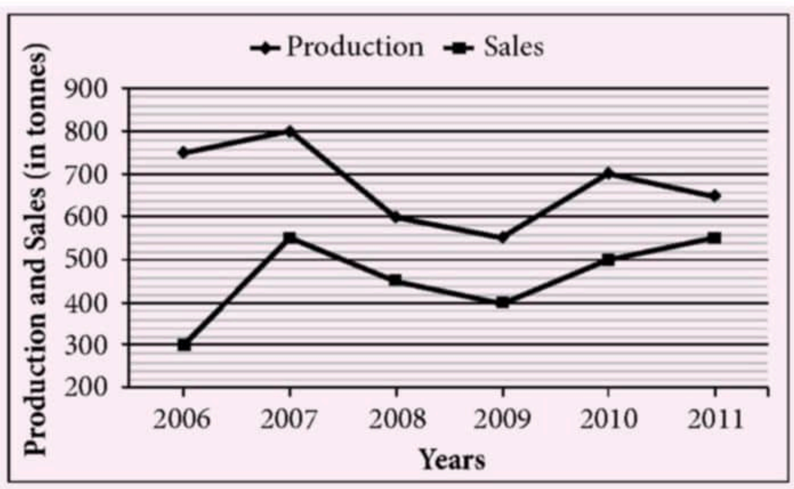
**14.** Three faces of a fair die are yellow, two faces red and one blue. The die is tossed three times. The probability

that the colours yellow, red and blue, appear in the first, second and third tosses respectively is
(a) 1/36 (b) 5/36 (c) 7/36 (d) 1/2

**15.** A survey was conducted on 100 people to find out whether they had read recent issues of Golmal, a monthly magazine. The summarized information regarding readership in three months is given below:
Only September: 18
September but not August: 23
September and July: 8
September: 28
July: 48
July and August: 10
None of the three months: 24
What is the number of surveyed people who have read exactly two consecutive issues (out of the three)?
(a) 7 (b) 9 (c) 12 (d) 14

***Directions Q.* (16-20): Answer the questions on the basis of the information given below:**

The graph given below represents the production (in tonnes) and sales (in tonnes) of company A from 2006-2011.




The given table represents the respective ratio of the production (in tonnes) of Company *A* to the production (in tonnes) of Company *B*, and the respective ratio of the sales (in tonnes) of Company *A* to the sales (in tonnes) of Company *B*.

| Years | Production | Sales |
|---|---|---|
| 2006 | 5 : 4 | 2 : 3 |
| 2007 | 8 : 7 | 11 : 12 |
| 2008 | 3 : 4 | 9 : 14 |
| 2009 | 11 : 12 | 4 : 5 |
| 2010 | 14 : 13 | 10 : 9 |
| 2011 | 13 : 14 | 1 : 1 |

**16.** What is the approximate percentage increase in the production of Company *A* (in tonnes) from the year 2009 to the production of Company *A* (in tonnes) in the year 2010?
(a) 18 (b) 38 (c) 23 (d) 27

**17.** The sales of Company *A* in the year 2009 was approximately what percent of the production of Company *A* in the same year?
(a) 65 (b) 73 (c) 79 (d) 83

**18.** What is the average production of company *B* (in tonnes) from the year 2006 to the year 2011?
(a) 574 (b) 649 (c) 675 (d) 593

**19.** What is the respective ratio of the total production (in tonnes) of Company *B* to the total sales (in tonnes) of Company *B*?
(a) 81 : 64 (b) 64 : 55 (c) 71 : 81 (d) 81 : 65

**20.** What is the respective ratio of production of Company *B* (in tonnes) in the year 2006 to production of Company *B* (in tonnes) in the year 2008?
(a) 2 : 5 (b) 4 : 5 (c) 3 : 4 (d) 3 : 5

## SOLUTIONS

**1. (b) :** Marks scored by Akash in Subject *C*

$$= \frac{73 + 0.56 \times 150 + x}{450}$$

$$\therefore \frac{73 + 84 + x}{450} = \frac{54}{100} \Rightarrow 157 + x = \frac{54}{100} \times 450$$

$$\Rightarrow x = 243 - 157 = 86$$

**2. (b) :** Let the amount received by *A* and *B* be *x* and *y* respectively.

$$\text{Then, } \frac{x \times 15 \times 3}{100} = 2 \times \frac{y \times 12 \times 5}{100} \Rightarrow \frac{x}{y} = \frac{2 \times 12 \times 5}{15 \times 3} = \frac{8}{3}$$

**3. (a) :** Profit% = [20 × 15 + 40 × 19 + 16 × 25]%

$$= \frac{300 + 760 + 400}{100} = \frac{1460}{100} = 14.60\%$$

∴ Profit = CP × Profit%

$$\Rightarrow \text{CP} = \frac{\text{Profit}}{\text{Profit\%}} \Rightarrow \frac{6570}{14.60} = ₹\,450$$

Thus, cost price of each cow is ₹ 450.

**4. (c) :** Assume CP = ₹ 1000/1100 g
M.P. = ₹ 1200 and SP = ₹ 960/900 g
So, SP / 1000 g = ₹ 1173.33. So, profit = ₹ 173.33
Profit percentage = 17.33%

**5. (d) :** Initial quantity of milk $= \frac{2}{3} \times 120 = 80$

Initial quantity of water $= \frac{1}{3} \times 120 = 40$

Now, let *x* litres of water is added.

So, $\frac{80}{40+x} = \frac{1}{2} \Rightarrow x = 120$

**6. (b) :** Let the number of students be 100.
Average scores of these 100 students is given as 52.
So, the question says weighted average of 20% is 80 and 25% is 31.
Let the mean of 55% be '$x$'.

$\therefore$ $5200 = 1600 + 775 + 55(x) \Rightarrow x = \frac{2825}{55}$

$\Rightarrow$ $x = 51.36$ *i.e.*, 51.4 (approx.)

**7. (c) :** Given expression

$$= \frac{1}{1+\frac{x^b}{x^a}+\frac{x^c}{x^a}} + \frac{1}{1+\frac{x^a}{x^b}+\frac{x^c}{x^b}} + \frac{1}{1+\frac{x^b}{x^c}+\frac{x^a}{x^c}}$$

$$= \frac{x^a}{x^a+x^b+x^c} + \frac{x^b}{x^b+x^a+x^c} + \frac{x^c}{x^c+x^b+x^a}$$

$$= \frac{x^a+x^b+x^c}{x^a+x^b+x^c} = 1$$

**8. (c) :** Let length of candle be 12 units [LCM of (6, 4)]
Burning/hr for first candle = 12/6 = 2 units/hr
Burning/hr for second candle = 12/4 = 3 units/hr
Let after $x$ hrs, the length of first is twice the length of second.

$\therefore$ $12 - 2x = 2(12 - 3x)$

$\Rightarrow$ $12 - 2x = 24 - 6x \Rightarrow 4x = 12 \Rightarrow x = 3$ hours

**9. (d) :** Total work = 32 × 12 = 384 units
So, 1 men can do 1 unit/day
1 woman can do 1/2 unit/day

1 child can do 1/4 unit/day
So, 64 women and 128 children will do
64 × 1/2 + 128 × 1/4 = 32 + 32 = 64 units

So, they will finish total work in $\frac{384}{64}$ *i.e.*, 6 days.

**10. (b) :** When the bus does not stop, it covers 54 km in 1 hour.
But when we include the stoppages, the bus covers only 45 km in hour *i.e.*, 9 km less.

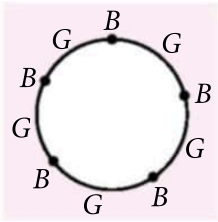
To cover this 9 km, bus will take $\frac{9}{54} = \frac{1}{6}$ hours or 10 minutes without stoppages.
Hence, the bus stops for 10 minutes.

**11. (b) :** Since, diagonal of square = side $\sqrt{2}$ = $3\sqrt{2}$ cm
Diameter of circle = $\sqrt{2}$ cm

$\therefore$ Required area = $\frac{\text{Area of square} - \text{Area of circle}}{2}$

$$= \frac{9-\pi/2}{2} = \left(\frac{18-\pi}{4}\right)\text{cm}^2$$

**12. (d) :** Five boys can be arranged in a circle in (5–1)! ways.
After that girls can be arranged in the five gaps shown as '$G$' in 5! ways.




$\therefore$ Total number of ways = 4! × 5!
= 2880

**13. (d) :** Total number of possible outcome $n(S) = {}^{20}C_2$
= 190

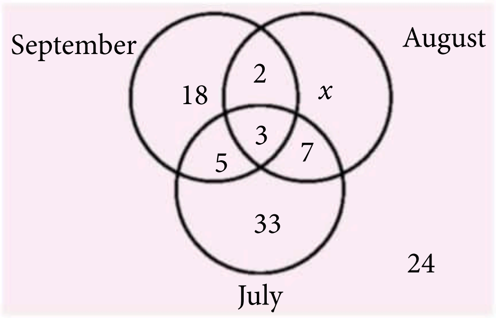
Favourable number of outcomes = ${}^{13}C_2 + {}^{7}C_2$
= 78 + 21 = 99

$\therefore$ $P$(Both balls are of same colour) = 99/190

**14. (a) :** Probability of yellow face appearing on the die,
$P(A) = 3/6 = 1/2$
Probability of red face appearing $P(B) = 2/6 = 1/3$
Probability of blue face appearing $P(C) = 1/6$
Probability that yellow, red and blue appear in the first, second and third tosses = $P(A) \times P(B) \times P(C)$

$$= \frac{1}{2} \times \frac{1}{3} \times \frac{1}{6} = \frac{1}{36}$$

**15. (b) :**




So, number of people reading the newspaper in consecutive months *i.e.*, July and August & August and September is 2 + 7 = 9

**16. (d) :** Required percentage increase

$$= \frac{700-550}{550} \times 100 \approx 27\%$$

**17. (b) :** Required percentage $= \frac{400}{550} \times 100 \approx 73\%$

**18. (c) :** Total production of company B

$$= \left(\frac{4}{5}\times 750 + \frac{7}{8}\times 800 + \frac{4}{3}\times 600 + \frac{12}{11}\times 550 + \frac{13}{14}\times 700 + \frac{14}{13}\times 650\right) \text{tonnes}$$

= (600 + 700 + 800 + 600 + 650 + 700) tonnes
= 4050 tonnes

Required average $= \frac{4050}{6} = 675$ tonnes

**19. (d) :** Required ratio = 4050 : 3250 = 81 : 65

**20. (c) :** Required ratio = 600 : 800 = 3 : 4

# Quiz CLUB

Unlock Your Knowledge!


**1.** Evaluate : $\int_{-\pi/4}^{\pi/4} \frac{e^x \cdot \sec^2 x}{e^{2x}-1} dx$

**2.** If $y=\sqrt{x^2+\sqrt{x^2+\sqrt{x^2+\ldots \text{ to } \infty}}}$, then find the value of $\frac{dy}{dx}$.

**3.** If $\alpha$ and $\beta$ are different complex numbers with $|\beta| = 1$, then find the value of $\left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right|$.

**4.** Evaluate :
$$\tan\left(\frac{\pi}{4}+\frac{1}{2}\cos^{-1} x\right)+\tan\left(\frac{\pi}{4}-\frac{1}{2}\cos^{-1} x\right)$$

**5.** If unit vector $\vec{c}$ makes an angle $\frac{\pi}{3}$ with $\hat{i}+\hat{j}$, then find the minimum and maximum values of $(\hat{i}\times\hat{j})\cdot\vec{c}$.

**6.** Find the values of $b$ for which the function $f(x) = \sin x - bx + c$ is decreasing for $x \in R$.

**7.** The mean and variance of $n$ values of a variable $x$ are 0 and $\sigma^2$, respectively. If the variable $y = x^2$, then find the mean of $y$.

**8.** Evaluate : $\lim_{x\to 0} \frac{\sin 3x}{\sin 2x}$

**9.** Find the distance between the vertex of the parabola $y = x^2 - 4x + 3$ and the centre of the circle $x^2 = 9 - (y - 3)^2$.

**10.** For any square matrix $A$, if $A^2 - A + I = O$, then find the inverse of $A$.

**11.** Find the number of discontinuous functions $y(x)$ on $[-2, 2]$ satisfying $x^2 + y^2 = 4$.

**12.** Calculate area lying between the parabola $y^2 = 4ax$ and its latus rectum.

**13.** Suppose that five good fuses and two defective ones have been mixed up. To find the defective fuses, we test them one-by-one, at random and without replacement. What is the probability that we are lucky and find both of the defective fuses in the first two tests?

**14.** Find the equation of the line where length of the perpendicular segment from the origin to the line is 4 and the inclination of the perpendicular segment with the positive direction of $x$-axis is 30°.

**15.** Find the sum of 20 terms of the series
$1 + (1 + 3) + (1 + 3 + 5) + (1 + 3 + 5 + 7) + \ldots$ .

**16.** Find the least positive integer $n$ such that
${}^{n-1}C_3 + {}^{n-1}C_4 > {}^{n}C_3$.

**17.** Evaluate : $2 \cos 22\frac{1^\circ}{2} \cdot \cos 67\frac{1^\circ}{2}$

**18.** What is the probability that an ordinary year has 53 Tuesdays?

**19.** Evaluate : $\int |x|\, dx$

**20.** Find the range of the function $f(x)=\frac{x^2+8}{x^2+4}$, $x \in R$.

*Readers can send their responses at editor@mtg.in or post us with complete address by 10th of every month. Winners' names and answers will be published in next issue.*

## MONTHLY TEST DRIVE CLASS XII ANSWER KEY

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **1.** | (d) | **2.** | (b) | **3.** | (c) | **4.** | (d) | **5.** | (c) |
| **6.** | (b) | **7.** | (a,b,c) | **8.** | (b,d) | **9.** | (a,c,d) | **10.** | (a,c,d) |
| **11.** | (b, c) | **12.** | (a,d) | **13.** | (a,b,c,d) | **14.** | (a) | **15.** | (b) |
| **16.** | (b) | **17.** | (1) | **18.** | (8) | **19.** | (5) | **20.** | (7) |

# GK CORNER

Enhance Your General Knowledge with Current Updates!

## SUMMITS AND CONFERENCES

1. The 46th session of the World Heritage Committee (WHC) meeting was held in New Delhi's Bharat Mandapam from July 21–31, 2024 and was proudly hosted by India for the first time. In India's illustrious history, which dates back to 1977, this momentous occasion signified a turning point in their engagement with the World Heritage Convention.

2. On the 29th July 2024, the Foreign Ministers of India, Japan, Australia and the secretary of the United States of America met at the Quad Foreign Ministers' Meeting in Tokyo, Japan. It is an informal strategic forum, which came into response to December 2004 Indian Ocean tsunami. Its one of the primary objective is to work for a free, open, prosperous and inclusive Indo - Pacific region.

3. The World Leaders Summit 2024 is a pivotal event bringing together influential figures at the historic House of Lords in the UK Parliament. This conference offers a great chance for international cooperation and communication on pressing global concerns like economic inequality, climate change and technological advancements.

4. The 9th Governing Council meeting of NITI Aayog was held at Rashtrapati Bhawan Cultural Centre, New Delhi on July 27. It was attended by Chief Ministers/ Lt. Governors representing 20 states and 6 UTs, chaired by PM Narendra Modi. He emphasised on cooperation and collective effort of all states and the centre to work together in order to achieve the vision of Viksit Bharat @2047.

5. India will host the first World Audio Visual and Entertainment Summit from 20-24 November alongside the International Film Festival of India in Goa. Information and Broadcasting Minister Ashwini Vaishnaw promising to strengthen the ecosystem for the protection of intellectual property rights in the media and entertainment sector.

6. India will lead the Asian Disaster Preparedness Centre (ADPC) for 2024-2025, taking over from China on July 25, 2024 in Bangkok, Thailand. India is playing a global and regional leadership role in the field of Disaster Risk Reduction (DRR) and building climate resilience in Asia and the Pacific region.

7. Global India AI Summit 2024 is successfully held by the Union Ministry of Electronics and Information Technology on July 3-4, 2024. The purpose of this two-day event was to promote ethical and inclusive AI advancement in areas like Compute Capacity, Foundational Models, Datasets, Application Development, Future Skills, Startup Financing and Safe AI.

**8** The 24$^{th}$ Meeting of the SCO Council of Heads of State (SCO Summit) was held on 4 July 2024 in Astana, under the presidency of Kazakhstan. External Affairs Minister, Dr. S. Jaishankar led the Indian delegation to Astana for the Summit.

**9** Following a meeting between representatives from India and Bangladesh in the nation's capital, Prime Minister Narendra Modi declared that India will introduce an e-medical visa facility for citizens of Bangladesh wishing to receive medical care in India. In addition, India intends to establish a new consulate in Bangladesh's Rangpur to serve the people in the north-west region of the country.

**10** The 112$^{th}$ annual International Labour Conference was held in Geneva, Switzerland 187 ILO's Member States, representing employers, workers and governments delegates, discussed a wide range of topics such as : a standard-setting discussion on protection against biological hazards, recurring discussion on the strategic goal of fundamental rights and principles at work and general discussion on decent work and the care economy. Members of the Governing Body for the 2024-27 term of office were likewise elected by the Conference.

## Test Yourself!

**1.** Where was the Quad Foreign Ministers' Meeting held?
(a) Canberra, Australia (b) Tokyo, Japan
(c) Delhi, India (d) Washington D.C, US

**2.** Who among the following is not included in NITI Aayog?
(a) President
(b) Chief Ministers of all the States and Union Territories with legislature
(c) Prime Minister
(d) Ex-Officio Members

**3.** Recently, 112$^{th}$ annual International Labour Conference was held in Geneva, Switzerland. Which topics were discussed in the conference?
(a) Standard-setting on biological hazard protection
(b) Recurring on the strategic goal of fundamental rights and principles at work
(c) General on decent work
(d) All of these

**4.** Which facility is declared by India recently for Bangladeshi citizens?
(a) e-learning (b) e-Governance
(c) e-medical (d) e-food services

**5.** Which country hosted the meeting of SCO 2024?
(a) Uzbekistan (b) Tajikistan
(c) Kyrgyzstan (d) Kazakhstan

**6.** What is the standard form of ADPC?
(a) Australian Disaster Preparedness Corporation
(b) Asian Disaster Preparedness Corporation
(c) Asian Disaster Preparedness Centre
(d) Australian Disaster Preparedness Centre

**7.** How many countries are included in SCO?
(a) 6 member states and 2 observers from Eurasia
(b) 7 member states and 3 observers from Eurasia
(c) 8 member states and 4 observers from Eurasia
(d) 9 member states and 4 observers from Eurasia

**8.** Where will India host the first World Audio Visual and Entertainment Summit from 20-24 November alongside the International Film Festival of India?
(a) Goa (b) Maharashtra
(c) Karnataka (d) Delhi

**9.** Which session of the World Heritage Committee meeting was recently held in New Delhi's Bharat Mandapam?
(a) 43$^{rd}$ (b) 44$^{th}$
(c) 45$^{th}$ (d) 46$^{th}$

**10.** Which countries are included in Quad Foreign Ministers Meeting?
(a) India, Japan, US and Australia
(b) Africa, Korea, UK and Australia
(c) India, Japan, US and Africa
(d) India, Japan, UK and Africa

### Answer Key

1. (b) 2. (a) 3. (d) 4. (c) 5. (d)
6. (c) 7. (d) 8. (a) 9. (d) 10. (a)

# Challenging PROBLEMS For JEE

ON

## Quadratic Equations & Complex Numbers

### SINGLE OPTION CORRECT TYPE

**1.** Let $z$ be a complex number such that $\left|3z+\frac{1}{z}\right|=1$ and $\arg(z) = \phi$ then the maximum value of $\cos^2\phi$ equals

(a) $\frac{1}{2}$ (b) $\frac{2}{3}$ (c) $\frac{1}{6}$ (d) $\frac{1}{12}$

**2.** The mirror image of the curve given by $\text{Arg}\left(\frac{z+i}{z-1}\right)=\frac{-\pi}{4}$ in the line $(1+i)\bar{z}+(i-1)z=0$ is

(a) $\text{Arg}\left(\frac{z+1}{z-i}\right)=-\frac{\pi}{4}$ (b) $\text{Arg}\left(\frac{z+1}{z-i}\right)=\frac{\pi}{4}$

(c) $\text{Arg}\left(\frac{z+1}{iz+1}\right)=\frac{\pi}{4}$ (d) None of these

**3.** If $z_1, z_2, z_3 \in C$ satisfy the system of equation given by $|z_1| = |z_2| = |z_3| = 1$, $z_1 + z_2 + z_3 = 1$ and $z_1 \cdot z_2 \cdot z_3 = 1$ such that $\text{Im}(z_1) < \text{Im}(z_2) < \text{Im}(z_3)$, then find the value of $[|z_1+z_2^2+z_3^3|]$ where [ ] denotes the greatest integer function.

(a) 1 (b) 2 (3) 3 (d) 4

### INTEGER ANSWER TYPE

**4.** Let $P(x) = x^2 + bx + c$, where $b$ and $c$ are integer. If $P(x)$ is a factor of both $x^4 + 6x^2 + 25$ and $3x^4 + 4x^2 + 28x + 5$, find the value of $P(1)$.

**5.** Let $a$, $b$, $c$ be the three roots of the equation $x^3 + x^2 - 333x - 1002 = 0$. If $P = a^3 + b^3 + c^3$ then the value of $\frac{P}{2006} =$

**6.** The number of the distinct real roots of the equation $(x + 1)^5 = 2(x^5 + 1)$ is

**7.** The equation $2(\log_3 x)^2 - |\log_3 x| + a = 0$ has exactly four real solutions if $a \in \left(0, \frac{1}{K}\right)$, then the value of $K$ is

**8.** Let $\alpha$, $\beta$ and $\gamma$ be the roots of equation $f(x) = 0$, where $f(x) = x^3 + x^2 - 5x - 1$. Then the value of $|[\alpha] + [\beta] + [\gamma]|$, where $[\cdot]$ denotes the greatest integer function, is equal to

**9.** $f(x)$ is a polynomial of $6^{th}$ degree and $f(x) = f(2 - x)$ $\forall\, x \in R$. If $f(x) = 0$ has 4 distinct real roots and two real and equal roots then sum of roots of $f(x) = 0$ is

**10.** If the roots of the equation $x^3 - ax^2 + 14x - 8 = 0$ are all real and positive, then the minimum value of $[a]$ (where $[a]$ is the greatest integer of $a$) is

**11.** If $x, y, z > 0$ and $x(1-y)>\frac{1}{4}$, $y(1-z)>\frac{1}{4}$, $z(1-x)>\frac{1}{4}$, then the number of ordered triplets $(x, y, z)$ satisfying the above inequalities is/are

**12.** Find the greatest integral value of $a$ such that $\sqrt{9-a^2+2ax-x^2}>\sqrt{16-x^2}$ for at least one positive $x$.

**13.** If the equation $x^4 + px^3 + qx^2 + rx + 5 = 0$ has four positive real roots, then the minimum value of $pr/10$ is

**14.** If roots $x_1$ and $x_2$ of $x^2+1=x/a$ satisfy $\left|x_1^2-x_2^2\right|>\frac{1}{a}$, then $a\in\left(-\frac{1}{2},0\right)\cup\left(0,\frac{1}{\sqrt{k}}\right)$ the numerical quantity $k$ must be equal to

**15.** Sum of all roots of the equation

$$\underbrace{\sqrt{x+2\sqrt{x+2\sqrt{x+.....+2\sqrt{x+2\sqrt{3x}}}}}}_{n\,\text{radical signs}}=x$$

must be equal to

**16.** If a complex number $\alpha$ satisfies the equation $(39z-12\bar{z})^4=-1$ then the value of

$\left[\frac{|\text{Re}(\alpha)|^{-1}+|\text{Im}(\alpha)|^{-1}}{13}\right]$ (where [ . ] is GIF)

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**17.** If $\alpha$, $\beta$ and $\gamma$ are three distinct real values such that $\frac{\sin\alpha+\sin\beta+\sin\gamma}{\sin(\alpha+\beta+\gamma)}=\frac{\cos\alpha+\cos\beta+\cos\gamma}{\cos(\alpha+\beta+\gamma)}=2$ and $\cos(\alpha+\beta)+\cos(\beta+\gamma)+\cos(\gamma+\alpha)=a$, then find the value of $\lim_{x\to a}\frac{\sqrt{x^2-a^2}}{\sqrt{x-a}+(\sqrt{x}-\sqrt{a})}$.

## COMPREHENSION TYPE

**Passage-1**

Let $x_1$, $x_2$, $x_3$, $x_4$ be the roots (real or complex) of the equation $x^4+ax^3+bx^2+cx+d=0$. Let $x_1+x_2=x_3+x_4$ and $a, b, c, d \in R$.

**18.** If $a=2$, then the value of $b-c$ is

(a) $-1$ (b) 1 (c) $-2$ (d) 2

**19.** If $b<0$, then how many different real values of '$a$' we may have?

(a) 3 (b) 2 (c) 1 (d) 0

**20.** If $b+c=1$ and $a\neq-2$, then for real values of '$a$', the value of $c\in$

(a) $\left(-\infty, \frac{1}{4}\right)$ (b) $(-\infty, 3)$

(c) $(-\infty, 1)$ (d) $(-\infty, 4)$

**Passage-2**

Let $(a+\sqrt{b})^{Q(x)}+(a-\sqrt{b})^{Q(x)-2\lambda}=A$ where $\lambda\in N$, $A\in R$ and $a^2-b=1$.

$\therefore\ (a+\sqrt{b})(a-\sqrt{b})=1 \Rightarrow (a+\sqrt{b})=(a-\sqrt{b})^{-1}$

and $(a-\sqrt{b})=(a+\sqrt{b})^{-1}$

*i.e.*, $(a\pm\sqrt{b})=(a+\sqrt{b})^{\pm1}$ or $(a-\sqrt{b})^{\pm1}$

**21.** If $\alpha$, $\beta$ are the roots of the equation $1!+2!+3!+....+(x-1)!+x!=k^2$ and $k\in I$, where $\alpha<\beta$ and if $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ are the roots of the equation $(a+\sqrt{b})^{x^2-[1+2\alpha+3\alpha^2+4\alpha^3+5\alpha^4]}+(a-\sqrt{b})^{x^2+[-5\beta]}=2a$ where $a^2-b=1$ and $[\cdot]$ denotes G.I.F., then the value of $|\alpha_1+\alpha_2+\alpha_3+\alpha_4-\alpha_1\alpha_2\alpha_3\alpha_4|$ is

(a) 216 (b) 221

(c) 224 (d) 209

**22.** If $\left(\sqrt{(49+20\sqrt{6})}\right)^{\sqrt{a\sqrt{a\sqrt{a.....\infty}}}}+\left(5-2\sqrt{6}\right)^{x^2+x-3-\sqrt{x\sqrt{x\sqrt{x.....\infty}}}}=10$ where $a=x^2-3$, then $x$ is

(a) $-\sqrt{2}$ (b) $\sqrt{2}$ (c) $-2$ (d) 2

**Passage–3**

Consider the quadratic equation $ax^2-bx+c=0$, $a, b, c\in N$. If the given equation has two real and distinct roots $\alpha$ and $\beta$ belonging to the interval $(1, 2)$ then

**23.** The value of $(\alpha-1)(\beta-1)(2-\alpha)(2-\beta)\in$

(a) $R$ (b) $\left(0, \frac{1}{16}\right)$ (c) $\left[0, \frac{1}{16}\right]$ (d) $(-\infty, 0)$

**24.** The minimum value of $(a-b+c)(4a-2b+c)$ is

(a) 1 (b) 2 (c) 4 (d) 5

**25.** The minimum value of $a$ is

(a) 2 (b) 3 (c) 4 (d) 5

## SOLUTIONS

**1. (d) :** $z=r(\cos\phi+i\sin\phi)$

$\Rightarrow \left|\left(3r+\frac{1}{r}\right)\cos\phi+i\left(3r-\frac{1}{r}\right)\sin\phi\right|=1$

$\Rightarrow \left(3r+\frac{1}{r}\right)^2\cos^2\phi+\left(3r-\frac{1}{r}\right)^2\sin^2\phi=1$

$\Rightarrow \cos^2\phi\left\{\left(3r+\frac{1}{r}\right)^2-\left(3r-\frac{1}{r}\right)^2\right\}=1-\left(3r-\frac{1}{r}\right)^2$

$\Rightarrow 12\cos^2\phi\le1 \Rightarrow \cos^2\phi\le\frac{1}{12}$

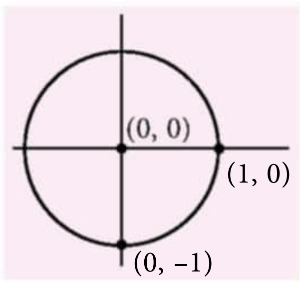
**2. (b) :** Clearly, $z$ lies on $|z|=1$

$\Rightarrow z\bar{z}=1$

Given line is $x=y$

The image of $z$ in $x=y$ is $i\bar{z}$ *i.e.* $\frac{i}{z}$ (as $|z|=1$)

$\therefore$ Required curve is




$\arg\left(\frac{\frac{i}{z}+i}{\frac{i}{z}-1}\right)=-\frac{\pi}{4}$

$\Rightarrow \arg\left(\frac{i(1+z)}{i-z}\right)=-\frac{\pi}{4}$

$\Rightarrow \arg(-i)+\arg\left(\frac{z+1}{z-i}\right)=-\frac{\pi}{4}\Rightarrow\arg\left(\frac{z+1}{z-i}\right)=\frac{\pi}{4}$

**3. (b) :** $z_1z_2+z_2z_3+z_3z_1=z_1z_2z_3\left(\frac{1}{z_1}+\frac{1}{z_2}+\frac{1}{z_3}\right)$

$=1\left(\frac{|z_1|^2}{z_1}+\frac{|z_2|^2}{z_2}+\frac{|z_3|^2}{z_3}\right)=\bar{z}_1+\bar{z}_2+\bar{z}_3$

$= \left(\overline{z_1+z_2+z_3}\right) = 1$

$\therefore$ The cubic equation with roots $z_1$, $z_2$ and $z_3$ in $z$ will be

$(z-z_1)(z-z_2)(z-z_3) = z^3 - 1z^2 + 1z - 1 = 0$

$\Rightarrow\ (z-1)(z^2+1) = 0 \Rightarrow z = 1, \pm i$

$\Rightarrow\ \text{Im}(z_1) < \text{Im}(z_2) < \text{Im}(z_3)$

$\Rightarrow\ z_1 = -i, z_2 = 1, z_3 = i$

$\therefore\ |z_1 + z_2^2 + z_3^3| = |-i + 1^2 + i^3| = |1-2i| = \sqrt{5}$

$\therefore\ \left[|z_1 + z_2^2 + z_3^3|\right] = [\sqrt{5}] = 2$

**4. (4) :** Given, $P(x)$ divides both of them

$\Rightarrow\ P(x)$ also divides

$(3x^4 + 4x^2 + 28x + 5) - 3(x^4 + 6x^2 + 25)$

$= -14x^2 + 28x - 70 = -14(x^2 - 2x + 5)$

which is a quadratic. Hence $P(x) = x^2 - 2x + 5$

$\therefore\ P(1) = 4$

**5. (1) :** Let $\alpha$ be the root of the given cubic where $\alpha$ can take values $a, b, c$.

Hence $\alpha^3 + \alpha^2 - 333\alpha - 1002 = 0$

or $\alpha^3 = 1002 + 333\alpha - \alpha^2$

$\therefore\ \Sigma\alpha^3 = \Sigma 1002 + 333\Sigma\alpha - \Sigma\alpha^2$

$= 3006 + 333\Sigma\alpha - [(\Sigma\alpha)^2 - 2\Sigma\alpha_1\alpha_2]$

But $\Sigma\alpha = -1; \Sigma\alpha_1\alpha_2 = -333$

$\therefore\ a^3 + b^3 + c^3 = 3006 - 333 - [1 + 666]$

$= 3006 - 333 - 667 = 3006 - 1000 = 2006 = P$

**6. (3) :** Given, $(x+1)^5 = 2(x^5+1)$, clearly, $x = -1$ is a real root of the equation.

Let $f(x) = \dfrac{(x+1)^5}{(x^5+1)}\quad (x \neq -1)$

$\Rightarrow\ f'(x) = \dfrac{5(x+1)^4(1-x^4)}{(x^5+1)^2} \Rightarrow x = 1$ is maximum

As, $f(0) = 1$ and $f(1) = 16$

And $\lim\limits_{x\to\pm\infty} f(x) = 1 \Rightarrow f(x) = 2$ has two solutions

$\therefore$ Given equation has three solutions.

**7. (8) :** On putting $\log_3 x = t$, we get

$2t^2 - |t| + a = 0$ ...(i)

If $t > 0$, then $2t^2 - t + a = 0$ ...(ii)

If $t < 0$, then $2t^2 + t + a = 0$ ...(iii)

If (i) has four roots then (ii) must have both roots positive and (iii) has both roots negative.

Now, (ii) has both roots positive *i.e.*, $t = \dfrac{1 \pm \sqrt{1-8a}}{4} > 0$

$\Rightarrow\ 1 - 8a > 0 \Rightarrow a < \dfrac{1}{8}$

and $1 - \sqrt{1-8a} > 0 \Rightarrow a > 0 \Rightarrow a < \dfrac{1}{8}$ and $a > 0$

$\Rightarrow\ a \in \left(0, \dfrac{1}{8}\right)$ on taking intersection.

Similarly, if (iii) has both roots negative, if $D > 0$, $a/2 > 0$.

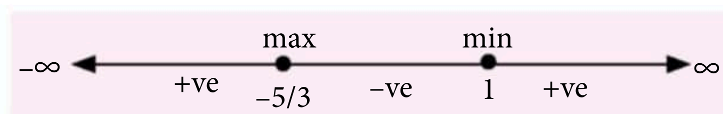
$\therefore\ a \in \left(0, \dfrac{1}{8}\right) \Rightarrow K = 8$

**8. (3) :** Given $f(x) = x^3 + x^2 - 5x - 1$

$\therefore\ f'(x) = 3x^2 + 2x - 5.$

The roots of $f'(x) = 0$ are $-\dfrac{5}{3}$ and 1

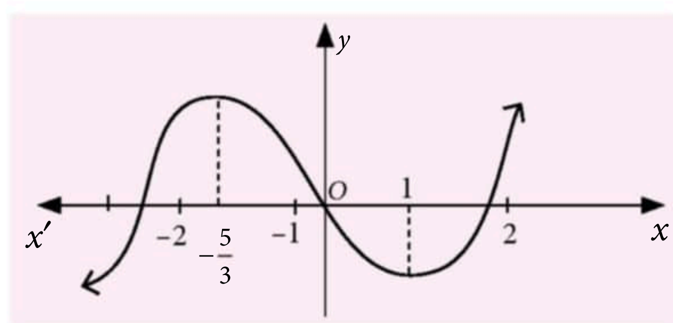
Writing the sign scheme for $f'(x)$,




Also, $f(-\infty) = -\infty < 0, f(\infty) = \infty > 0$

$f(1) = -4, f\left(-\dfrac{5}{3}\right) = \dfrac{148}{27}$

Now, graph of $y = f(x)$ is as follows




$f(-3) = -27 + 9 + 15 - 1 = -4 < 0$

$f(-2) = -8 + 4 + 10 - 1 = 5 > 0$

$f(-1) = 4 > 0, f(0) = -1 < 0$

$f(2) = 1 > 0, f(1) = -4 < 0$

$\Rightarrow\ -3 < \alpha < -2, -1 < \beta < 0, 1 < \gamma < 2$

$\therefore\ |[\alpha] + [\beta] + [\gamma]| = |-3 - 1 + 1| = 3$

**9. (6) :** Let $\alpha$ be a root of $f(x) = 0$.

$\Rightarrow\ f(\alpha) = f(2-\alpha) = 0$

When $\alpha \neq 2 - \alpha$, sum of roots = 4

When $\alpha = 2 - \alpha$ *i.e.*, $\alpha = 1$, sum of roots = 2

$\therefore$ Total sum = 6

**10. (6) :** $f(x) = x^3 - ax^2 + 14x - 8 = 0$

Let $\alpha$, $\beta$ and $\gamma$ be the roots of the equation.

Then $\dfrac{\alpha+\beta+\gamma}{3} \geq (\alpha\beta\gamma)^{1/3} \Rightarrow \dfrac{a}{3} \geq (8)^{1/3}\ \therefore\ a \geq 6$

**11. (0) :** Multiplying the inequalities, we get

$$xyz(1-x)(1-y)(1-z) > \frac{1}{64} \quad \text{...(1)}$$

Now, for $t > 0$, $t(1-t) = t - t^2 = \frac{1}{4} - \left(\frac{1}{2} - t\right)^2 \le \frac{1}{4}$

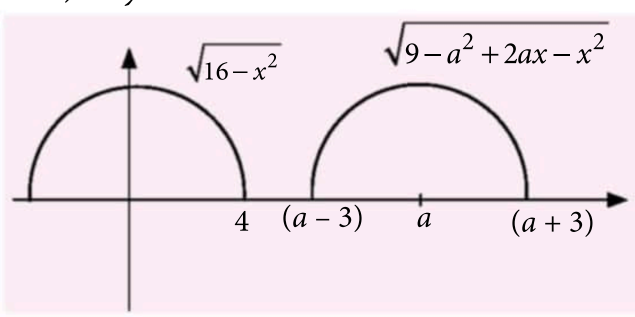
$$\Rightarrow \quad x(1-x)y(1-y)z(1-z) \le \frac{1}{64} \quad \text{...(2)}$$

(1) and (2) are contradictory to each other.

∴ No solution.

**12. (6) :** $y = \sqrt{9 - a^2 + 2ax - x^2}$

$\Rightarrow \ (x-a)^2 + y^2 = 9$




For given inequality to hold for positive $x$.

$a - 3 < 4 \ \Rightarrow \ a < 7 \ \Rightarrow \ a = 6$

**13. (8) :** Let $\alpha, \beta, \gamma, \delta$ be four positive real roots of given equation.

Then $\alpha + \beta + \gamma + \delta = -p$

$\Sigma\alpha\beta = q$ ; $\Sigma\alpha\beta\gamma = -r$ ; $\alpha\beta\gamma\delta = 5$

Using A.M. ≥ G.M. ; $\frac{\alpha+\beta+\gamma+\delta}{4} \ge (\alpha\beta\gamma\delta)^{1/4}$

Also, $\frac{\Sigma\alpha\beta\gamma}{4} \ge (\alpha^3\beta^3\gamma^3\delta^3)^{1/4}$

$$\Rightarrow \ \frac{(\Sigma\alpha)\cdot(\Sigma\alpha\beta\gamma)}{16} \ge (\alpha\beta\gamma\delta) \ \Rightarrow pr \ge 80$$

**14. (5) :** $|x_1 + x_2||x_1 - x_2| > \frac{1}{a}$

$$\Rightarrow \ \left|\frac{1}{a}\right|\left|\sqrt{\frac{1}{a^2} - 4}\right| > \frac{1}{a} \quad \text{...(i)}$$

The inequation (i) has meaning if $\frac{1}{a^2} - 4 > 0$

If $a \in \left(-\frac{1}{2}, 0\right)$ then (i) is automatically satisfied

If $a \in \left(0, \frac{1}{2}\right)$ then (i) becomes equivalent to $\sqrt{\frac{1}{a^2} - 4} > 1$

(on cancelling $\frac{1}{a} > 0$ )

$$\Rightarrow \ -\frac{1}{\sqrt{5}} < a < \frac{1}{\sqrt{5}}$$

but $a \in \left(0, \frac{1}{2}\right)$ was assumed $\Rightarrow \ a \in \left(0, \frac{1}{\sqrt{5}}\right)$

Thus all the values of $a$ lie in the interval

$$\left(-\frac{1}{2}, 0\right) \cup \left(0, \frac{1}{\sqrt{5}}\right) \ \Rightarrow \ k = 5$$

**15. (3) :** Rewrite the given equation

$$\sqrt{x + 2\sqrt{x + 2\sqrt{x + \ldots\ldots + 2\sqrt{x + 2\sqrt{x + 2x}}}}} = x \quad \text{...(1)}$$

On replacing the last letter $x$ on the L.H.S of (1) by the value of $x$ expressed by (1) we obtain,

$$x = \sqrt{x + 2\sqrt{x + 2\sqrt{x + \ldots\ldots + 2\sqrt{x + 2x}}}} \quad (2n \text{ radicals})$$

Further, replace the last letter $x$ by the same expression, we can write


Samurai Sudoku puzzle consists of five overlapping sudoku grids. The standard sudoku rules apply to each 9 × 9 grid. Place digits from 1 to 9 in each empty cell. Every row, every column and every 3 × 3 box should contain one of each digit.

| | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 9 | | 2 | | | 8 | | 4 | | | | 9 | 8 | 6 | | | 2 | | | 4 |
| | 2 | | 8 | 4 | 6 | | 9 | | | | | | | | 9 | 8 | 4 | 6 | | |
| 4 | | 8 | | | | 6 | | | | | | 7 | 2 | 4 | | | | 8 | 9 | |
| | 6 | 2 | | | 8 | 4 | | | | | | | | 2 | 8 | | 9 | 4 | | |
| | | | | | 4 | | 8 | | | | | 4 | 9 | | 2 | 5 | | | 8 | 6 |
| 8 | 4 | | | 6 | 9 | 2 | | | | | | | | | 4 | 6 | | 9 | | 2 |
| | | 9 | 6 | 8 | | 5 | 4 | 7 | | | | 2 | 6 | 8 | | 4 | | | | |
| | | 4 | | 9 | | 1 | 6 | 8 | 5 | 2 | 4 | 3 | 7 | 9 | | | | | 4 | 8 |
| 6 | 8 | | 4 | 1 | | 3 | 2 | 9 | 8 | | | 1 | 4 | 5 | | | 8 | 2 | | |
| | | | | | | 8 | | 6 | 2 | | | | | 4 | | | | | | |
| | | | | | | 4 | 9 | 2 | | 8 | | | | | | | | | | |
| | | | | | | | | | | 4 | | | 2 | | | | | | | |
| 6 | | | | | | 2 | 8 | 4 | | | 9 | 7 | 1 | 3 | | | 4 | 8 | | |
| | 4 | 2 | 8 | | | 9 | 7 | 3 | | | 2 | 5 | 8 | 6 | | 2 | | 9 | 4 | |
| | 8 | | | | 4 | 6 | | | | | | 4 | 9 | 2 | | | | | | |
| | 6 | | 4 | 8 | | | | | | | | | | | | 9 | 8 | | | 4 |
| 8 | | | | | | 4 | | | | | | | | | | 4 | | | | |
| 2 | 1 | 4 | | | | | 6 | 8 | | | | 1 | 4 | | | | | | | 8 |
| 4 | | | | 2 | | | | | | | | 9 | | | 8 | 6 | | 4 | | |
| | | | | 4 | | 8 | | 2 | | | | 8 | 2 | | 4 | | 9 | 6 | 5 | |
| | 2 | | | | 8 | | 4 | 6 | | | | | | 4 | 2 | | | | 8 | |

*Readers can send their responses at editor@mtg.in or post us with complete address. Winners' name with their valuable feedback will be published in next issue.*

$$x=\sqrt{x+2\sqrt{x+2\sqrt{x+.......}}}$$

$$=\lim_{n\to\infty}\sqrt{x+2\sqrt{x+2\sqrt{x+........+2\sqrt{x+2x}}}}$$

It follows that $x=\sqrt{x+2\sqrt{x+2\sqrt{x+.........}}}$

$$=\sqrt{x+2(\sqrt{x+2\sqrt{x+.........}})}=\sqrt{x+2x}$$

$$\Rightarrow\ x=\sqrt{x+2x}\Rightarrow x^2=3x\Rightarrow x(x-3)=0$$

$\Rightarrow\ x=0$ or $x=3$ $\quad\therefore$ Sum of roots = 3

**16. (8) :** $\alpha=a+ib\ \Rightarrow\ 27a+(51b)i=e^{i\frac{(2k+1)\pi}{4}}$

Solutions : $a=\pm\frac{1}{27\sqrt{2}},\ b=\pm\frac{1}{51\sqrt{2}}$

$\therefore$ Required value $=\left[\frac{27\sqrt{2}+51\sqrt{2}}{13}\right]=8$

**17. (2) :** Given,

$$\frac{\sin\alpha+\sin\beta+\sin\gamma}{\sin(\alpha+\beta+\gamma)}=\frac{\cos\alpha+\cos\beta+\cos\gamma}{\cos(\alpha+\beta+\gamma)}$$

$$=\frac{\sin(\alpha+\beta+\gamma)(\sin\alpha+\sin\beta+\sin\gamma)+\cos(\alpha+\beta+\gamma)(\cos\alpha+\cos\beta+\cos\gamma)}{\sin(\alpha+\beta+\gamma)\sin(\alpha+\beta+\gamma)+\cos(\alpha+\beta+\gamma)\cos(\alpha+\beta+\gamma)}$$

$\left(\because\ \frac{a}{b}=\frac{c}{d}=\frac{xa+yc}{xb+yd}\right)$ (by ratio and proportion)

$$=\frac{\cos(\alpha+\beta)+\cos(\beta+\gamma)+\cos(\gamma+\alpha)}{1}=2\ \Rightarrow\ a=2$$

**(18 - 20) :**

Let $x^4+ax^3+bx^2+cx+d$

$=(x-x_1)(x-x_2)(x-x_3)(x-x_4)$

Let $(x-x_1)(x-x_2)=x^2+px+q$

and $(x-x_3)(x-x_4)=x^2+px+r$

$\therefore\ q=x_1x_2$ and $r=x_3x_4$

$\therefore\ x^4+ax^3+bx^2+cx+d$

$=x^4+2px^3+(p^2+q+r)x^2+p(q+r)x+qr$

$\therefore\ a=2p,\ b=p^2+q+r,\ c=p(q+r),\ d=qr$

Clearly, $a^3-4ab+8c=0$ ....(i)

**18. (b) :** If $a=2\Rightarrow b-c=1$

**19. (c) :** Investigating the nature of the cubic equation of '$a$'.

Let $f(a)=a^3-4ab+8c$

$f'(a)=3a^2-4b$

If $b<0\ \Rightarrow\ f'(a)>0$

$\therefore$ The equation $a^3-4ab+8c=0$ hence only one real root.

**20. (a) :** Substituting $c=1-b$ in (i), we have

$(a+2)[(a-1)^2+3-4b]=0\ \Rightarrow\ 4b-3>0$

$$\Rightarrow\ b>\frac{3}{4}\ \Rightarrow\ c<\frac{1}{4}$$

**(21 - 22) :**

**21. (c) :** For $x\geq 4$, the last digit of $1!+2!+...+x!$ is 3

For $x<4$, the given equation has only solutions $x=1$, $K=\pm1$ and $x=3$, $K=\pm3$; $\alpha=1$, $\beta=3$

$$(a+\sqrt{b})^{x^2-15}+(a-\sqrt{b})^{x^2-15}=2a$$

$$\therefore\ x^2-15=\pm1\ \Rightarrow\ x=\pm4,\pm\sqrt{14}$$

$$\alpha_1=-4,\ \alpha_2=4,\alpha_3=-\sqrt{14},\ \alpha_4=\sqrt{14}$$

$$\therefore\ |\alpha_1+\alpha_2+\alpha_3+\alpha_4-\alpha_1\alpha_2\alpha_3\alpha_4|=|0-16\times14|=224$$

**22. (d) :** $a^{\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+....}=a$

$\sqrt{x\sqrt{x\sqrt{x....\infty}}}=x$ and $\sqrt{49+20\sqrt{6}}=5+2\sqrt{6}$

$x^2-3>0$ and $x>0\ \Rightarrow\ x>\sqrt{3}$

$$(5+2\sqrt{6})^{\sqrt{a\sqrt{a\sqrt{a....\infty}}}}+(5-2\sqrt{6})^{x^2+x-3-\sqrt{x\sqrt{x\sqrt{x...\infty}}}}=10$$

$$\Rightarrow\ (5+2\sqrt{6})^{x^2-3}+(5-2\sqrt{6})^{x^2-3}=10$$

$$\therefore\ x^2-3=1\ \Rightarrow\ x=2\ (\because\ x>\sqrt{3})$$

**(23 - 25) :**

Given, $1<\alpha<2$ ; $1<\beta<2$

$\Rightarrow\ 0<\alpha-1<1$ and $0<(\beta-1)<1$

Similarly, $0<2-\alpha<1$ and $0<2-\beta<1$

Apply AM $\geq$ GM for $\alpha-1$ and $2-\alpha$

$$\frac{\alpha-1+2-\alpha}{2}\geq\sqrt{(\alpha-1)(2-\alpha)}$$

$$\Rightarrow\ (\alpha-1)(2-\alpha)\leq\frac{1}{4}$$

Similarly, $(\beta-1)(2-\beta)\leq\frac{1}{4}$

$$\therefore\ 0\leq(\alpha-1)(\beta-1)(2-\alpha)(2-\beta)\leq\frac{1}{16}$$

$$\Rightarrow\ (\alpha-1)(\beta-1)(2-\alpha)(2-\beta)\in\left[0,\ \frac{1}{16}\right]$$

**23. (c)** **24. (a)** **25. (d)**

# 10 BEST PROBLEMS for JEE Advanced

**1.** Let $f(x)=\lim\limits_{m\to 0}\frac{1}{m^4}\cdot\int\limits_0^m \frac{(e^{x+t}-e^x)(\log(1+t))^2}{3+2t^3}\,dt$

then $f(\log 3)=$

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) $\frac{1}{12}$

**2.** Let $f(x)=\begin{cases} e^{\{x^2\}}-1, & x>0\\ 0, & x=0\\ \dfrac{\sin x-\tan x+\cos x-1}{2x^2+\tan x+\log(x+2)}, & x<0\end{cases}$

Lines $L_1$ and $L_2$ represent tangent and normal to curve $y = f(x)$ at $x = 0$. Consider the family of circles touching both lines $L_1$ and $L_2$. Then the ratio of the radii of two orthogonal circles of this family is

(a) $2+\sqrt{2}$ (b) $2+\sqrt{3}$

(c) $2-\sqrt{2}$ (d) $2-\sqrt{6}$

**3.** There are four seats numbered 1, 2, 3, 4 in a room and four persons having tickets corresponding to these seats (one person having one ticket). Now the person having the ticket number 1, enters into the room and sits on any of the seat at random. Then the person having the ticket number 2, enters in room. If his seat is empty then he sits on his seat otherwise he sits on any of the empty seat at random. Similarly the other persons sit. Probability that the person having ticket numbered 4 gets the seat number 4 is

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{8}$ (d) $\frac{1}{16}$

**4.** Let $f(x)=\log\left(\frac{x^2+e}{x^2+1}\right)$ and

$g(x)=\sqrt{\sin f(x)}+\sqrt{\cos f(x)}$, then range of $g(x)$ is

(a) $(1, 2^{3/4}]$ (b) $(2^{1/2}, 2^{3/4}]$

(c) $(\pi^{1/2}, \pi^{3/4})$ (d) $(e^{1/2}, \pi^{1/2}]$

**5.** The number of subsets with three elements that can be formed from the set {1, 2, ...., 20} so that 4 is a factor of the product of the three numbers in the subset is

(a) 700 (b) 795 (c) 800 (d) 825

**6.** If $z$ be a complex number satisfying, $|z|^2 + 2(z + \bar{z}) + 3i(z - \bar{z}) + 4 = 0$ then complex number $z + 3 + 2i$ will lie on

(a) circle with centre $1 - 5i$ and radius 4

(b) circle with centre $1 + 5i$ and radius 4

(c) circle with centre $1 + 5i$ and radius 3

(d) circle with centre $1 - 5i$ and radius 3

**7.** Given three vectors $\vec{a}, \vec{b}, \vec{c}$.

Define $\vec{u}=(\vec{b}\cdot\vec{c})\vec{a}-(\vec{c}\cdot\vec{a})\vec{b},\ \vec{v}=(\vec{a}\cdot\vec{c})\vec{b}-(\vec{a}\cdot\vec{b})\vec{c}$, $\vec{w}=(\vec{b}\cdot\vec{a})\vec{c}-(\vec{b}\cdot\vec{c})\vec{a}$. If $\vec{a}, \vec{b}, \vec{c}$ form a triangle, then

(a) $\vec{u}, \vec{v}, \vec{w}$ also form a similar triangle with $\vec{a}, \vec{b}, \vec{c}$.

(b) $\vec{u}, \vec{v}, \vec{w}$ also form a congruent triangle with $\vec{a}, \vec{b}, \vec{c}$.

(c) $\vec{u}, \vec{v}, \vec{w}$ forms an isosceles triangle.

(d) $\vec{u}, \vec{v}, \vec{w}$ does not form a triangle.

**8.** For positive integers $n$ and $k$, we have

$\frac{\left({}^nC_{n-1}\right)^6+\left({}^{n-2}C_k\right)^6+\left({}^{n+3}C_{n+1}\right)^3}{3\left({}^{n-2}C_k\right)^2\left({}^{n+3}C_2\right)}=n^2$, then $n+k=$

(a) 8 (b) 0 (c) 16 (d) 60

**9.** $f:\left(0,\frac{\pi}{2}\right)\to A,\ f(x)=\log_e(\sin x^{\sin x}+1)$,

then the minimum value of $f(x)$ is

(a) $\log_e 2$ (b) $\log_e\left(\left(\frac{1}{e}\right)^{1/e}+1\right)$

(c) $\log_e((e)^2 + 1)$ (d) 2

**10.** $a$, $b$ are complex numbers on a unit circle centered at origin with $a \neq -b$, then $\text{Im}\left[\frac{4ab}{(a+b)^2}\right]=$

(a) 0 (b) 1 (c) $-1$ (d) 2

## SOLUTIONS

**1. (c) :** $f(x)=e^x\cdot\lim_{m\to 0}\dfrac{\int_0^m \dfrac{(e^t-1)\cdot(\log(1+t))^2}{2t^3+3}dt}{m^4}\quad\left(\dfrac{0}{0}\right)$

$\Rightarrow f(x)=e^x\cdot\lim_{m\to 0}\dfrac{(e^m-1)\cdot(\log(1+m))^2}{(3+2m^3)\cdot 4m^3}$

$\Rightarrow f(x)=e^x\cdot\lim_{m\to 0}\dfrac{e^m-1}{m}\cdot\left(\dfrac{\log(1+m)}{m}\right)^2\cdot\dfrac{1}{4}\cdot\dfrac{1}{3+2m^3}$

$\Rightarrow f(x)=\dfrac{e^x}{12}$, so, $f(\log 3)=\dfrac{3}{12}=\dfrac{1}{4}$

**2. (b) :** L.H.D. $=\lim_{h\to 0}\left[\dfrac{-\sin h+\tan h+\cos h-1}{2h^2-\tan h+\log(2-h)}\right]\times\dfrac{1}{-h}=0$

and R.H.D. $=\lim_{h\to 0}\dfrac{e^{h^2}-1-0}{h}=0$

Hence, $L_1=y=0$ and $L_2=x=0$

Let $r_1$ and $r_2$ be the radius of two orthogonal circles.

$\Rightarrow r_1^2+r_2^2=d^2=2(r_1-r_2)^2$

$\Rightarrow r_1^2+r_2^2-4r_1r_2=0$

$\Rightarrow \left(\dfrac{r_1}{r_2}\right)^2-4\left(\dfrac{r_1}{r_2}\right)+1=0\Rightarrow\dfrac{r_1}{r_2}=2\pm\sqrt{3}$

**3. (a) : Way 1 :** 1 take 1, 2 take 2, 3 take 3, and 4 take 4

$\therefore\ P(\text{way } 1)=\dfrac{1}{4}\times 1\times 1\times 1=\dfrac{1}{4}$

**Way 2 :** 1 take 2, 2 take 1, 3 take 3, 4 take 4

$\therefore\ P(\text{way } 2)=\dfrac{1}{4}\times\dfrac{1}{3}\times 1\times 1=\dfrac{1}{12}$

**Way 3 :** 1 take 2, 2 take 3, 3 take 1, 4 take 4

$\therefore\ P(\text{way } 3)=\dfrac{1}{4}\times\dfrac{1}{3}\times\dfrac{1}{2}\times 1=\dfrac{1}{24}$

**Way 4 :** 1 take 3, 2 take 2, 3 take 1, 4 take 4

$\therefore\ P(\text{way } 4)=\dfrac{1}{4}\times 1\times\dfrac{1}{2}\times 1=\dfrac{1}{8}$

Required probability $=\dfrac{1}{4}+\dfrac{1}{12}+\dfrac{1}{24}+\dfrac{1}{8}=\dfrac{1}{2}$

**4. (a) :** Notice that $\dfrac{x^2+e}{x^2+1}\in(1,e]$

Hence, $f(x)\in(0,1]$

So, $g(x)=\sqrt{\sin\alpha}+\sqrt{\cos\alpha},\ \alpha\in(0,1]$ where $\alpha$ is $f(x)$

$g'(x)=0$ gives $\alpha=\pi/4$

So, $g(x)\in(1,2^{3/4}]$

**5. (b) :** There are in all ${}^{20}C_3$ subsets. All of these, when multiplied will have 4 as a factor, except in two cases.

**Case (i) :** All elements are odd = ${}^{10}C_3$

**Case (ii) :** 2 elements are odd and 1 element is even but not multiple of 4 = ${}^{10}C_2\times{}^5C_1$

Hence, total number of subsets

$={}^{20}C_3-({}^{10}C_3)-({}^{10}C_2\times{}^5C_1)=795$

**6. (c) :** Given equation is

$|z|^2+z(2+3i)+\bar{z}(2-3i)+4=0$, which represents a circle with centre $-(2-3i)$, radius = 3

Let $\omega=z+3+2i=z+2-3i+1+5i$,

$|\omega-1-5i|=|z+2-3i|=3$

so, $\omega$ lies on circle whose centre is $1+5i$ and radius is 3.

**7. (a) :** Clearly $\vec{u}+\vec{v}+\vec{w}=\vec{0}$.

Hence, $\vec{u},\vec{v},\vec{w}$ form a triangle.

Now, consider $\vec{u}\cdot\vec{c}=(\vec{b}\cdot\vec{c})(\vec{a}\cdot\vec{c})-(\vec{c}\cdot\vec{a})(\vec{b}\cdot\vec{c})=0$

*i.e.*, $\vec{u}$ and $\vec{c}$ are orthogonal. Similarly, $\vec{v}$ is orthogonal to $\vec{a}$ and $\vec{w}$ to $\vec{b}$. Hence the sides of the triangle formed with $\vec{u},\vec{v},\vec{w}$ are perpendicular to the sides of the triangle formed with $\vec{a},\vec{b},\vec{c}$. This shows that the two triangles have equal angles and hence are similar.

**8. (a) :** Applying A.M.-G.M. inequality on $({}^nC_{n-1})^6$, $({}^{n-2}C_k)^6$ and $({}^{n+3}C_{n+1})^3$. We have A.M. = G.M. from the given condition in question.

So, all the given numbers should be same.

*i.e.* $({}^nC_{n-1})^6=({}^{n-2}C_k)^6=({}^{n+3}C_{n+1})^3$

*i.e.* $n^2=({}^{n-2}C_k)^2=({}^{n+3}C_2)$;

$\therefore\ n=6$ and $k=2$

Hence, $n+k=6+2=8$

**9. (b) :** Let $h(x)=\log(x^x+1)$, for $0<x<1$. Then, $h'(x)>0$ if $x>\dfrac{1}{e}$. Thus, for $\dfrac{1}{e}<x<1$, function will be increasing. Also, $h'(x)<0$, if $x<\dfrac{1}{e}$. So, at $x=\dfrac{1}{e}$, there is minimum.

$f_{\min}=\log_e\left(\left(\dfrac{1}{e}\right)^{1/e}+1\right)$

**10. (a) :** $|a|=|b|=1$. Let $t=\dfrac{a}{b}\Rightarrow|t|=1\Rightarrow\bar{t}=\dfrac{1}{t}$

Also, let $z=\dfrac{4ab}{(a+b)^2}\ \Rightarrow\ \dfrac{4}{z}=\dfrac{a}{b}+\dfrac{b}{a}+2$

$\because\ \dfrac{a}{b}+\dfrac{b}{a}+2$ is purely real $\therefore\ \dfrac{4}{z}$ is also purely real

Hence, $\text{Im}[z]=0$


# CBSE Warm-up!

CLASS-XI

*Chapterwise practice questions for CBSE Exams as per the latest pattern and syllabus by CBSE for the academic session 2024-25.*

## Series-4 Linear Inequalities / Permutations and Combinations

***General Instructions : Read the following instructions very carefully and strictly follow them :***

(i) This question paper contains **38** questions. **All** questions are **compulsory.**

(ii) This question paper is divided into **five** Sections – **A, B, C, D** and **E.**

(iii) In **Section A,** Questions no. **1** to **18** are multiple choice questions (MCQs) and Questions no. **19 and 20** are Assertion-Reason based questions of **1** mark each.

(iv) In **Section B,** Questions no. **21** to **25** are very short answer (VSA) type questions, carrying **2** marks each.

(v) In **Section C,** Questions no. **26** to **31** are short answer (SA) type questions, carrying **3** marks each.

(vi) In **Section D,** Questions no. **32** to **35** are long answer (LA) type questions, carrying **5** marks each.

(vii) In **Section E,** Questions no. **36** to **38** are case study based questions, carrying **4** marks each.

(viii) There is no overall choice. However, an internal choice has been provided in **2** questions in Section **B, 3** questions in Section **C, 2** questions in Section **D** and **2** questions in Section E.

(ix) Use of calculators is **not** allowed.

**Time Allowed :** 3 hours **Maximum Marks :** 80

### SECTION - A

**This section comprises multiple choice questions (MCQs)**

**1.** Solve : $37 - (3x + 5) \geq 9x - 8(x - 3)$

(a) $(-\infty, 2)$ (b) $(-\infty, 2]$
(c) $(-\infty, -2]$ (d) $[2, \infty)$

**2.** The values of $x$ for which $-11 \leq 4x - 3 \leq 13$ is

(a) $-4 \leq x \leq 5$ (b) $-2 \leq x \leq 4$
(c) $-8 \leq x \leq 16$ (d) $-11 \leq x \leq 10$

**3.** Solve : $\frac{3x-4}{2} \leq \frac{x+1}{3} - 1$

(a) $(\infty, 8/7)$ (b) $x \leq 8/7$
(c) $(-\infty, 8/7]$ (d) Both (b) and (c)

**4.** Solve : $x - 3 \geq 4 - \frac{7x}{2}$ and $4 - \frac{7x}{2} \geq 18$

(a) $(-\infty, -4)$ (b) $[14/9, \infty)$
(c) $(-\infty, -4] \cup [14/9, \infty)$
(d) $(-\infty, -4) \cup (14/9, \infty)$

**5.** Solution set of the inequations $2x - 1 \leq 3$ and $3x + 1 \geq -5$ is $[-k, k]$, then $k =$

(a) 1 (b) 2 (c) 3 (d) 4

**6.** If $|3x - 5| \leq 2$, then

(a) $-1 \leq x \leq \frac{7}{3}$ (b) $1 \leq x \leq \frac{9}{3}$
(c) $1 \leq x \leq \frac{7}{3}$ (d) $-1 \leq x \leq \frac{9}{3}$

**7.** Five persons entered the lift cabin on the ground floor of an 8-floor house. Suppose each of them can leave the cabin independently at any floor beginning with the first. The total number of ways in which each of the five persons can leave the cabin at any one of the 7-floor is

(a) $5^7$ (b) 2520 (c) $7^5$ (d) 35

**8.** How many numbers are there between 99 and 1000 having 7 in the unit's place?

(a) 93 (b) 95 (c) 90 (d) 97

9. Find the number of different signals that can be generated by arranging at least 2 flags in order (one below the other) on a vertical staff, if five different flags are available.
(a) 312 (b) 313 (c) 315 (d) 320

10. The number of words that can be formed by using the letters of the word 'MATHEMATICS' that start as well as end with T is
(a) 90720 (b) 28060
(c) 713090 (d) none of these

11. Find the numbers of arrangements of the letters of word SALOON, if the two O's do not come together.
(a) 230 (b) 220 (c) 250 (d) 240

12. Evaluate 4! – 3!.
(a) 18 (b) 10 (c) 20 (d) 30

13. If ${}^nC_r + {}^nC_{r+1} = {}^{n+1}C_x$, then $x =$
(a) $r$ (b) $r-1$ (c) $n$ (d) $r+1$

14. In how many ways can a student choose a programme of 5 courses if 9 courses are available and 2 specific courses are compulsory for every student?
(a) 126 (b) 35 (c) 89 (d) 21

15. Find $n$, ${}^{n-1}P_3 : {}^nP_4 = 1 : 9$.
(a) 7 (b) 8 (c) 9 (d) 10

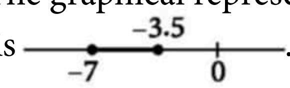
16. There are 8 lamps in a hall. Each one of them can be switched on independently. The number of ways in which the hall can be illuminated is
(a) 8 ! (b) 16 (c) 255 (d) $2^8$

17. The number of numbers greater than 3000, which can be formed by using the digits 0, 1, 2, 3, 4, 5 without repetition, is
(a) 1240 (b) 1280 (c) 1320 (d) 1380

18. The letters of the word COCHIN are permuted and all the permutations are arranged in alphabetical order as in English dictionary. The number of words that appear before the word COCHIN is
(a) 360 (b) 192 (c) 96 (d) 48

**Questions number 19 and 20 are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) options as given below.**

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is **not** the correct explanation of the Assertion (A).
(c) Assertion (A) is true, but Reason (R) is false.
(d) Assertion (A) is false, but Reason (R) is true.

19. **Assertion (A) :** If $-5 \le 2x + 9 \le 2$, then $x \in [-7, -3.5]$.
**Reason (R) :** The graphical representation of $-5 \le 2x + 9 \le 2$ is 

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20. **Assertion (A) :** If a license plate contains three letters of English alphabet followed by any three digits, then the total number of different car licence plates is $(26)^3 \times 900$ (if repetitions are allowed).
**Reason (R) :** The number of permutations of $n$ different things taken $r$ at a time when each things may be repeated any number of times is $n^r$.

## SECTION - B

**This section comprises very short answer (VSA) type questions of 2 marks each.**

21. Solve the linear inequality
$$\frac{x}{4} > \frac{5x-2}{3} - \frac{7x-3}{5},\ x \in R.$$

22. Solve the inequality $\frac{x}{3} - \frac{x-2}{4} > \frac{x-1}{5}$, where $x$ belongs to $R$.

**OR**

Solve the following inequation :
$$\frac{2x-3}{4} + 19 \ge 13 + \frac{4x}{3}$$

23. For a set of five true or-false questions, no student has written all the correct answers, and no two students have given the same sequence of answers. What is the maximum number of students in the class, for this to be possible?

**OR**

How many words (with or without meaning) of three distinct English alphabets are there?

24. If ${}^nC_{r-1} : {}^nC_r : {}^nC_{r+1} = 2 : 3 : 4$, then find $(n, r)$.

25. If ${}^nC_2 - {}^nC_1 = 35$, then find the value of $n$.

## SECTION - C

**This section comprises short answer (SA) type questions of 3 marks each.**

26. Find the pairs of consecutive even positive integers which are larger than 5 and are such that their sum is less than 20.

**27.** Solve and represent the solution on number line :

$$\frac{5x+1}{3}-(3x+4)\leq 4x+7,$$

$$\frac{x+5}{2}+\frac{x-3}{3}\leq\frac{7x+4}{2}-\frac{x+7}{3}.$$

**OR**

Solve the following system of inequations :

$$\frac{5x}{4}+\frac{3x}{8}>\frac{39}{8},\ \frac{2x-1}{12}-\frac{x-1}{3}<\frac{3x+1}{4}$$

**28.** How many words can be made by using all letters of the word "MATHEMATICS" in which all vowels are never together?

**OR**

If all the letters of the word 'MOTHER' are written in all possible orders and the words so formed are arranged in a dictionary order, then find the rank of the word 'MOTHER'.

**29.** If ${}^nP_r = {}^nP_{r+1}$ and ${}^nC_r = {}^nC_{r-1}$, find the values of $n$ and $r$.

**OR**

There are 15 points in a plane out of which only 6 are in a straight line.

(i) How many different straight lines can be made?

(ii) How many triangles can be made?

**30.** In a school there are 15 teachers (including physical education teacher) and 5 captains of different games. The Principal wants to form a game committee consisting of physical education teacher, 2 other teachers and 2 captains. In how many ways can he do it?

**31.** A boy has 3 library tickets and 8 books of his interest in the library. Of these 8 books he does not want to borrow mathematics part II unless mathematics part I is also borrowed. In how many ways can he choose the three books to be borrowed?

## SECTION - D

**This section comprises long answer (LA) type questions of 5 marks each.**

**32.** How many litres of water will have to be added to 1125 litres of the 45% solution of acid so that resulting mixture will contain more than 25% but less than 30% acid content?

**33.** (i) How many different words can be formed with the letters of the word HARYANA?

(ii) How many of these begin with H and end with N?

(iii) In how many of these H and N are together?

**OR**

In how many ways three girls and nine boys can be seated in two vans, each having numbered seats, 3 in the front and 4 at the back? How many seating arrangements are possible if 3 girls sit together in a back row on adjacent seats?

**34.** If ${}^nC_r : {}^nC_{r+1} = 1:2$ and ${}^nC_{r+1} : {}^nC_{r+2} = 2:3$, determine the values of $n$ and $r$.

**OR**

From a class of 12 boys and 10 girls, 10 students are to be chosen for a competition, including atleast 4 boys and 4 girls. The two girls who won the prizes last year should be included. In how many ways can the selection be made?

**35.** To receive Grade '$A$' in the course, one must obtain an average of 90 marks or more in five examinations (each of 100 marks). If Sunita's marks in first four examinations are 87, 92, 94 and 95, find minimum marks that Sunita must obtain in fifth examination to get grade '$A$' in the course.

## SECTION - E

### Case Study 1

**36.** A company produces certain items. The manager in the company maintain record about cost and revenue of these items on daily basis. The cost and revenue functions of product are given by $C(x) = 15x + 3000$ and $R(x) = 45x + 1500$ respectively, where $x$ is the number of items produced and sold.

On the basis of the above information, answer the following questions :

(i) How many items must be sold to gain some profit?

(ii) How many items must be produced, so that the cost is less than 4500 rupees?

(iii) If number of items produced lies between 200 to 250, then what will be the cost?

**OR**

If number of items sold lies between 300 to 400, then what will be the revenue?

### Case Study 2

**37.** On sunday, Riyan and Ravi decided to play with cards. Riyan found a deck of cards at his home but he observed that 1 card is missing in 52 cards.

After checking on cards he found that the missing card is an ace of spade. After that they start playing with the remaining cards and Riyan asked Ravi to choose 4 cards from the 51 cards.

On the basis of the above information, answer the following questions :

(i) In how many ways Ravi can select all the 4 cards from same suit?

(ii) Find number of ways of selecting two red cards and two black cards.

**Case Study 3**

**38.** Raveena, a class 12 student invited her friends Bharti, Ravi, Aarush and Ekta for her birthday party. After cutting cake they all want to take a group photograph sitting in a row.

On the basis of the above information, answer the following questions :

(i) Find the number of distinct photographs that can be clicked.

(ii) In how many of the photographs Raveena be sitting in middle?

(iii) If Ravi and Ekta were not sitting together, then how many photographs can be taken?

**OR**

If Ekta can sit on any place except the middle one, then how many distinct photographs are possible?

## SOLUTIONS

**1. (b) :** The inequality is $37 - (3x + 5) \geq 9x - 8(x - 3)$
On simplifying, we get $37 - 3x - 5 \geq 9x - 8x + 24$
or $32 - 3x \geq x + 24$
$\Rightarrow -3x - x \geq 24 - 32 \Rightarrow -4x \geq -8$
Dividing both sides by $-4$, we get $x \leq 2$
$\therefore$ The solution is $(-\infty, 2]$.

**2. (b) :** We have, $-11 \leq 4x - 3 \leq 13 \Rightarrow -8 \leq 4x \leq 16$
$\Rightarrow -2 \leq x \leq 4$

**3. (d) :** We have, $x\left(\frac{3}{2} - \frac{1}{3}\right) \leq \frac{1}{3} - 1 + \frac{4}{2}$
$\Rightarrow \frac{7x}{6} \leq \frac{4}{3} \Rightarrow x \leq \frac{8}{7}$

**4. (c) :** We have, $x - 3 \geq 4 - \frac{7x}{2} \Rightarrow \left(1 + \frac{7}{2}\right)x \geq 4 + 3$
$\Rightarrow \frac{9x}{2} \geq 7 \Rightarrow x \geq \frac{14}{9}$ ...(i)
and $4 - \frac{7x}{2} \geq 18 \Rightarrow -\frac{7x}{2} \geq 18 - 4 = 14$
$\Rightarrow x \leq -14 \times \frac{2}{7} = -4$ ...(ii)
From (i) and (ii), we get $\Rightarrow x \in (-\infty, -4] \cup \left[\frac{14}{9}, \infty\right)$

**5. (b) :** We have, $2x - 1 \leq 3 \Rightarrow 2x \leq 4$
$\Rightarrow x \leq 2$ and $3x + 1 \geq -5 \Rightarrow 3x \geq -6 \Rightarrow x \geq -2$
Hence, $x \in [-2, 2]$
$\therefore k = 2$

**6. (c) :** Given, $|3x - 5| \leq 2$
$\Rightarrow -2 \leq 3x - 5 \leq 2 \Rightarrow 3 \leq 3x \leq 7 \Rightarrow 1 \leq x \leq \frac{7}{3}$

**7. (c) :** Total number of ways $= 7 \times 7 \times 7 \times 7 \times 7 = 7^5$

**8. (c) :** Number between 99 to 1000 having 7 in unit's place $= 10 + 10 + 10 + 10 + 10 + 10 + 10 + 10 + 10 = 90$

**9. (d) :** For 2 flags, $5 \times 4 = 20$
For 3 flags, number of signals $= 5 \times 4 \times 3 = 60$
For 4 flags, number of signals $= 5 \times 4 \times 3 \times 2 = 120$
For 5 flags, number of signals $= 5 \times 4 \times 3 \times 2 \times 1 = 120$
Thus, total number of signals $= 20 + 60 + 120 + 120 = 320$

**10. (a) :** T_ _ _ _ _ _ _ _ _T
In the word MATHEMATICS, there are total of 11 letters, out of which there are 2T, 2A, 2M and one each of H, E, I C, S.
$\therefore$ Required number of words formed $= \frac{9!}{2! \times 2!} = 90720$

**11. (d) :** Total number of arrangements are $\frac{6!}{2!} = 360$
The number of ways in which two O's come together $= 5! = 120$
Hence, required number of ways $= 360 - 120 = 240$

**12. (a) :** We have, $4! - 3! = (4 \times 3 \times 2 \times 1) - (3 \times 2 \times 1)$
$= 24 - 6 = 18$

**13. (d) :** We have, ${}^nC_r + {}^nC_{r+1} = {}^{n+1}C_x$

$\Rightarrow {}^{n+1}C_{r+1} = {}^{n+1}C_x \quad \Rightarrow n + 1 = x + r + 1$

or $r + 1 = x \Rightarrow x = n - r$ or $x = r + 1$

**14. (b) :** Total number of courses = 9
Number of compulsory courses = 2
So, the student will choose 3 courses out of 7 courses [non-compulsory courses].
∴ Required number of ways

$= {}^7C_3 = \frac{7!}{3!4!} = \frac{7 \times 6 \times 5}{6} = 35$

**15. (c) :** Given, ${}^{n-1}P_3 : {}^nP_4 = 1 : 9$

$\Rightarrow \frac{(n-1)!}{(n-4)!} \times \frac{(n-4)!}{n!} = \frac{1}{9}$

$\Rightarrow \frac{(n-1)!}{n!} = \frac{1}{9} \Rightarrow \frac{(n-1)!}{n(n-1)!} = \frac{1}{9} \quad \Rightarrow n = 9$

**16. (c) :** Each of the 8 switches can be handled in 2 ways *i.e.* either switched on or switched off. Hence, total number of ways of handling the 8 switches is $2^8$ which includes the case of all the 8 lamps are off.
∴ Number of ways of illuminating the room is $2^8 - 1 = 255$

**17. (d) :** Number of 4 digit numbers = $3 \cdot 5 \cdot 4 \cdot 3 = 180$
Number of 5 digit numbers = $5 \cdot 5 \cdot 4 \cdot 3 \cdot 2 = 600$
Number of 6 digit numbers = $5 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 600$
∴ Required number of ways = 180 + 600 + 600 = 1380

**18. (c) :** Number of words starting with CC is 4!, CH is 4!, CI is 4! and CN is 4!
The next word is COCHIN.
There are 4(4!) = 96 words before COCHIN.

**19. (a) :** We have, $-5 \le 2x + 9 \le 2 \quad \Rightarrow -14 \le 2x \le -7$

$\Rightarrow -7 \le x \le \frac{-7}{2} \quad \therefore x \in \left[-7, \frac{-7}{2}\right]$

**20. (d) :** Total letters = 26 (*i.e., A, B, C,..., Y, Z*) and total digits = 10 (*i.e.,* 0, 1, 2, 3, ..., 9)
∵ Repetition of letters is allowed.
∴ The three space for letters can be filled in $26 \times 26 \times 26 = (26)^3$ ways and three digit numbers on plate in 999 ways. (*i.e.,* 001, 002, ..., 999)
∴ Required number of ways = $(26)^3 \times 999$

**21.** We have, $\frac{x}{4} > \frac{5x-2}{3} - \frac{7x-3}{5}$

$\Rightarrow \frac{x}{4} > \frac{25x - 10 - 21x + 9}{15}$

$\Rightarrow \frac{x}{4} > \frac{4x-1}{15} \quad \Rightarrow 15x > 16x - 4 \Rightarrow x < 4$

$\therefore x \in (-\infty, 4)$

**22.** We have, $\frac{x}{3} - \frac{x-2}{4} > \frac{x-1}{5}$

$\Rightarrow \frac{4x - 3x + 6}{12} > \frac{x-1}{5} \Rightarrow \frac{x+6}{12} > \frac{x-1}{5}$

$\Rightarrow 5x + 30 > 12x - 12 \Rightarrow 7x < 42 \Rightarrow x < 6$

$\therefore x \in (-\infty, 6)$

**OR**

We have, $\frac{2x-3}{4} + 19 \ge 13 + \frac{4x}{3}$

$\Rightarrow \frac{2x-3}{4} - \frac{4x}{3} \ge 13 - 19 \Rightarrow \frac{6x - 9 - 16x}{12} \ge -6$

$\Rightarrow -10x - 9 \ge -72 \quad \Rightarrow -10x \ge -72 + 9$

$\Rightarrow -10x \ge -63 \quad \Rightarrow x \le \frac{63}{10}$

Thus, solution set of given inequation is $\left(-\infty, \frac{63}{10}\right]$.

**23.** Since questions I, II, III and IV can be answered in 2 ways.
Hence, total number of possible different answers
$= 2 \times 2 \times 2 \times 2 \times 2 = 32$
There is only one sequence of all correct answers. Thus, the total number of sequences are 32 – 1 = 31. [Since no student has written all correct answers]
Now, as no two students have given the same sequence of answers, hence the maximum number of students in the class = 31.

**OR**

There are 26 distinct English alphabets.
First alphabet can be chosen in 26 ways.
Second alphabet can be chosen in 25 ways.
Third alphabet can be chosen in 24 ways.
∴ Total number of three letter words = $26 \times 25 \times 24$
= 15600

**24.** We have, $\frac{{}^nC_{r-1}}{{}^nC_r} = \frac{2}{3}$

$\Rightarrow \frac{r}{n-r+1} = \frac{2}{3} \quad \Rightarrow 3r = 2n - 2r + 2$

$\Rightarrow 2n - 5r + 2 = 0$ ...(i)

Also, $\frac{{}^nC_r}{{}^nC_{r+1}} = \frac{3}{4} \quad \Rightarrow \frac{r+1}{n-r} = \frac{3}{4} \quad \Rightarrow 4r + 4 = 3n - 3r$

$\Rightarrow 3n - 7r - 4 = 0$ ...(ii)

Solving (i) and (ii), we get $r = 14$ and $n = 34$
$\therefore (n, r) = (34, 14)$

**25.** Here, ${}^nC_2 - {}^nC_1 = 35$

$\Rightarrow \dfrac{n(n-1)}{2} - n = 35 \quad \Rightarrow \quad n^2 - n - 2n = 70$

$\Rightarrow n^2 - 3n - 70 = 0 \quad \Rightarrow \quad (n - 10)(n + 7) = 0$

$\Rightarrow n = 10$ or $n = -7$

$\Rightarrow n = 10$ ($\because$ $n$ cannot be negative)

**26.** Let the consecutive even positive integers be $x$ and $x + 2$.

$\therefore$ $x > 5$ and $x + 2 > 5$

$\Rightarrow x > 5$ and $x > 3 \quad \Rightarrow \quad x > 5$ ...(i)

Also, $x + (x + 2) < 20$

$\Rightarrow 2x < 20 - 2 \quad \Rightarrow 2x < 18 \quad \Rightarrow x < 9$ ...(ii)

From (i) and (ii), we get $5 < x < 9$

$\Rightarrow x = 6, 7, 8$

But $x$ is even positive integer

$\therefore$ $x = 6$ and 8

Thus, two pairs of even positive integers are 6, 8 and 8, 10.

**27.** We have, $\dfrac{5x+1}{3} - (3x+4) \le 4x+7$

$\Rightarrow 5x + 1 - 9x - 12 \le 12x + 21 \quad \Rightarrow -4x - 11 \le 12x + 21$

$\Rightarrow 16x \ge -32 \quad \Rightarrow x \ge -2$ ...(i)

Also, $\dfrac{x+5}{2} + \dfrac{x-3}{3} \le \dfrac{7x+4}{2} - \dfrac{x+7}{3}$

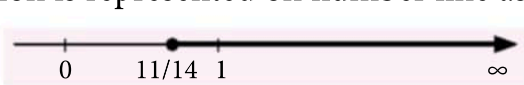
$\Rightarrow 3x + 15 + 2x - 6 \le 21x + 12 - 2x - 14$

$\Rightarrow 5x + 9 \le 19x - 2$

$\Rightarrow 14x \ge 11 \quad \Rightarrow x \ge \dfrac{11}{14}$ ...(ii)

From (i) and (ii), we get $x \ge \dfrac{11}{14}$

The solution is represented on number line as follows :




**OR**

We have, $\dfrac{5x}{4} + \dfrac{3x}{8} > \dfrac{39}{8} \quad \Rightarrow \quad \dfrac{10x+3x}{8} > \dfrac{39}{8}$

$\Rightarrow 13x > 39 \quad \Rightarrow x > 3 \quad \Rightarrow x \in (3, \infty)$ ....(i)

Also, $\dfrac{2x-1}{12} - \dfrac{x-1}{3} < \dfrac{3x+1}{4}$

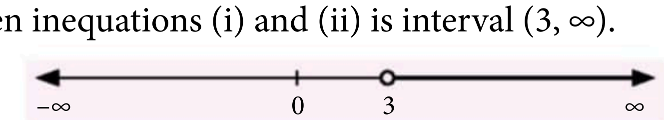
$\Rightarrow \dfrac{(2x-1)-4(x-1)}{12} < \dfrac{3x+1}{4}$

$\Rightarrow -2x + 3 < 3(3x + 1) \quad \Rightarrow \quad -2x + 3 < 9x + 3$

$\Rightarrow -2x - 9x < 3 - 3 \quad \Rightarrow \quad -11x < 0 \quad \Rightarrow \quad x > 0$

$\Rightarrow x \in (0, \infty)$ ....(ii)

We observe that the intersection of the solution sets of given inequations (i) and (ii) is interval $(3, \infty)$.




**28.** In given word, 'MATHEMATICS' there are 2M's, 2A's, 2T's and rest all are different.

So, total arrangements for the letters of the word

'MATHEMATICS' $= \dfrac{11!}{2!\,2!\,2!} = 4989600$

$\therefore$ Total arrangements when all the vowels (2A's, 1E and 1I) are together $= \dfrac{8!}{2!\,2!} \times \dfrac{4!}{2!}$

$= \dfrac{8\times7\times6\times5\times4\times3}{2\times1} \times \dfrac{4\times3}{1} = 120960$

Hence, total arrangements when all the vowels are not together = 4989600 – 120960 = 4868640

**OR**

There are 6 letters in the word 'MOTHER'

Hence, total no. of words starting with E = 5!

Total no. of words starting with H = 5!

Total no. of words starting with ME = 4!

Total no. of words starting with MH = 4!

Total no. of words starting with MOE = 3!

Total no. of words starting with MOH = 3!

Total no. of words starting with MOR = 3!

Total no. of words starting with MOTE = 2!

Thus, next word formed is MOTHER

Total no. of words before MOTHER

$= 2 \times 5! + 2 \times 4! + 3 \times 3! + 2! = 308$

$\therefore$ Rank of the word 'MOTHER' = 308 + 1 = $309^{\text{th}}$

**29.** Here, ${}^nP_r = {}^nP_{r+1}$

$\Rightarrow \dfrac{n!}{(n-r)!} = \dfrac{n!}{(n-r-1)!}$

$\Rightarrow \dfrac{1}{n-r} = 1 \quad \Rightarrow n - r = 1$ ...(i)

Also, ${}^nC_r = {}^nC_{r-1}$

$\Rightarrow \dfrac{n!}{(n-r)!\,r!} = \dfrac{n!}{(n-r+1)!(r-1)!}$

$\Rightarrow \dfrac{1}{(n-r)!\,r(r-1)!} = \dfrac{1}{(n-r+1)(n-r)!(r-1)!}$

$\Rightarrow \dfrac{1}{r} = \dfrac{1}{n-r+1}$

$\Rightarrow n - 2r = -1$ ...(ii)

From (i) and (ii), we get

$n = 3$ and $r = 2$

**OR**

(i) Number of straight lines formed joining the 15 points, taking 2 at a time = ${}^{15}C_2 = 105$

Number of straight lines formed by joining the 6 points, taking 2 at a time = ${}^6C_2 = 15$

But, 6 collinear points when joined pairwise give only one line

$\therefore$ Required number of straight lines = 105 – 15 + 1 = 91

(ii) Number of triangles formed by joining the points, taking 3 at a time = ${}^{15}C_3 = 455$

Number of triangles formed by joining the 6 points, taken 3 at a time = ${}^{6}C_3 = 20$

But, 6 collinear points cannot form a triangle when taken 3 at a time.

$\therefore$ Required number of triangles = 455 – 20 = 435

**30.** Total number of teachers = 15

Total number of captains = 5

Excluding physical education teacher, 2 other teachers out of 15 and 2 captains out of 5 are to be choosen.

$\therefore$ Required number of ways = ${}^{14}C_2 \times {}^{5}C_2 = 910$

**31.** **Case I :** When mathematics part I is borrowed, then part II is also borrowed. Hence, the number of ways of selecting 1 book from 6 books is ${}^{6}C_1 = 6$

**Case II :** When mathematics part I is not borrowed, then part II is not borrowed. So, he has to select three books out of remaining 6 books.

$\therefore$ Total number of ways of selection is ${}^{6}C_3 = 20$

Hence, total number of ways of selection = 6 + 20 = 26

**32.** Quantity of acid in the given mixture

$= 45\% \text{ of } 1125 = \frac{45}{100} \times 1125 = 506.25$

Let $x$ litres of water be added.

Then, volume of new mixture = $(1125 + x)$ litres

According to question, we have

$25\% \text{ of } (1125 + x) \le 506.25 \le 30\% \text{ of } (1125 + x)$

$\Rightarrow \frac{25}{100}(1125 + x) \le 506.25 \le \frac{30}{100}(1125 + x)$

$\Rightarrow 28125 + 25x \le 50625 \le 33750 + 30x$

$\therefore 28125 + 25x \le 50625$ and $50625 \le 33750 + 30x$

$\Rightarrow 25x \le 22500$ and $30x \ge 16875$

$\Rightarrow x \le 900$ and $x \ge 562.5$

$\therefore 562.5 \le x \le 900$

**33.** (i) There are 7 letters in the word 'HARYANA' out of which 3 are A's and remaining all are different.

So, total number of words $= \frac{7!}{3!} = 840.$

(ii) After fixing H in first place and N in last place, we have 5 letters out of which three A's are alike.

So, total number of words $= \frac{5!}{3!} = 20.$

(iii) Considering H and N together we have, 7–2 + 1 = 6 letters out of which three A's are alike. These six letters can be arranged in $\frac{6!}{3!}$ ways. But H and N can be arranged among themselves in 2! ways.

$\therefore$ Required number of words $= \frac{6!}{3!} \times 2! = 240$

**OR**

Total number of persons = 3 girls + 9 boys = 12

Total number of numbered seats = 2 × 3 + 4 × 2 = 14

So, total number of ways in which 12 persons can be seated on 14 seats = Number of arrangements of 14 seats by taking 12 at a time = ${}^{14}P_{12}$.

Three girls can be seated together in a back row on adjacent seats.

So, the total number of ways in which three girls can be seated together in a back row on adjacent seats = 4 × 3!

Now, 9 boys are to be seated on remaining 11 seats, which can be done in ${}^{11}P_9$ ways.

Hence, by the fundamental principle of counting, the total number of seating arrangements in ${}^{11}P_9 \times 4 \times 3!$.

**34.** We have, ${}^{n}C_r : {}^{n}C_{r+1} = 1 : 2$ and ${}^{n}C_{r+1} : {}^{n}C_{r+2} = 2 : 3$

Now, $\frac{{}^{n}C_r}{{}^{n}C_{r+1}} = \frac{1}{2} \Rightarrow \frac{\frac{n!}{r!(n-r)!}}{\frac{n!}{(r+1)!(n-r-1)!}} = \frac{1}{2}$

$\Rightarrow \frac{(r+1)!(n-r-1)!}{r!(n-r)!} = \frac{1}{2} \Rightarrow \frac{(r+1)}{(n-r)} = \frac{1}{2}$

$\Rightarrow 2r + 2 = n - r \Rightarrow n = 3r + 2$ ...(i)

Similarly, $\frac{{}^{n}C_{r+1}}{{}^{n}C_{r+2}} = \frac{2}{3} \Rightarrow \frac{\frac{n!}{(r+1)!(n-r-1)!}}{\frac{n!}{(r+2)!(n-r-2)!}} = \frac{2}{3}$

$\Rightarrow \frac{(r+2)!(n-r-2)!}{(r+1)!(n-r-1)!} = \frac{2}{3} \Rightarrow \frac{r+2}{n-r-1} = \frac{2}{3}$

$\Rightarrow 3r + 6 = 2n - 2r - 2 \Rightarrow 2n = 5r + 8$ ...(ii)

Solving (i) and (ii), we get $r = 4$ and $n = 14$

**OR**

There are 12 boys and 10 girls in the class. We have to select 10 students for a competition including atleast 4 boys and 4 girls. Two girls who were last year's winner are to be included. Since, two girls are already selected now we are left with 8 girls out of which atleast 2 girls are to be selected.

We can make selection in the following ways:

| Choice | Boys | Girls (+2 particular girls) |
|---|---|---|
| I | 4 | 4 + 2 |
| II | 5 | 3 + 2 |
| III | 6 | 2 + 2 |

First choice can be made ${}^{12}C_4 \times {}^{8}C_4$

$$= \frac{12\times11\times10\times9}{4\times3\times2\times1}\times\frac{8\times7\times6\times5}{4\times3\times2\times1} = 34650 \text{ ways}$$

Second choice can be made in ${}^{12}C_5 \times {}^{8}C_3$

$$= \frac{12\times11\times10\times9\times8}{5\times4\times3\times2\times1}\times\frac{8\times7\times6}{3\times2\times1} = 44352 \text{ ways}$$

Third choice can be made in ${}^{12}C_6 \times {}^{8}C_2$

$$= \frac{12\times11\times10\times9\times8\times7}{6\times5\times4\times3\times2\times1}\times\frac{8\times7}{2\times1} = 25872 \text{ ways}$$

Hence, total number of possible selections

= 34650 + 44352 + 25872 = 104874

**35.** Let Sunita obtained $x$ marks in the fifth examination.

∴ Average marks of 5 examinations

$$= \frac{87+92+94+95+x}{5} = \frac{368+x}{5}$$

This average must be atleast 90

$$\therefore \quad \frac{368+x}{5} \geq 90$$

Multiplying both sides by 5

$368 + x \geq 5 \times 90 = 450$

Transposing 368 to R.H.S., we get

$x \geq 450 - 368 = 82$

∴ Sunita should obtain atleast 82 marks in the fifth examination.

**36. (i)** In order to realise some profit $R(x) > C(x)$

*i.e.*, $45x + 1500 > 15x + 3000$

$\Rightarrow \; 45x - 15x > 3000 - 1500 \; \Rightarrow 30x > 1500 \; \Rightarrow \; x > 50$

∴ To get some profit number of items produced and sold should be more than 50.

**(ii)** We have, $C(x) < 4500$

$\Rightarrow \; 15x + 3000 < 4500 \; \Rightarrow \; 15x < 1500 \; \Rightarrow \; x < 100$

**(iii)** Given that, $200 < x < 250$

$\Rightarrow \; 15 \times 200 < 15x < 15 \times 250 \; \Rightarrow \; 3000 < 15x < 3750$

$\Rightarrow \; 3000 + 3000 < 15x + 3000 < 3750 + 3000$

$\Rightarrow \; 6000 < C(x) < 6750$

Hence, cost will lies between 6000 and 6750 rupees.

**OR**

Given that, $300 < x < 400$

$\Rightarrow \; 300 \times 45 < 45x < 400 \times 45 \; \Rightarrow \; 13500 < 45x < 18000$

$\Rightarrow \; 13500 + 1500 < 45x + 1500 < 18000 + 1500$

$\Rightarrow \; 15000 < R(x) < 19500$

∴ Revenue will be between 15000 and 19500 rupees.

**37. (i)** There are 3 suits of 13 cards each and 1 suit of 12 cards.

∴ There are ${}^{13}C_4$ ways of choosing 4 diamond cards, ${}^{13}C_4$ ways of choosing 4 club cards, ${}^{13}C_4$ ways of choosing 4 heart cards and ${}^{12}C_4$ ways of choosing 4 spade cards

∴ Required number of ways

$= {}^{13}C_4 + {}^{13}C_4 + {}^{13}C_4 + {}^{12}C_4 = (3 \times {}^{13}C_4) + {}^{12}C_4$

$$= \frac{3\times13!}{4!9!} + \frac{12!}{4!8!} = 2145 + 495 = 2640$$

**(ii)** There are 26 red cards and 25 black cards, 2 red and 2 black cards need to be choosen.

∴ Required number of ways $= {}^{26}C_2 \times {}^{25}C_2$

$$= \frac{26\times25}{2\times1}\times\frac{25\times24}{2\times1} = 97500$$

**38. (i)** We have 5 people, then the numbers of distinct photographs of 5 people is same as the number of ways of filling 5 vacant places with 5 different elements.

∴ Total no. of distinct photographs = 5 × 4 × 3 × 2 × 1 = 120

**(ii)** Raveena is sitting in middle i.e., on third place.

∴ We have 4 choices for first place, 3 choices to fill second place, 2 choice for fourth place and 1 choice for fifth place.

∴ Total number of ways to get the photograph clicked

= 4 × 3 × 2 × 1 = 4! = 24

**(iii)** Let us treat Ravi and Ekta as a single person.

∴ Number of photographs in which they would be sitting together = 4! × 2 = 48

So, the numbers of photographs in which Ravi and Ekta were not sitting together = Total no. of possible photographs – No. of photographs in which they are sitting together = 120 – 48 = 72

**OR**

Ekta can not be seated at the middle place.

∴ For middle place we have 4 choices,
For first place we have 4 choices,
For second place we have 3 choices,
For fourth place we have 2 choices,
and for fifth place we have 1 choice.

∴ Total number of sitting arrangements
= 4 × 3 × 4 × 2 × 1 = 96

Therefore, 96 photographs can be taken in which Ekta will not be seated in middle.

Class XI

# Monthly test drive

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

## Series-3 Complex Numbers and Quadratic Equations

**Total Marks :** 80 **Time taken :** 60 Min.

### Only One Option Correct Type

**1.** One of the square roots of $6+4\sqrt{3}$ is

(a) $\sqrt{3}(\sqrt{3}+1)$ (b) $-\sqrt{3}(\sqrt{3}-1)$
(c) $\sqrt{3}(\sqrt{3}-1)$ (d) None of these

**2.** If $n$ is a positive integer then $(1+i\sqrt{3})^n+(1-i\sqrt{3})^n$ is equal to

(a) $2^{n+1}\cos n\pi/3$ (b) $\cos n\pi/3$
(c) $2^{n+1}\cos n\pi/6$ (d) $\cos n\pi/6$

**3.** If $|z_1| = 1$, $|z_2| = 2$, $|z_3| = 3$ and $|9z_1z_2 + 4z_1z_3 + z_2z_3| = 12$, then the value of $|z_1 + z_2 + z_3|$ is

(a) 3 (b) 4 (c) 8 (d) 2

**4.** Let $z = x + iy$ be a complex number such that $\arg\left(\frac{z-1}{z+1}\right)=\frac{\pi}{2}$. Then $x^2 + y^2 =$

(a) 2 (b) $\frac{1}{3}$ (c) 1 (d) $\frac{2}{3}$

**5.** The value of $\left|\sqrt{4+2\sqrt{3}}\right|-\left|\sqrt{4-2\sqrt{3}}\right|$ is

(a) 1 (b) 2 (c) 4 (d) 3

**6.** Let $z = x + iy$, where $x$ and $y$ are real. The points $(x, y)$ in the $X$-$Y$ plane for which $\frac{z+i}{z-i}$ is purely imaginary lie on

(a) a straight line (b) an ellipse
(c) a hyperbola (d) a circle

### One or More than One Option(s) Correct Type

**7.** If $z_1 = a + ib$ and $z_2 = c + id$ are complex numbers such that $|z_1| = |z_2| = 1$ and $\mathrm{Re}(z_1\bar{z}_2)=0$, then the pair of complex numbers $w_1 = a + ic$ and $w_2 = b + id$ satisfies :

(a) $|w_1| = 1$ (b) $|w_2| = 1$
(c) $\mathrm{Re}(w_1\bar{w}_2)=0$ (d) None of these

**8.** The equation $x^{\frac{3}{4}(\log_2 x)^2+\log_2 x-\frac{5}{4}}=\sqrt{2}$ has

(a) at least one real solution
(b) exactly three real solutions
(c) exactly one irrational solution
(d) complex roots

**9.** For real $x$, the function $\frac{(x-a)(x-b)}{x-c}$ will assume all real values provided

(a) $a > b > c$ (b) $a < b < c$
(c) $a > c > b$ (d) $a < c < b$.

### SOLUTIONS TO AUGUST 2024 QUIZ CLUB

| | | |
|---|---|---|
| **1.** $\frac{1}{4}$ | **2.** $3\sqrt{30}$ | **3.** 13 |
| **4.** 2 | **5.** $\frac{8}{3}$ sq. units | **6.** $\frac{e^{x^2}}{2}+C$ |
| **7.** 8 | **8.** 0 | **9.** 3 |
| **10.** 3 | **11.** 24 | **12.** $\frac{3}{7}$ |
| **13.** 20 | **14.** $\frac{3}{2}$ | **15.** 8 units |
| **16.** $y^2 = -12x$ | **17.** $\frac{3}{5}$ units | **18.** 27 |
| **19.** $88\sqrt{3}$ | **20.** 100 | |

10. The complex number $z$ satisfying the equation $|z-i| = |z+1| = 1$ is
(a) 0 (b) $1+i$
(c) $-1+i$ (d) $1-i$

11. If $a, b \in \{1, 2, 3\}$ and the equation $ax^2 + bx + 1 = 0$ has real roots, then
(a) $a > b$
(b) $a \le b$
(c) number of possible ordered pairs $(a, b)$ is 3
(d) $a < b$

12. If the equation $x^2 + y^2 - 10x + 21 = 0$ has real roots $x = \alpha$ and $y = \beta$, then
(a) $3 \le x \le 7$ (b) $3 \le y \le 7$
(c) $-2 \le y \le 2$ (d) $-2 \le x \le 2$

13. Let $\sin\alpha$, $\cos\alpha$ be the roots of the equation $x^2 - bx + c = 0$. Then which of the following statements is/are correct?
(a) $c \le \frac{1}{2}$ (b) $b \le \sqrt{2}$
(c) $c > \frac{1}{2}$ (d) $b > \sqrt{2}$

## Comprehension Type

### Paragraph for Q. No. 14 and 15

Let $A$, $B$, $C$ be three sets of complex numbers as defined below
$A = \{z : \text{Im } z \ge 1\}$
$B = \{z : |z - 2 - i| = 3\}$
$C = \{z : \text{Re}((1-i)z) = \sqrt{2}\}$.

14. The number of elements in the set $A \cap B \cap C$ is
(a) 0 (b) 1 (c) 2 (d) $\infty$

15. Let $z$ be any point in $A \cap B \cap C$. Then, $|z + 1 - i|^2 + |z - 5 - i|^2$ lies between
(a) 25 and 29 (b) 30 and 34
(c) 35 and 39 (d) 40 and 44

## Matrix Match Type

16. Match the statements in column-I with those in column-II [Note : Here $z$ takes the values in the complex plane and Im $z$ and Re $z$ denote respectively, the imaginary part and the real part of $z$]

| | Column-I | | Column-II |
|---|---|---|---|
| (P) | The set of points $z$ satisfying $\|z - i\|z\|\| = \|z + i\|z\|\|$ is contained in or equal to | (1) | an ellipse with eccentricity 4/5 |
| (Q) | The set of points $z$ satisfying $\|z+4\| + \|z-4\| = 10$ is contained in or equal to | (2) | the set of points $z$ satisfying Im $z = 0$ |
| (R) | If $\|\omega\| = 2$, then the set of points $z = \omega - 1/\omega$ is contained in or equal to | (3) | the set of points $z$ satisfying $\|\text{Im } z\| \le 1$ |
| (S) | If $\|\omega\| = 1$, then the set of points $z = \omega + 1/\omega$ is contained in or equal to | (4) | the set of points $z$ satisfying $\|\text{Re } z\| \le 1$ |
| | | (5) | the set of points $z$ satisfying $\|z\| \le 3$ |

| | P | Q | R | S |
|---|---|---|---|---|
| (a) | 2, 3 | 1 | 1 | 2, 3, 5 |
| (b) | 2, 4 | 2 | 3 | 2, 4 |
| (c) | 1, 3 | 1 | 2 | 1, 3, 4 |
| (d) | None of these | | | |

## Numerical Answer Type

17. The sum of all the real roots of the equation $|x-2|^2 + |x-2| - 2 = 0$ is ______.

18. The smallest value of $k$, for which both the roots of the equation $x^2 - 8kx + 16(k^2 - k + 1) = 0$ are real, distinct and have values at least 4, is ______.

19. If $\omega$ ($\ne 1$) be a cube root of unity and $(1+\omega^2)^n = (1+\omega^4)^n$, then the least positive integral value of $n$ is ______.

20. If $|z+4| \le 3$, then the maximum value of $|z+1|$ is ______.

◆◆

*Keys are published in this issue. Search now!* ☺

**SELF CHECK**

No. of questions attempted ……
No. of questions correct ……
Marks scored in percentage ……

**Check your score! If your score is**

| | | |
|---|---|---|
| > 90% | EXCELLENT WORK ! | You are well prepared to take the challenge of final exam. |
| 90-75% | GOOD WORK ! | You can score good in the final exam. |
| 74-60% | SATISFACTORY ! | You need to score more next time. |
| < 60% | NOT SATISFACTORY! | Revise thoroughly and strengthen your concepts. |

# CBSE Warm-up!

**CLASS-XII**

*Chapterwise practice questions for CBSE Exams as per the latest pattern and syllabus by CBSE for the academic session 2024-25.*

## Series-4 Continuity and Differentiability

***General Instructions : Read the following instructions very carefully and strictly follow them :***

(i) This question paper contains **38** questions. **All** questions are **compulsory.**
(ii) This question paper is divided into **five** Sections – **A, B, C, D** and **E.**
(iii) In **Section A,** Questions no. **1** to **18** are multiple choice questions (MCQs) and Questions no. **19 and 20** are Assertion-Reason based questions of **1** mark each.
(iv) In **Section B,** Questions no. **21** to **25** are very short answer (VSA) type questions, carrying **2** marks each.
(v) In **Section C,** Questions no. **26** to **31** are short answer (SA) type questions, carrying **3** marks each.
(vi) In **Section D,** Questions no. **32** to **35** are long answer (LA) type questions, carrying **5** marks each.
(vii) In **Section E,** Questions no. **36** to **38** are case study based questions, carrying **4** marks each.
(viii) There is no overall choice. However, an internal choice has been provided in **2** questions in Section **B, 3** questions in Section **C, 2** questions in Section **D** and **2** questions in Section E.
(ix) Use of calculators is **not** allowed.

**Time Allowed :** 3 hours **Maximum Marks :** 80

### SECTION - A

**This section comprises multiple choice questions (MCQs)**

**1.** If $f(x)$ is continuous at $x = 0$, where

$$f(x)=\begin{cases}\dfrac{e^{5x}-e^{2x}}{\sin 3x}, & \text{for } x\neq 0\\ K, & \text{for } x=0\end{cases}$$, then find the value of $K$.

(a) $-1$ (b) 0 (c) 4 (d) 1

**2.** If $f(x)=\begin{cases}k(x^2-2), & \text{for } x\le 0\\ 4x+1, & \text{for } x>0\end{cases}$ is continuous at $x = 0$, then find the value of $k$.

(a) 1 (b) $\frac{1}{2}$ (c) 0 (d) $-\frac{1}{2}$

**3.** If $f(x)=\begin{cases}1+x, & \text{for } x\le 2\\ 5-x, & \text{for } x>2\end{cases}$, then

(a) $f(x)$ is continuous and differentiable at $x = 2$
(b) $f(x)$ is continuous but not differentiable at $x = 2$
(c) $f(x)$ is everywhere differentiable
(d) $f(x)$ is not continuous at $x = 2$

**4.** If $y=\tan^{-1}\left(\dfrac{a+b\tan x}{b-a\tan x}\right)$, then find $\dfrac{dy}{dx}$.

(a) $\dfrac{1}{1+x^2}$ (b) $-1$ (c) 1 (d) 0

**5.** Differentiate $\log(e^x \cdot \sin^5 x)$ w.r.t. $x$.

(a) $1 - 5\cot x$ (b) $1 + \cot x$
(c) $1 - 5\cot x^5$ (d) $1 + 5\cot x$

**6.** Differentiate $e^{\log \tan x}$ w.r.t. $x$.

(a) $\sec^2 x$ (b) $-\sec^2 x$
(c) $\sec x \tan x$ (d) $-\sec x \tan x$

**7.** Differentiate $\log(1 + x^2)$ w.r.t. $\tan^{-1} x$.

(a) $-2x$ (b) $\dfrac{1}{1+x^2}$
(c) $2x$ (d) $\dfrac{2x}{1+x^2}$

8. If $x = e^{\cos 2t}$ and $y = e^{\sin 2t}$, then find $\frac{dy}{dx}$.

(a) $\frac{y\log x}{x\log y}$ (b) $\frac{x\log x}{y\log y}$

(c) $-\frac{y\log x}{x\log y}$ (d) $-\frac{x\log x}{y\log y}$

9. If $x = a\cos nt - b\sin nt$, then $\frac{d^2x}{dt^2} =$

(a) $n^2x$ (b) $-n^2x$ (c) $-nx$ (d) $nx$

10. If $x = a\sin t - b\cos t$, $y = a\cos t + b\sin t$, then $\frac{d^2y}{dx^2} =$

(a) $\frac{x^2+y^2}{y^3}$ (b) $\frac{y^3}{x^2+y^2}$

(c) $\frac{-y^3}{x^2+y^2}$ (d) $-\frac{x^2+y^2}{y^3}$

11. If $f(x) = \frac{\sqrt{4+x}-2}{x}$, $x \neq 0$ is continuous at $x = 0$, then find $f(0)$.

(a) 2 (b) $\frac{1}{4}$ (c) 0 (d) $\frac{1}{2}$

12. The function $f(x) = \frac{4-x^2}{4x-x^3}$ is

(a) discontinuous at only one point

(b) discontinuous at exactly two points

(c) discontinuous at exactly three points

(d) None of these

13. If $f(x) = \begin{cases} x^2+2x, & x \le 0 \\ ax+b, & x > 0 \end{cases}$, then values of $a$ and $b$ such that $f(x)$ is continuous and differentiable at $x = 0$ are

(a) $a = 1, b = 0$ (b) $a = 2, b = 0$

(c) $a = 0, b = 2$ (d) $a = 0, b = 1$

14. If $y = \sqrt{\frac{1-\cos x}{1+\cos x}}$, then find $\frac{dy}{dx}$.

(a) $\frac{1}{2}\sec^2\frac{x}{2}$ (b) $-\frac{1}{2}\sec^2\frac{x}{2}$

(c) $\sec^2\frac{x}{2}$ (d) $-\sec^2\frac{x}{2}$

15. If $y = e^{\sin x}\sin(e^x)$, then find $dy/dx$.

(a) $e^{\sin x}(e^x\sin(e^x) + \sin x\cos(e^x))$

(b) $e^{\sin x}(e^x\cos(e^x) - \cos x\cos(e^x))$

(c) $e^{\sin x}(e^x\cos(e^x) + \cos x\sin(e^x))$

(d) $-e^{\sin x}(e^x\cos(e^x) + \cos x\sin(e^x))$

16. If $f(x) = \log\left[e^x\left(\frac{3-x}{3+x}\right)^{1/3}\right]$, then $f'(1)$ is equal to

(a) 3/4 (b) 2/3 (c) 1/3 (d) 1/2

17. If $y = e^{m\sin^{-1}x}$ and $(1-x^2)\left(\frac{dy}{dx}\right)^2 = Ay^2$, then $A =$

(a) $m$ (b) $-m$ (c) $m^2$ (d) $-m^2$

18. Find $\frac{dy}{dx}$, when $x = e^\theta(\sin\theta + \cos\theta)$, $y = e^\theta(\sin\theta - \cos\theta)$.

(a) $-\tan\theta$ (b) $\cot\theta$ (c) $-\cot\theta$ (d) $\tan\theta$

**Questions number 19 and 20 are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) options as given below.**

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(c) Assertion (A) is true, but Reason (R) is false.

(d) Assertion (A) is false, but Reason (R) is true.

19. **Assertion (A) :** Consider the function $f(x) = [\sin x]$, $x \in [0, \pi]$. $f(x)$ is not continuous at $x = \frac{\pi}{2}$.

**Reason (R) :** A function $f(x)$ is continuous at $x = a$, if $\lim_{x\to a} f(x)$ exists and is equal to $f(a)$.

20. **Assertion (A) :** Consider the function

$$f(x) = \begin{cases} x^2, & x \ge 1 \\ x+1, & x < 1 \end{cases}.$$

$f$ is derivable at $x = 1$ as $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x)$.

**Reason (R) :** If a function $f$ is derivable at a point '$a$', then it is continuous at '$a$'.

## SECTION - B

**This section comprises very short answer (VSA) type questions of 2 marks each.**

**21.** Determine the value of the constant '$k$' so that the function $f(x)=\begin{cases}\dfrac{kx}{|x|}, & \text{if } x<0\\ 3, & \text{if } x\geq 0\end{cases}$ is continuous at $x = 0$.

**OR**

Determine the value of '$k$' for which the following function is continuous at $x = 3$.

$$f(x)=\begin{cases}\dfrac{(x+3)^2-36}{x-3}, & x\neq 3\\ k, & x=3\end{cases}$$

**22.** Let $f(x) = x|x|$, for all $x \in R$ check its differentiability at $x = 0$.

**OR**

Find $\dfrac{dy}{dx}$ at $x = 1$, $y = \dfrac{\pi}{4}$ if $\sin^2 y + \cos xy = K$.

**23.** If $x = a\sec\theta$, $y = b\tan\theta$, then find $\dfrac{dy}{dx}$ at $\theta=\dfrac{\pi}{3}$.

**24.** Find the derivative of $\sin^2 x$ w.r.t. $e^{\cos x}$.

**25.** If $x = t^2 + 1$, $y = 2at$, then find $\dfrac{d^2y}{dx^2}$ at $t = a$.

## SECTION - C

**This section comprises short answer (SA) type questions of 3 marks each.**

**26.** Find the values of $p$ and $q$, for which

$$f(x)=\begin{cases}\dfrac{1-\sin^3 x}{3\cos^2 x}, & \text{if } x<\pi/2\\ p, & \text{if } x=\pi/2\\ \dfrac{q(1-\sin x)}{(\pi-2x)^2}, & \text{if } x>\pi/2\end{cases}$$

is continuous at $x = \pi/2$.

**OR**

Find the value of $k$, for which

$$f(x)=\begin{cases}\dfrac{\sqrt{1+kx}-\sqrt{1-kx}}{x}, & \text{if } -1\leq x<0\\ \dfrac{2x+1}{x-1}, & \text{if } 0\leq x<1\end{cases}$$

is continuous at $x = 0$.

**27.** Show that the function $f(x)=\begin{cases}x^2 & \text{for } x<1\\ 2-x & \text{for } x\geq 1\end{cases}$ is continuous but not differentiable at $x = 1$.

**OR**

If $\cos y = x\cos(a + y)$, where $\cos a \neq \pm 1$, prove that $\dfrac{dy}{dx}=\dfrac{\cos^2(a+y)}{\sin a}$.

**28.** If $y=e^{x^2\cos x}+(\cos x)^x$, then find $\dfrac{dy}{dx}$.

**OR**

If $\log(x^2+y^2)=2\tan^{-1}\left(\dfrac{y}{x}\right)$, show that $\dfrac{dy}{dx}=\dfrac{x+y}{x-y}$.

**29.** If $x^y = e^{x-y}$, prove that $\dfrac{dy}{dx}=\dfrac{\log x}{(1+\log x)^2}$.

**30.** If $x = a\cos\theta$; $y = b\sin\theta$, then find $\dfrac{d^2y}{dx^2}$.

**31.** If $x = a\sec^3\theta$, $y = a\tan^3\theta$, then find $\dfrac{d^2y}{dx^2}$ at $\theta=\dfrac{\pi}{4}$.

## SECTION - D

**This section comprises long answer (LA) type questions of 5 marks each.**

**32.** If $f(x)=\begin{cases}\dfrac{1-\cos 4x}{x^2}, & \text{when } x<0\\ a, & \text{when } x=0\\ \dfrac{\sqrt{x}}{\left(\sqrt{16+\sqrt{x}}\right)-4}, & \text{when } x>0\end{cases}$

and $f$ is continuous at $x = 0$, find the value of $a$.

**OR**

If $y=\tan^{-1}\left(\dfrac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right), x^2\leq 1$

then find $\dfrac{dy}{dx}$.

**33.** Show that the function $f(x) = |x - 1| + |x + 1|$, for all $x \in R$, is not differentiable at the points $x = -1$ and $x = 1$.

**OR**

If $y = \sin(2\sin^{-1}x)$, then find the value of $(1 - x^2)\,y_2$.

**34.** If $x = \sin t$, $y = \sin pt$, prove that

$$(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}+p^2y=0.$$

**35.** If $x^m y^n = (x + y)^{m+n}$, prove that $\dfrac{d^2y}{dx^2}=0$.

## SECTION - E

### Case Study 1

**36.** A city's transportation is studying the traffic flow at a toll booth on a major highway. The toll booth experiences different traffic patterns depending on the time of a day, leading to variations in the rate of vehicle entry. The department uses a piecewise function to model the rate of cars entering the toll booth per minute $T(t)$, where $t$ represent time in minutes past 8 am.

The function is defined as follows :

$$T(t)=\begin{cases}20, & \text{if } 0\le t<30\\ 30, & \text{if } 30\le t<60\\ 25+0.5(t-60), & \text{if } t\ge 60\end{cases}$$

On the basis of above information, answer the following questions :

(i) Evaluate : $\lim_{t\to 30} T(t)$

(ii) Check whether the function $T(t)$ is continuous or not at $x = 30$.

(iii) Check whether the function is differentiable or not at $x = 60$.

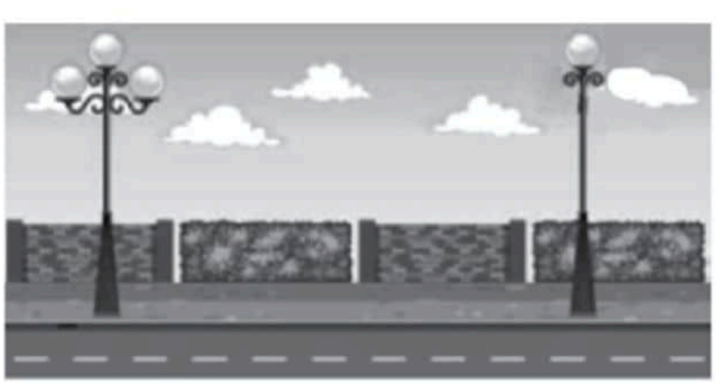
**OR**

Find the value of $T'(70)$.

### Case Study 2

**37.** In a street, two lamp posts are 600 feet apart. The light intensity at a distance $d$ from the first (stronger) lamp post is $\frac{1000}{d^2}$, the light intensity at distance $d$ from the second (weaker) lamp post is $\frac{125}{d^2}$ (in both cases the light intensity is inversely proportional to the square of the distance to the light source).

The combined light intensity is the sum of the two light intensities coming from both lamp posts.



On the basis of above information, answer the following questions :

(i) If you are in between the lamp posts, at distance $x$ feet from the stronger light, then find the formula for the combined light intensity coming from both lamp posts as function of $x$.

(ii) If $I(x)$ denotes the combined light intensity, then find the value of $I'(100)$.

### Case Study 3

**38.** The shape of the roller coaster track is modeled using two functions. The height $H(x)$ of the track above the ground at a horizontal position $x$ is given by

$$H(x)=\begin{cases}-x^2+4x+1 & \text{if } x\le 1\\ 2x^2-3x+1 & \text{if } x>1\end{cases}$$

Additionally, the speed $S(x)$ of the roller coaster at position $(x)$ is given by

$$S(x)=\frac{x^3-3x^2+2x}{x-1}$$

On the basis of above information, answer the following questions :

(i) Find $\lim_{x\to 1} H(x)$.

(ii) Check the continuity of the function $H(x)$ at $x = 1$.

(iii) Check whether the function $S(x)$ at $x = 1$ is continuous or not.

**OR**

Find the left hand derivative of $S(x)$ at $x = 1$.

## SOLUTIONS

**1. (d) :** Since, $f(x)$ is continuous at $x = 0$, then

$$f(0)=\lim_{x\to 0} f(x)=\lim_{x\to 0}\frac{e^{5x}-e^{2x}}{\sin 3x}=\lim_{x\to 0}\frac{e^{2x}(e^{3x}-1)}{\sin 3x}$$

$$=\frac{\lim_{x\to 0} e^{2x}\left(\frac{e^{3x}-1}{3x}\right)}{\lim_{x\to 0}\left(\frac{\sin 3x}{3x}\right)}=\frac{e^0\times 1}{1}=1$$

**2. (d) :** $\lim_{x\to 0^-} f(x)=\lim_{x\to 0} k(x^2-2)$

$=k\lim_{x\to 0}(x^2-2)=k(0-2)=-2k$

$\lim_{x\to 0^+} f(x)=\lim_{x\to 0}(4x+1)=4(0)+1=1$

Since $f$ is continuous at $x = 0$.

$\therefore \quad \lim_{x\to 0^-} f(x)=\lim_{x\to 0^+} f(x) \quad \therefore \ -2k=1 \Rightarrow k=-\frac{1}{2}$

**3. (b)**

**4. (c) :** Let $y=\tan^{-1}\left(\frac{a+b\tan x}{b-a\tan x}\right)$

$$\Rightarrow\ y=\tan^{-1}\left(\frac{\frac{a}{b}+\tan x}{1-\frac{a}{b}\tan x}\right)$$

$$\Rightarrow\ y=\tan^{-1}\left(\frac{a}{b}\right)+\tan^{-1}(\tan x)$$

$$\Rightarrow\ y=\tan^{-1}\left(\frac{a}{b}\right)+x\quad\therefore\quad\frac{dy}{dx}=1$$

**5. (d)**

**6. (a) :** Let $y = e^{\log\tan x} = \tan x$ $\quad[\because\ a^{\log_a x}=x]$

$$\therefore\ \frac{dy}{dx}=\frac{d}{dx}(\tan x)=\sec^2 x$$

**7. (c) :** Let $u = \log(1+x^2)$, $v=\tan^{-1}x$

$\therefore$ $u$ and $v$ becomes parametric functions, where $x$ is parameter.

$$\therefore\ \frac{du}{dx}=\frac{1}{1+x^2}\times 2x=\frac{2x}{1+x^2}$$

$$\frac{dv}{dx}=\frac{1}{1+x^2}\quad\therefore\quad\frac{du}{dv}=\frac{\frac{du}{dx}}{\frac{dv}{dx}}=\frac{\frac{2x}{1+x^2}}{\frac{1}{1+x^2}}=2x$$

**8. (c) :** We have, $x=e^{\cos 2t}$

$$\therefore\ \frac{dx}{dt}=e^{\cos 2t}\times(-\sin 2t)(2)$$

Also, $y=e^{\sin 2t}$ $\quad\therefore\ \frac{dy}{dt}=e^{\sin 2t}\times(\cos 2t)(2)$

$$\text{Now, }\frac{dy}{dx}=\frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)}=\frac{2\cos 2t\cdot e^{\sin 2t}}{-2\sin 2t\cdot e^{\cos 2t}}$$

$$=\frac{-\log(e^{\cos 2t})\cdot y}{\log(e^{\sin 2t})\cdot x}=\frac{-y\log x}{x\log y}$$

**9. (b) :** $x = a\cos(nt) - b\sin(nt)$

$$\therefore\ \frac{dx}{dt}=a(-\sin nt)(n)-b(\cos nt)(n)$$

$= -n[a\sin nt + b\cos nt]$

$$\frac{d^2x}{dt^2}=-n[a\cos nt(n)+b(-\sin nt)(n)]$$

$= -n^2(a\cos nt - b\sin nt) = -n^2x$

**10. (d)**

**11. (b) :** Given, $f(x)$ is continuous at $x = 0$.

$$\therefore\ f(0)=\lim_{x\to 0}f(x)=\lim_{x\to 0}\frac{\sqrt{4+x}-2}{x}$$

$$=\lim_{x\to 0}\left(\frac{\sqrt{4+x}-2}{x}\times\frac{\sqrt{4+x}+2}{\sqrt{4+x}+2}\right)$$

$$=\lim_{x\to 0}\frac{4+x-4}{x(\sqrt{4+x}+2)}=\lim_{x\to 0}\frac{x}{x(\sqrt{4+x}+2)}$$

$$=\lim_{x\to 0}\frac{1}{\sqrt{4+x}+2}=\frac{1}{2+2}=\frac{1}{4}$$

**12. (c) :** We have, $f(x)=\frac{4-x^2}{4x-x^3}=\frac{4-x^2}{x(2-x)(2+x)}$

So, $f(x)$ is discontinuous at $x = 0, 2, -2$.

**13. (b) :** $\because$ $f(x)$ is continuous at $x = 0$.

$$\lim_{h\to 0}f(0-h)=\lim_{h\to 0}f(0+h)=f(0)$$

$$\Rightarrow\ \lim_{h\to 0}[(-h)^2+2(-h)]=\lim_{h\to 0}[ah+b]\ \Rightarrow\ b=0$$

Also, since $f(x)$ is differentiable at $x = 0$.

$$\therefore\ \lim_{h\to 0}\frac{f(0+h)-f(0)}{h}=\lim_{h\to 0}\frac{f(0-h)-f(0)}{-h}$$

$$\Rightarrow\ \lim_{h\to 0}\frac{(ah+b)-0}{h}=\lim_{h\to 0}\frac{[(-h)^2+2(-h)]-0}{-h}$$

$(\because b=0)$

$$\Rightarrow\ \lim_{h\to 0}\frac{ah+0}{h}=\lim_{h\to 0}(2-h)$$

$\Rightarrow\ a=2$

**14. (a) :** Let $y=\sqrt{\frac{1-\cos x}{1+\cos x}}=\sqrt{\frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}}}=\tan\left(\frac{x}{2}\right)$

$$\therefore\ \frac{dy}{dx}=\frac{d}{dx}(\tan(x/2))=\sec^2\frac{x}{2}\cdot\frac{d}{dx}\left(\frac{x}{2}\right)$$

$$=\sec^2\frac{x}{2}\cdot\frac{1}{2}=\frac{1}{2}\sec^2\frac{x}{2}$$

**15. (c) :** Let $y=e^{\sin x}\sin(e^x)$

$$\therefore\ \frac{dy}{dx}=\frac{d}{dx}\left\{e^{\sin x}\sin\left(e^x\right)\right\}$$

$$=e^{\sin x}\frac{d}{dx}\left(\sin e^x\right)+\sin\left(e^x\right)\cdot\frac{d}{dx}\left(e^{\sin x}\right)$$

$$=e^{\sin x}\cdot\cos\left(e^x\right)\cdot\frac{d}{dx}(e^x)+\sin\left(e^x\right)\cdot e^{\sin x}\cdot\frac{d}{dx}(\sin x)$$

$$=e^{\sin x}\cdot\cos\left(e^x\right)\cdot e^x+\sin\left(e^x\right)\cdot e^{\sin x}\cdot\cos x$$

$$=e^{\sin x}\left[e^x\cos\left(e^x\right)+\cos x\cdot\sin\left(e^x\right)\right]$$

**16. (a) :** $f(x)=\log\left[e^x\left(\frac{3-x}{3+x}\right)^{1/3}\right]$

$\Rightarrow f(x)=\log e^x+\frac{1}{3}\log\left(\frac{3-x}{3+x}\right)$

$=x+\frac{1}{3}[\log(3-x)-\log(3+x)]$

$\Rightarrow f'(x)=1+\frac{1}{3}\left[\frac{-1}{3-x}-\frac{1}{3+x}\right]=\frac{7-x^2}{9-x^2}$

$\therefore \quad f'(1)=\frac{7-1}{9-1}=\frac{6}{8}=\frac{3}{4}$

**17. (c) :** We have, $y=e^{m\sin^{-1}x}$.

Differentiating w.r.t. $x$, we get

$\frac{dy}{dx}=e^{m\sin^{-1}x}\frac{d}{dx}[m\sin^{-1}x]$

$\Rightarrow \frac{dy}{dx}=e^{m\sin^{-1}x}\times\frac{m}{\sqrt{1-x^2}}=\frac{my}{\sqrt{1-x^2}}$ ...(i)

Since, $(1-x^2)\left(\frac{dy}{dx}\right)^2=Ay^2$

$\Rightarrow (1-x^2)\frac{m^2y^2}{(1-x^2)}=Ay^2$ [From (i)]

$\Rightarrow m^2y^2=Ay^2 \Rightarrow A=m^2$

**18. (d) :** We have, $x=e^{\theta}(\sin\theta+\cos\theta)$

$\therefore \frac{dx}{d\theta}=e^{\theta}(\cos\theta-\sin\theta)+(\sin\theta+\cos\theta)e^{\theta}$

$=e^{\theta}(\cos\theta-\sin\theta+\sin\theta+\cos\theta)=2e^{\theta}\cos\theta$

Also, $y=e^{\theta}(\sin\theta-\cos\theta)$

$\therefore \frac{dy}{d\theta}=e^{\theta}(\cos\theta+\sin\theta)+(\sin\theta-\cos\theta)e^{\theta}$

$=e^{\theta}(\cos\theta+\sin\theta+\sin\theta-\cos\theta)=2e^{\theta}\sin\theta$

Now, $\frac{dy}{dx}=\frac{\left(\frac{dy}{d\theta}\right)}{\left(\frac{dx}{d\theta}\right)}=\frac{2e^{\theta}\sin\theta}{2e^{\theta}\cos\theta}=\tan\theta$

**19. (d) :** We know that for all

$x\in\left[0,\frac{\pi}{2}\right)\cup\left(\frac{\pi}{2},\pi\right], 0<\sin x<1$

$\Rightarrow [\sin x]=0 \quad \therefore \lim_{x\to\frac{\pi}{2}}[\sin x]=0$

Also, $f\left(\frac{\pi}{2}\right)=\left[\sin\frac{\pi}{2}\right]=1 \Rightarrow \lim_{x\to\frac{\pi}{2}}f(x)\neq f\left(\frac{\pi}{2}\right)$

$\therefore$ $f$ is not continuous at $x=\frac{\pi}{2}$.

Hence, assertion (A) is false and reason (R) is true.

**20. (d) :** $\because \lim_{x\to1^-}f(x)=\lim_{x\to1}(x+1)=2$

and $\lim_{x\to1^+}f(x)=\lim_{x\to1}x^2=1$

$\Rightarrow \lim_{x\to1^-}f(x)\neq\lim_{x\to1^+}f(x)$

$\Rightarrow$ $f$ is not continuous at $x=1$.

$\Rightarrow$ $f$ is not derivable at $x=1$.

**21.** We have, $f(x)=\begin{cases}\frac{kx}{|x|}, & x<0\\ 3, & x\geq0\end{cases}$

L.H.L. $=\lim_{x\to0^-}f(x)=\lim_{x\to0}\frac{kx}{-x}=-k$

R.H.L. $=\lim_{x\to0^+}f(x)=\lim_{x\to0}3=3$

Since, $f(x)$ is continuous at $x=0$.

$\therefore$ L.H.L. = R.H.L. $=f(0) \Rightarrow -k=3 \Rightarrow k=-3$

**OR**

Given, $f(x)$ is continuous at $x=3$.

So, $\lim_{x\to3}f(x)=f(3) \Rightarrow \lim_{x\to3}\frac{(x+3)^2-36}{x-3}=k$

$\Rightarrow \lim_{x\to3}\frac{(x+3)^2-6^2}{x-3}=k \Rightarrow \lim_{x\to3}\frac{(x+3+6)(x+3-6)}{x-3}=k$

$\Rightarrow 3+3+6=k \Rightarrow k=12$

**22.** To check the differentiability of $f(x)=x|x|$ at $x=0$.

$Lf'(0)=\lim_{h\to0}\frac{f(0-h)-f(0)}{-h}=\lim_{h\to0}\frac{-h|-h|-0}{-h}=\lim_{h\to0}h=0$

$Rf'(0)=\lim_{h\to0}\frac{f(0+h)-f(0)}{h}$

$=\lim_{h\to0}\frac{f(h)-f(0)}{h}=\lim_{h\to0}\frac{h|h|-0}{h}=\lim_{h\to0}|h|=0$

Hence, L.H.D. = R.H.D., so $f(x)=x|x|$ is differentiable at $x=0$.

**OR**

We have, $\sin^2y+\cos xy=K$

Differentiating w.r.t. $x$ on both sides, we get

$2\sin y\cos y\frac{dy}{dx}+(-\sin xy)\left(x\frac{dy}{dx}+y\right)=0$

($\because$ Product rule : $(uv)'=u'v+uv'$)

$\Rightarrow \frac{dy}{dx}=\frac{y\sin xy}{\sin 2y-x\sin xy}$

$$\Rightarrow \left[\frac{dy}{dx}\right]_{\left(1,\frac{\pi}{4}\right)} = \frac{\frac{\pi}{4}\sin\frac{\pi}{4}}{\sin\frac{\pi}{2} - \sin\frac{\pi}{4}} = \frac{\pi}{4(\sqrt{2}-1)}$$

**23.** We have, $x = a\sec\theta$ and $y = b\tan\theta$

$$\therefore \frac{dx}{d\theta} = a\sec\theta\tan\theta \text{ and } \frac{dy}{d\theta} = b\sec^2\theta$$

$$\text{Now, } \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{b\sec^2\theta}{a\sec\theta\tan\theta} = \frac{b}{a}\cdot\frac{1}{\cos\theta}\cdot\frac{\cos\theta}{\sin\theta}$$

$$= \frac{b}{a}\cdot\frac{1}{\sin\theta}$$

$$\therefore \left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{3}} = \frac{b}{a}\cdot\frac{1}{\sin\frac{\pi}{3}} = \frac{b}{a}\cdot\frac{1}{\frac{\sqrt{3}}{2}} = \frac{2b}{a\sqrt{3}}$$

**24.** Let $u = \sin^2 x$ and $v = e^{\cos x}$

$$\therefore \frac{du}{dx} = 2\sin x\cos x \text{ and } \frac{dv}{dx} = e^{\cos x}(-\sin x)$$

$$\text{Now, } \frac{du}{dv} = \frac{du}{dx}\cdot\frac{dx}{dv} = \frac{2\sin x\cos x}{e^{\cos x}(-\sin x)} = \frac{-2\cos x}{e^{\cos x}}$$

**25.** Given, $x = t^2 + 1$ and $y = 2at$

$$\Rightarrow \frac{dx}{dt} = 2t \text{ and } \frac{dy}{dt} = 2a$$

$$\therefore \frac{dy}{dx} = \frac{a}{t}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{-a}{t^2}\cdot\frac{dt}{dx} = \frac{-a}{2t^3}$$

$$\therefore \left.\frac{d^2y}{dx^2}\right|_{t=a} = \frac{-a}{2a^3} = \frac{-1}{2a^2}$$

**26.** $\because$ $f(x)$ is continuous at $\pi/2$.

$$\therefore \lim_{x\to\pi/2^-} f(x) = \lim_{x\to\pi/2^+} f(x) = f(\pi/2) \quad \text{...(i)}$$

$$\text{Now, } \lim_{x\to\pi/2^-} f(x) = \lim_{h\to 0} f\left(\frac{\pi}{2}-h\right)$$

$$= \lim_{h\to 0}\frac{1-\sin^3\left(\frac{\pi}{2}-h\right)}{3\cos^2\left(\frac{\pi}{2}-h\right)} = \lim_{h\to 0}\frac{1-\cos^3 h}{3\sin^2 h}$$

$$= \lim_{h\to 0}\frac{(1-\cos h)(1+\cos^2 h+\cos h)}{3(1-\cos h)(1+\cos h)}$$

$$= \lim_{h\to 0}\frac{(1+\cos^2 h+\cos h)}{3(1+\cos h)} = \frac{1+1+1}{3(1+1)} = \frac{1}{2}$$

$$\text{and } \lim_{x\to\pi/2^+} f(x) = \lim_{h\to 0} f\left(\frac{\pi}{2}+h\right)$$

$$= \lim_{h\to 0}\frac{q\left[1-\sin\left(\frac{\pi}{2}+h\right)\right]}{\left[\pi-2\left(\frac{\pi}{2}+h\right)\right]^2} = \lim_{h\to 0}\frac{q(1-\cos h)}{4h^2}$$

$$= \frac{q}{4}\times\lim_{h\to 0}\frac{2\sin^2\frac{h}{2}}{\frac{h^2}{4}\times 4} = \frac{q}{4}\times\frac{2}{4} = \frac{q}{8} \text{ and } f(\pi/2) = p$$

$$\therefore \frac{1}{2} = \frac{q}{8} = p \Rightarrow p = \frac{1}{2} \text{ and } q = 4 \quad \text{[From (i)]}$$

**OR**

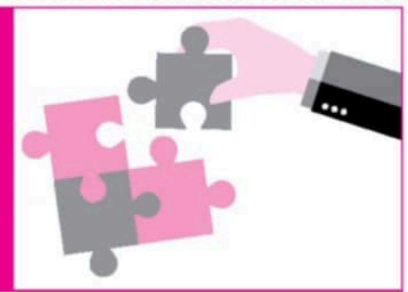
$\because$ $f(x)$ is continuous at $x = 0$

$$\therefore \lim_{x\to 0^+} f(x) = f(0) = \lim_{x\to 0^-} f(x) \quad \text{...(i)}$$

$$\text{Now, } f(0) = \frac{2\times 0+1}{0-1} = -1$$

$$\lim_{x\to 0^+} f(x) = \lim_{h\to 0} f(0+h) = \lim_{h\to 0}\frac{2h+1}{h-1} = -1$$

# PUZZLE CORNER



## NUMBER MAZE

A NUMBER MAZE is an 5 × 5 grid of positive integers. A token starts in the upper left corner, your goal is to move the token to the lower-right corner. On each turn, you are allowed to move the token up, down, left or right; the distance you may move the token is determined by the number on its current square. For example, if the token is on a square labelled 3, then you may move the token three steps up, three steps down, three steps left or three steps right. However, you are never allowed to move the token out the edge of the board.

| | | | | |
|---|---|---|---|---|
| (3) | 5 | 2 | 4 | 6 |
| 5 | 3 | 1 | 5 | 3 |
| 7 | 8 | 3 | 1 | 4 |
| 4 | 5 | 7 | 2 | 3 |
| 3 | 1 | 3 | 2 | ★ |

*Readers can send their responses at editor@mtg.in or post us with complete address. Winners' names with their valuable feedback will be published in next issue.*

$$\lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h) = \lim_{h\to 0}\frac{\sqrt{1-kh}-\sqrt{1+kh}}{-h}$$

$$= \lim_{h\to 0}\frac{\sqrt{1-kh}-\sqrt{1+kh}}{-h}\times\frac{\sqrt{1-kh}+\sqrt{1+kh}}{\sqrt{1-kh}+\sqrt{1+kh}}$$

$$= \lim_{h\to 0}\frac{(1-kh)-(1+kh)}{-h[\sqrt{1-kh}+\sqrt{1+kh}]}$$

$$= \lim_{h\to 0}\frac{2k}{\sqrt{1-kh}+\sqrt{1+kh}} = \frac{2k}{2} = k$$

$\therefore$ From (i), we get $k = -1$

**27.** At $x = 1$, $\lim_{x\to 1^-} f(x) = \lim_{x\to 1} x^2 = 1$

and $\lim_{x\to 1^+} f(x) = \lim_{x\to 1} 2 - x = 1$

Also, $f(1) = 2 - 1 = 1$

$\because \lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) = f(x)$

$\therefore$ $f(x)$ is continuous at $x = 1$

Now, L.H.D. $= \lim_{x\to 1^-}\frac{f(x)-f(1)}{x-1}$

$$= \lim_{x\to 1}\frac{x^2-1}{x-1} = \lim_{x\to 1}(x+1) = 2$$

$$\text{R.H.D.} = \lim_{x\to 1^+}\frac{f(x)-f(1)}{x-1} = \lim_{x\to 1}\frac{(2-x)-1}{x-1} = -1$$

$\because$ L.H.D. $\neq$ R.H.D.

$\therefore$ $f(x)$ is not differentiable at $x = 1$.

**OR**

We have, $\cos y = x\cos(a + y)$

$$\Rightarrow x = \frac{\cos y}{\cos(a+y)}$$

Differentiating w.r.t. $y$ on both sides, we get

$$\frac{dx}{dy} = \frac{\cos(a+y)\left(\frac{d}{dy}\cos y\right) - \cos y\left(\frac{d}{dy}\cos(a+y)\right)}{\cos^2(a+y)}$$

$$\Rightarrow \frac{dx}{dy} = \frac{\cos(a+y)(-\sin y)+\cos y\sin(a+y)}{\cos^2(a+y)}$$

$$\Rightarrow \frac{dx}{dy} = \frac{\cos y\sin(a+y)-\cos(a+y)\sin y}{\cos^2(a+y)}$$

$$= \frac{\sin[(a+y)-y]}{\cos^2(a+y)} = \frac{\sin a}{\cos^2(a+y)}$$

$$\therefore \frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$$

**28.** We have, $y = e^{x^2\cos x} + (\cos x)^x = e^{x^2\cos x} + e^{x(\ln\cos x)}$

$$\therefore \frac{dy}{dx} = e^{x^2\cos x}\frac{d}{dx}(x^2\cos x) + e^{x\ln\cos x}\frac{d}{dx}(x\ln\cos x)$$

$$= e^{x^2\cos x}(2x\cos x - x^2\sin x) + e^{x\ln\cos x}\left(\ln\cos x - \frac{x}{\cos x}\sin x\right)$$

$$= e^{x^2\cos x}(2x\cos x - x^2\sin x) + (\cos x)^x(\ln\cos x - x\tan x)$$

**OR**

Given, $\log(x^2+y^2) = 2\tan^{-1}\left(\frac{y}{x}\right)$

Differentiating w.r.t. $x$ on both sides, we get

$$\frac{1}{x^2+y^2}\left(2x+2y\frac{dy}{dx}\right) = 2\times\frac{1}{1+\frac{y^2}{x^2}}\frac{d}{dx}\left(\frac{y}{x}\right)$$

$$\Rightarrow \frac{2x}{x^2+y^2}+\frac{2y}{x^2+y^2}\frac{dy}{dx} = \frac{2x^2}{x^2+y^2}\left(\frac{1}{x}\frac{dy}{dx}+y\left(\frac{-1}{x^2}\right)\right)$$

$$\Rightarrow \frac{dy}{dx}\left[\frac{2y}{x^2+y^2}-\frac{2x}{x^2+y^2}\right] = \frac{2x^2}{x^2+y^2}\left[\frac{-y}{x^2}-\frac{1}{x}\right]$$

$$\Rightarrow \frac{2(y-x)}{x^2+y^2}\frac{dy}{dx} = \frac{-2x^2}{x^2+y^2}\left(\frac{y+x}{x^2}\right) \Rightarrow \frac{dy}{dx} = \frac{x+y}{x-y}$$

**29.** We have, $x^y = e^{x-y}$

Taking log on both sides, we get

$y\log x = x - y \Rightarrow y(\log x + 1) = x \Rightarrow y = \frac{x}{\log x + 1}$

Differentiating w.r.t. $x$ on both sides, we get

$$\frac{dy}{dx} = \frac{(\log x+1)\cdot 1 - x\left(\frac{1}{x}\right)}{(1+\log x)^2} = \frac{\log x + 1 - 1}{(1+\log x)^2} = \frac{\log x}{(1+\log x)^2}$$

**30.** We have, $x = a\cos\theta$, $y = b\sin\theta$

$$\therefore \frac{dx}{d\theta} = -a\sin\theta, \frac{dy}{d\theta} = b\cos\theta$$

So, $\frac{dy}{dx} = \frac{dy}{d\theta}\times\frac{d\theta}{dx} = \frac{b\cos\theta}{-a\sin\theta} = -\frac{b}{a}\cot\theta$

Now, $\frac{d^2y}{dx^2} = -\frac{b}{a}(-\text{cosec}^2\theta)\frac{d\theta}{dx}$

$$= \frac{b}{a}\text{cosec}^2\theta\left(-\frac{1}{a}\text{cosec}\,\theta\right) = -\frac{b}{a^2}\text{cosec}^3\theta$$

**31.** Here, $x = a\sec^3\theta$

$$\Rightarrow \frac{dx}{d\theta} = a\cdot 3\sec^2\theta\cdot\sec\theta\tan\theta = 3a\sec^3\theta\tan\theta$$

and $y = a\tan^3\theta$

$$\Rightarrow \frac{dy}{d\theta} = a\cdot 3\tan^2\theta\cdot\sec^2\theta$$

$$\therefore \quad \frac{dy}{dx}=\frac{dy/d\theta}{dx/d\theta}=\frac{3a\tan^2\theta\sec^2\theta}{3a\sec^3\theta\tan\theta} = \frac{\tan\theta}{\sec\theta}=\sin\theta$$

On differentiating w.r.t. $x$, we get

$$\frac{d^2y}{dx^2}=\cos\theta\cdot\frac{d\theta}{dx}=\frac{\cos\theta}{3a\sec^3\theta\tan\theta}=\frac{1}{3a}\cos^4\theta\cdot\cot\theta$$

$$\therefore \quad \left.\frac{d^2y}{dx^2}\right|_{\theta=\frac{\pi}{4}}=\frac{1}{3a}\cos^4\frac{\pi}{4}\cdot\cot\frac{\pi}{4}=\frac{1}{3a}\cdot\left(\frac{1}{\sqrt{2}}\right)^4\cdot 1$$

$$=\frac{1}{3a}\cdot\frac{1}{4}=\frac{1}{12a}$$

**32.** $\because$ $f(x)$ is continuous at $x = 0$.

$$\therefore \quad \lim_{x\to 0^+} f(x)=f(0)=\lim_{x\to 0^-} f(x) \quad \text{...(i)}$$

Now, $f(0) = a$

$$\lim_{x\to 0^+} f(x)=\lim_{h\to 0} f(0+h)=\lim_{h\to 0}\frac{\sqrt{h}}{\sqrt{16+\sqrt{h}}-4}$$

$$=\lim_{h\to 0}\frac{\sqrt{h}}{\sqrt{16+\sqrt{h}}-4}\times\frac{\sqrt{16+\sqrt{h}}+4}{\sqrt{16+\sqrt{h}}+4}$$

$$=\lim_{h\to 0}\frac{\sqrt{h}\left(\sqrt{16+\sqrt{h}}+4\right)}{16+\sqrt{h}-4^2}=\lim_{h\to 0}\left(\sqrt{16+\sqrt{h}}+4\right)=8$$

$$\lim_{x\to 0^-} f(x)=\lim_{h\to 0} f(0-h)=\lim_{h\to 0}\frac{1-\cos 4(-h)}{(-h)^2}$$

$$=\lim_{h\to 0}\frac{1-\cos 4h}{h^2}=\lim_{h\to 0}\frac{2\sin^2 2h}{h^2}=8\lim_{h\to 0}\left(\frac{\sin 2h}{2h}\right)^2=8$$

$\therefore \quad a = 8$ [From (i)]

**OR**

We have, $y=\tan^{-1}\left(\dfrac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right), x^2\le 1$

Putting $x^2 = \cos\theta \Rightarrow \theta = \cos^{-1}(x^2)$, we get

$$y=\tan^{-1}\left(\frac{\sqrt{1+\cos\theta}+\sqrt{1-\cos\theta}}{\sqrt{1+\cos\theta}-\sqrt{1-\cos\theta}}\right)$$

$$=\tan^{-1}\left(\frac{\cos\frac{\theta}{2}+\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}-\sin\frac{\theta}{2}}\right)=\tan^{-1}\left(\frac{1+\tan\frac{\theta}{2}}{1-\tan\frac{\theta}{2}}\right)$$

$$=\tan^{-1}\left(\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)\right)=\frac{\pi}{4}+\frac{\theta}{2}\Rightarrow y=\frac{\pi}{4}+\frac{1}{2}\cos^{-1}(x^2)$$

Differentiating w.r.t. $x$ on both sides, we get

$$\frac{dy}{dx}=-\frac{1\times 2x}{2\sqrt{1-x^4}}=\frac{-x}{\sqrt{1-x^4}}$$

**33.** The given function is $f(x) = |x-1| + |x+1|$

$$=\begin{cases}-(x-1)-(x+1), & x<-1\\ -(x-1)+x+1, & -1\le x\le 1\\ x-1+x+1, & x>1\end{cases}=\begin{cases}-2x, & x<-1\\ 2, & -1\le x\le 1\\ 2x, & x>1\end{cases}$$

At $x = 1$,

$$f'(1^-)=\lim_{h\to 0}\frac{f(1-h)-f(1)}{-h}=\lim_{h\to 0}\frac{2-2}{-h}=0$$

$$f'(1^+)=\lim_{h\to 0}\frac{f(1+h)-f(1)}{h}$$

$$=\lim_{h\to 0}\frac{2(1+h)-2}{h}=\lim_{h\to 0}\frac{2h}{h}=2$$

$\therefore \quad f'(1^-)\ne f'(1^+)$

$\Rightarrow \quad f$ is not differentiable at $x = 1$

At $x = -1$, $f'(-1^-)=\lim_{h\to 0}\dfrac{f(-1-h)-f(-1)}{-h}$

$$=\lim_{h\to 0}\frac{-2(-1-h)-(2)}{-h}=\lim_{h\to 0}\frac{2h}{-h}=-2$$

$$f'(-1^+)=\lim_{h\to 0}\frac{f(-1+h)-f(-1)}{h}=\lim_{h\to 0}\frac{2-2}{h}=0$$

$\therefore \quad f'(-1^-)\ne f'(-1^+)$

$\Rightarrow \quad f$ is not differentiable at $x = -1$.

**OR**

We have, $y = \sin(2\sin^{-1}x)$

$$\Rightarrow \quad y=\sin\left[\sin^{-1}\left(2x\sqrt{1-x^2}\right)\right]$$

$$\left[\because\ 2\sin^{-1}x=\sin^{-1}2x\sqrt{1-x^2}\right]$$

$$\Rightarrow \quad y=2x\sqrt{1-x^2} \quad \text{...(i)}$$

$$\Rightarrow \quad y_1=2x\times\frac{-2x}{2\sqrt{1-x^2}}+2\sqrt{1-x^2}=\frac{-4x^2+2}{\sqrt{1-x^2}} \quad \text{...(ii)}$$

$$\therefore \quad y_2=\frac{\sqrt{1-x^2}(-8x)-(-4x^2+2)\times\frac{-2x}{2\sqrt{1-x^2}}}{1-x^2}$$

$$=\frac{4x^3-6x}{(1-x^2)\sqrt{1-x^2}} \quad \therefore \quad (1-x^2)y_2=\frac{4x^3-6x}{\sqrt{1-x^2}}$$

**34.** We have, $x = \sin t$ and $y = \sin pt$

$$\frac{dx}{dt}=\cos t \text{ and } \frac{dy}{dt}=p\cos pt \Rightarrow \frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{p\cos pt}{\cos t}$$

Differentiating w.r.t. $x$ on both sides, we get

$$\frac{d^2y}{dx^2}=\frac{-p^2\sin pt\cos t+p\cos pt\sin t}{\cos^2 t}\times\frac{dt}{dx}$$

$$\Rightarrow \frac{d^2y}{dx^2}=\frac{-p^2\sin pt\cos t+p\cos pt\sin t}{\cos^3 t}$$

$$\Rightarrow \frac{d^2y}{dx^2}=-\frac{p^2\sin pt\cos t}{\cos^3 t}+\frac{p\cos pt\sin t}{\cos^3 t}$$

$$\Rightarrow \frac{d^2y}{dx^2}=\frac{-p^2y}{\cos^2 t}+\frac{x\frac{dy}{dx}}{\cos^2 t}\Rightarrow \cos^2 t\frac{d^2y}{dx^2}=-p^2y+x\frac{dy}{dx}$$

$$\Rightarrow (1-\sin^2 t)\frac{d^2y}{dx^2}=-p^2y+x\frac{dy}{dx}$$

$$\Rightarrow (1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}+p^2y=0$$

**35.** Given, $x^my^n=(x+y)^{m+n}$

Taking log on both the sides, we get

$\log x^m+\log y^n=(m+n)\log(x+y)$

$\Rightarrow\ m\log x+n\log y=(m+n)\log(x+y)$

Differentiating w.r.t. $x$ on both sides, we get

$$m\cdot\frac{1}{x}+n\cdot\frac{1}{y}\cdot\frac{dy}{dx}=(m+n)\cdot\frac{1}{x+y}\left(1+\frac{dy}{dx}\right)$$

$$\Rightarrow \frac{dy}{dx}\left(\frac{n}{y}-\frac{m+n}{x+y}\right)=\frac{m+n}{x+y}-\frac{m}{x}$$

$$\Rightarrow \frac{dy}{dx}\left(\frac{nx+ny-my-ny}{y(x+y)}\right)=\frac{mx+nx-mx-my}{x(x+y)}$$

$$\Rightarrow \frac{dy}{dx}\left(\frac{nx-my}{y(x+y)}\right)=\frac{nx-my}{x(x+y)}\quad\therefore\quad \frac{dy}{dx}=\frac{y}{x}$$

Again, differentiating w.r.t. $x$ on both sides, we get

$$\frac{d^2y}{dx^2}=\frac{x\frac{dy}{dx}-y}{x^2}=\frac{x\left(\frac{y}{x}\right)-y}{x^2}=0\ \therefore\ \frac{d^2y}{dx^2}=0$$

**36.** (i) L.H.L $=\lim\limits_{t\to30^-}T(t)=20$

R.H.L $=\lim\limits_{t\to30^+}T(t)=30$

$\therefore$ L.H.L $\neq$ R.H.L

$\therefore$ Limit does not exist.

(ii) Since limit at $t=30$ does not exist. Therefore function $T(t)$ is not continuous at $x=30$.

(iii) L.H.L. $=\lim\limits_{t\to60^-}T(t)=30$

R.H.L $=\lim\limits_{t\to60^+}T(t)=\lim\limits_{t\to60^+}(25+0.5(t-60))$

$=25+0.5\times0=25$

$\Rightarrow$ L.H.L $\neq$ R.H.L

$\therefore$ Limit does not exist

Hence, function is neither continuous nor differentiable.

**OR**

For $t\geq60$, we have

$T(t)=25+0.5\,(t-60)$

$\therefore\ T'(t)=0.5\Rightarrow T'(70)=0.5$

**37.** (i) Since, the distance is $x$ feet from the stronger light, therefore the distance from the weaker light will be $600-x$.

So, the combined light intensity from both lamp posts is given by $\frac{1000}{x^2}+\frac{125}{(600-x)^2}$.

(ii) We have, $I(x)=\frac{1000}{x^2}+\frac{125}{(600-x)^2}$

$$\Rightarrow I'(x)=\frac{-2000}{x^3}+\frac{250}{(600-x)^3}$$

$$\therefore I'(100)=\frac{-2000}{(100)^3}+\frac{250}{(600-100)^3}$$

$$=\frac{-2000}{1000000}+\frac{250}{(500)^3}=\frac{-2}{1000}+\frac{250}{125000000}$$

$$=\frac{-1}{500}+\frac{1}{500000}=\frac{-1000+1}{500000}=\frac{-999}{500000}$$

**38.** (i) L.H.L. $=\lim\limits_{x\to1^-}H(x)=\lim\limits_{x\to1}(-x^2+4x+1)$

$=-1+4+1=4$

R.H.L. $=\lim\limits_{x\to1^+}H(x)=\lim\limits_{x\to1}(2x^2-3x+1)=2-3+1=0$

L.H.L. $\neq$ R.H.L.

$\therefore$ Limit does not exist at $x=1$.

(ii) Since, limit does not exist.

$\therefore$ Function $H(x)$ is not continuous at $x=1$.

(iii) $\lim\limits_{x\to1}\frac{x^3-3x^2+2x}{x-1}=\lim\limits_{x\to1}\frac{x(x^2-3x+2)}{x-1}$

$$=\lim_{x\to1}\frac{x(x-1)(x-2)}{x-1}=\lim_{x\to1}x(x-2)=1(1-2)=-1$$

$$S(x)=\frac{x^3-3x^2+2x}{x-1}=\frac{x(x-1)(x-2)}{x-1}=x(x-2)$$

$\therefore\ S(1)=-1$

$\therefore\ S(x)$ is continuous at $x=1$.

**OR**

L.H.D. at $x=1$

$$=\lim_{h\to0}\frac{S(1-h)-S(1)}{-h}=\lim_{h\to0}\frac{(1-h)(1-h-2)+1}{-h}$$

$$=\lim_{h\to0}\frac{(1-h)(-1-h)+1}{-h}=\lim_{h\to0}\frac{-(1-h^2)+1}{-h}$$

$$=\lim_{h\to0}\frac{-1+h^2+1}{-h}=\lim_{h\to0}(-h)=0$$

## Class XII

# Monthly test drive

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

## Series-3 Matrices and Determinants

**Total Marks :** 80 **Time taken :** 60 Min.

### Only One Option Correct Type

**1.** If $A(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 \end{pmatrix}$, then $A(\theta)^3$ will be a null matrix if and only if

(a) $\theta = (2k+1)\pi/3, k \in I$ (b) $\theta = (4k-1)\pi/3, k \in I$
(c) $\theta = (3k-1)\pi/4, k \in I$ (d) None of these

**2.** If $A$ is skew-symmetric matrix of order 2 and $B$, $C$ are matrices $\begin{bmatrix} 1 & 4 \\ 2 & 9 \end{bmatrix}$ and $\begin{bmatrix} 9 & -4 \\ -2 & 1 \end{bmatrix}$ respectively, then $A^3BC + A^5(B^2C^2) + A^7(B^3C^3) + ...... + A^{2n+1}(B^nC^n)$ is

(a) a symmetric matrix
(b) a skew-symmetric matrix
(c) an identity matrix (d) None of these

**3.** If $i = \sqrt{-1}$, $a = \frac{1+\sqrt{5}}{2}$ and $b = \frac{1-\sqrt{5}}{2}$, then which of the following is an idempotent matrix ?

(a) $\begin{bmatrix} a & i \\ i & -b \end{bmatrix}$ (b) $\begin{bmatrix} b & i \\ i & -a \end{bmatrix}$

(c) $\begin{bmatrix} a & i \\ i & b \end{bmatrix}$ (d) $\begin{bmatrix} a & b \\ b & a \end{bmatrix}$

**4.** If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, then $A^4 - 5A^3 - A^2 - 4A - I =$

(a) 0 (b) $I$ (c) $A$ (d) $A + I$

**5.** If $A = \begin{bmatrix} -1 & 3/2 \\ -1/2 & 1/2 \end{bmatrix}$, then $I + A + A^2 + \ldots \infty =$

(a) $\begin{bmatrix} 1 & -3 \\ 1 & 4 \end{bmatrix}$ (b) $\frac{2}{7}\begin{bmatrix} 1 & -3 \\ -1 & 4 \end{bmatrix}$

(c) $\frac{2}{7}\begin{bmatrix} 1 & 3 \\ -1 & 4 \end{bmatrix}$ (d) Undefined

**6.** The product of all the values of $t$, for which the system of the equations $(a-t)x + by + cz = 0$, $bx + (c-t)y + az = 0$, $cx + ay + (b-t)z = 0$ has non-trivial solution, is

(a) $\begin{vmatrix} a & -c & -b \\ -c & b & -a \\ -b & -a & c \end{vmatrix}$ (b) $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$

(c) $\begin{vmatrix} a & c & b \\ b & a & c \\ c & b & a \end{vmatrix}$ (d) None of these

### One or More than One Option(s) Correct Type

**7.** Which of the following statements is (are) true?

(a) Every square matrix can be expressed as the sum of a symmetric and skew- symmetric matrices in unique way.
(b) For a square matrix '$A$', $|\text{Adj } A| = |A|^{n-1}$, where '$n$' is the order of '$A$'.
(c) If '$A$' is a non singular square matrix, then $|A^{-1}| = \frac{1}{|A|}$.
(d) Two matrices '$A$' and '$B$' are said to be conformable for multiplication in the same order iff the number of rows of $A$ is equal to number of columns of $B$.

**8.** If the equations $x + y = 1$, $(c+2)x + (c+4)y = 6$, $(c+2)^2x + (c+4)^2y = 36$ are consistent, then $c =$

(a) 1 (b) 2 (c) 3 (d) 4

**9.** If the elements of a 2 × 2 matrix $A$ are real positive and distinct such that $\det(A + A^T)^T = 0$, then

Class XII

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(c) $\begin{bmatrix} a & i \\ i & b \end{bmatrix}$ (d) $\begin{bmatrix} a & b \\ b & a \end{bmatrix}$

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(a) 0 (b) $I$ (c) $A$ (d) $A + I$

**5.** If $A = \begin{bmatrix} -1 & 3/2 \\ -1/2 & 1/2 \end{bmatrix}$, then $I + A + A^2 + \ldots \infty =$

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**8.** If the equations $x + y = 1$, $(c+2)x + (c+4)y = 6$, $(c+2)^2x + (c+4)^2y = 36$ are consistent, then $c =$

(a) 1 (b) 2 (c) 3 (d) 4

**9.** If the elements of a $2 \times 2$ matrix $A$ are real positive and distinct such that $\det(A + A^T)^T = 0$, then

(a) $\det A > 0$ (b) $\det A \geq 0$
(c) $\det(A - A^T) > 0$ (d) $\det(AA^T) > 0$

**10.** If the matrix $A = \begin{bmatrix} 0 & 2\beta & \gamma \\ \alpha & \beta & -\gamma \\ \alpha & -\beta & \gamma \end{bmatrix}$ is orthogonal, then

(a) $\alpha = \pm 1/\sqrt{2}$ (b) $\alpha = \pm 1/\sqrt{3}$
(c) $\beta = \pm 1/\sqrt{6}$ (d) $\alpha = \pm\sqrt{3}\beta$

**11.** Let $A$ and $B$ two $3 \times 3$ matrices of real numbers, where $A$ is symmetric and $B$ is skew symmetric. If $(A+B)(A-B) = (A-B)(A+B)$ and $(AB)^T = (-1)^k AB$, then the possible value(s) of $k$ is/are

(a) 2 (b) 3 (c) 5 (d) 6

**12.** If $\alpha, \beta, \gamma$ are three real numbers and

$$A = \begin{bmatrix} 1 & \cos(\alpha-\beta) & \cos(\alpha-\gamma) \\ \cos(\beta-\alpha) & 1 & \cos(\beta-\gamma) \\ \cos(\gamma-\alpha) & \cos(\gamma-\beta) & 1 \end{bmatrix}, \text{ then}$$

(a) $A$ is symmetric (b) $A$ is orthogonal
(c) $A$ is skew-symmetric (d) $A^T - A = O$

**13.** If $D_1$ and $D_2$ are two $3 \times 3$ diagonal matrices, then

(a) $D_1 D_2$ is a diagonal matrix
(b) $D_1^2 + D_2^2$ is a diagonal matrix
(c) $D_1 D_2 = D_2 D_1$
(d) $D_1^n$ is a diagonal matrix $\forall\, n \in N$

## Comprehension Type

### Paragraph for Q. No. 14 and 15

Let $A = \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}^n = [a_{ij}]^n$, where $a_{ij}$ is any element of $i^{\text{th}}$ row and $j^{\text{th}}$ column of the matrix $A$. Then

**14.** The value of $\lim\limits_{n\to\infty} \dfrac{a_{12}}{(\sqrt{2}+1)^n}$ is

(a) $\dfrac{1}{2\sqrt{2}}$ (b) $\dfrac{\sqrt{2}-1}{2\sqrt{2}}$ (c) $\dfrac{\sqrt{2}+1}{2\sqrt{2}}$ (d) $\dfrac{\sqrt{2}+1}{2}$

**15.** The value of $\lim\limits_{n\to\infty} \dfrac{a_{22}}{(\sqrt{2}+1)^n}$ is

(a) $\dfrac{1}{2\sqrt{2}}$ (b) $\dfrac{\sqrt{2}-1}{2\sqrt{2}}$ (c) $\dfrac{\sqrt{2}+1}{2\sqrt{2}}$ (d) $\dfrac{\sqrt{2}+1}{2}$

## Matrix Match Type

**16.** If $p(\theta) = \begin{vmatrix} -\sqrt{2} & \sin\theta & \cos\theta \\ 1 & \cos\theta & \sin\theta \\ -1 & \sin\theta & -\cos\theta \end{vmatrix}, q(\theta) = \begin{vmatrix} \sin 2\theta & -1 & 1 \\ \cos 2\theta & 4 & -3 \\ 2 & 7 & -5 \end{vmatrix},$

$r(\theta) = \begin{vmatrix} \cos\theta & \sin\theta & \cos\theta \\ -\sin\theta & \cos\theta & \sin\theta \\ -\cos\theta & -\sin\theta & \cos\theta \end{vmatrix}, s(\theta) = \begin{vmatrix} \sec^2\theta & 1 & 1 \\ \cos^2\theta & \cos^2\theta & \csc^2\theta \\ 1 & \cos^2\theta & \cot^2\theta \end{vmatrix}$

Then match the following:

| Column-I | | Column-II | |
|---|---|---|---|
| (P) | $p(\theta)$ | (1) | [0, 1] |
| (Q) | $q(\theta)$ | (2) | $[0, 2\sqrt{2}]$ |
| (R) | $r(\theta)$ | (3) | [−2, 2] |
| (S) | $s(\theta)$ | (4) | $[-\sqrt{5}-2, \sqrt{5}-2]$ |

| | P | Q | R | S |
|---|---|---|---|---|
| (a) | 1 | 2 | 3 | 4 |
| (b) | 2 | 4 | 3 | 1 |
| (c) | 3 | 1 | 2 | 3 |
| (d) | 4 | 3 | 1 | 2 |

## Numerical Answer Type

**17.** If $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, then $\det(A^{2005})$ equals to ______.

**18.** If $(1 + ax + bx^2)^4 = a_0 + a_1x + a_2x^2 + \ldots + a_8x^8$, where $a, b, a_0, a_1, \ldots, a_8 \in R$ such that $a_0 + a_1 + a_2 \neq 0$ and $\Delta = \begin{vmatrix} a_0 & a_1 & a_2 \\ a_1 & a_2 & a_0 \\ a_2 & a_0 & a_1 \end{vmatrix} = 0$, then the value of $\dfrac{5a}{b}$ is ______.

**19.** Suppose a matrix $A$ satisfies $A^2 - 5A + 7I = O$. If $A^5 = aA + bI$, then the value of $a + \dfrac{3b}{7} + 21$ must be ______.

**20.** Let $a_r = r({}^7C_r)$, $b_r = (7-r)({}^7C_r)$ and $A_r = \begin{pmatrix} a_r & 0 \\ 0 & b_r \end{pmatrix}$. If $A = \sum\limits_{r=0}^{7} A_r = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$, then the value of $\dfrac{(a+b)}{128}$ must be ______.

◆◆

*Keys are published in this issue. Search now!* ☺

**SELF CHECK**

No. of questions attempted ……
No. of questions correct ……
Marks scored in percentage ……

**Check your score! If your score is**

| | | |
|---|---|---|
| > 90% | EXCELLENT WORK ! | You are well prepared to take the challenge of final exam. |
| 90-75% | GOOD WORK ! | You can score good in the final exam. |
| 74-60% | SATISFACTORY ! | You need to score more next time. |
| < 60% | NOT SATISFACTORY! | Revise thoroughly and strengthen your concepts. |

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