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Sample math lesson plans embedding a competency-based approach

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Contents

Introduction

A bank of resources has been created to support teachers to implement competency-based education principles into their teaching and assessment. These resources should be used together. You can view and download the following resources from cbseacademic.nic.in:

- Learning ladder for math
- Assessment specification for math
- Sample lesson plans

This document is a compilation of ten sample lesson plans for math from class VI to class X.

Using these sample lesson plans

You can use these lesson plans as they are written in your classes as you would any other lesson plan.

When you use the lesson plans, reflect on how:

- they place the student at the centre of learning
- they vary the teacher's role (e.g., as facilitator, mediator, assessor)
- they give students choice and/or autonomy
- they bring real-world problems or applications to the classroom
- they employ (informal) formative assessment
- they promote the use of higher order thinking skills
- they ensure all students are included in learning.

You can also use these plans as templates to develop your own learner-centred lessons that encourage students to develop their competencies and skills in your subject rather than merely accumulating knowledge.

What is competency-based education (CBE)?

There is no single global definition or unifying framework for CBE. However, an overarching principle is that competency-based education focuses on the student's demonstration of learning outcomes as central to the learning process.

There is also a focus on attaining proficiency in particular competencies to facilitate progression.

Self-paced, individualised learning is a further common theme as is the emphasis on the authenticity of the learning experience and real-world applications of knowledge and skills. Central to all definitions is the goal to empower students, providing a meaningful and positive learning experience.

Competency-based education within the context of languages is best articulated in the Council of Europe's Common European Framework of Reference for Languages, the CEFR[1]. Proficiency is described on a six-point scale which provide statements of what a language user can do at each of the levels: A1 (Breakthrough), A2 (Waystage), B1 (Threshold), B2 (Vantage), C1 (Advanced), C2 (Mastery).

A useful working definition of high-quality CBE in the context of K12 education is that developed by the Aurora Institute:

- Students are empowered daily to make important decisions about their learning experiences, how they will create and apply knowledge, and how they will demonstrate their learning.
- Assessment is a meaningful, positive, and empowering learning experience for students that yields timely, relevant, and actionable evidence.
- Students receive timely, differentiated support based on their individual learning needs.
- Students progress based on evidence of mastery, not seat time.
- Students learn actively using different pathways and varied pacing.
- Strategies to ensure equity for all students are embedded in the culture, structure, and pedagogy of schools and education systems.
- Rigorous, common expectations for learning (knowledge, skills, and dispositions) are explicit, transparent, measurable, and transferable.'[2]

1. CEFR framework. Published by the Council of Europe. Available at <https://www.coe.int/en/web/common-european-framework-reference-languages/home>.
2. Competency Works, 2019. What is Competency-based Education - An Updated Definition. [pdf] Published by: Aurora Institute. Available at: <https://aurora-institute.org/wp-content/uploads/what-is-competency-based-education-an-updated-definition-web.pdf>.

What are higher order thinking skills?

These are skills, such as analysis, evaluation and synthesis, that go beyond skills such as recall and understanding. These skills are designed to stretch secondary level students to develop the cognitive skills for further progression onto more advanced level study and prepare them for the workplace.

What does competency-based education look like?

Curriculum design: A core principle in the design of CBE curricula is that it should be grounded in real-world contexts covering topics with relevance to employment and daily life. Therefore, there is an emphasis on integrating higher order thinking skills, incorporating an interdisciplinary approach (linking within and between subjects), and including a focus on problem solving using learnt skills and knowledge. Subject content and developing mastery of the prerequisite knowledge remain key components of curriculum design.

There has also been a focus on integrating 21st century skills within secondary level qualification design, with explicit links to core skills such as numeracy, literacy and social and emotional skills development as well as global citizenship and developing global literacy to enable learners to be competent not only in the national context but also in the international labour market.

Teaching and learning: A general principle of CBE-oriented delivery comprises student-centred learning, with a focus on the teacher empowering the students to learn actively supported by feedback. Whilst traditional methods have emphasised the role of the teacher as the imparter of knowledge, and subsequently place emphasis on lecturing, dictation and drilling as techniques of classroom delivery, CBE seeks to place the student at the centre and actively engage the student in the learning process. CBE delivery is facilitated by the development of lesson plans based on learning outcomes and sharing learning outcomes with students at the outset to ensure mutual understanding of expectations. The use of formative assessment, particularly elements of peer and self-assessment, are key characteristics of competency-based approaches, where students are encouraged to reflect on their own work and identify areas for improvement

Assessment: Robust and valid assessment, allowing for evaluation of the full range of learning outcomes can be considered a core feature of good practice in CBE summative assessment. Data driven, CBE-oriented summative assessments should accurately gauge the extent to which the student can demonstrate the learning outcomes, including the key skills and knowledge on completion of the programme.

Assessing the application of knowledge and skills to real-world contexts and using authentic problems which draw on real-life data are key features of CBE assessment systems concerned with real world performance. Synoptic assessment is a further key feature of CBE. This encompasses the use of assessment tasks and questions which seek to assess multiple learning outcomes and/or topic areas from across the curriculum.

Competency-based assessments should be designed to be equitable[3], enabling evaluation of a wide range of ability levels of the target group of students, which at secondary level comprises a countrywide cohort aged 15 and 16. Maintaining a balance between accessibility on the one hand and providing opportunities to demonstrate higher order thinking skills on the other is one of the aspects to consider in designing competency-based secondary school level assessments.

3. Aurora Institute, 2017. How Systems of Assessment Aligned with Competency-based Education can Support Equity. [pdf] Published by: Aurora Institute. Available at: <https://aurora-institute.org/wp-content/uploads/how-systems-of-assessment-aligned-with-competency-based-education-can-support-equity-jan-2020-web.pdf>.

Lowest Common Multiple (LCM)

Learning Ladder Assessment Content	6N2b Highest common factor and lowest common multiple Note: This lesson focuses on lowest common multiple.
Lesson duration	40 minutes
Book reference	Mathematics Textbook for Class VI, Chapter 3

Time (mins)	Learning Outcomes What we want the learners to know, understand and be able to do.	Lesson Activities	Assessment How AfL strategies will be used How AOs will be embedded
10	Applies HCF or LCM in a particular situation Note: This introductory lesson focuses on LCMs for numbers 2 to 10. A later lesson will develop use of prime factorisation to find LCMs of larger numbers.	Ask learners to think back to their Class V learning about multiples. Assign a number from 2 to 10 to each learner (different numbers for different learners). Ask each learner to write the first ten multiples of their number. Have number lines and 1–100 grids available to support learners' thinking. Discuss answers and learners' approaches as a class. • *How would you explain to someone else how to find multiples of 5?* • *What about multiples of a bigger number such as 25?*	Self-assessment of prior understanding of how to find multiples Discussion to enable teacher to assess prior understanding and to inform next steps
20		Pair up learners with different starting numbers and ask them to compare the multiples in their lists: • *Are there any numbers which appear in both of your lists?* • *Which is the smallest number that appears in both of your lists?* Explain that we call multiples that appear in both lists 'common multiples' of the two starting numbers, and that the smallest common multiple is called the 'lowest common multiple'.	Sharing and feeding back on ideas to solve a problem to develop each other's learning

	Ask learners to pair up with someone different. After sharing their starting number, but without looking at their lists, ask learners to predict and then check the lowest common multiple. Repeat several times encouraging learners to share and discuss strategies. Discuss and reinforce strategies as a class. Building on learners' own contributions to the discussion, also make links to ideas of divisibility tests, for example: • *How do you know that 120 is a common multiple of 4 and 5?* Learners use divisibility tests to confirm that 120 is a multiple of 4 and 5. • *How do you know that 110 is not a common multiple of 4 and 5?* Learners use divisibility tests to confirm that 110 is not a multiple of 4 and therefore cannot be a common multiple of 4 and 5.	Using discussion to elicit evidence of learners' understanding Sharing of strategies to develop each other's learning Discussion of strategies so teacher can observe and develop learners' mathematical thinking.
10	Pose a problem with a real-life context that can be solved using the concept of lowest common multiples. Ask learners to work in pairs or small groups to discuss and solve it. For example: • *A driver needs to refuel his car every 6 days and his van every 8 days. He refuelled both his car and his van today. How many days will it be before he refuels both vehicles on the same day again?* • *Siri has a bag of cherries. If she shares them equally between 2, 3, 4, 5 or 6 people there will be 1 cherry left over.* What is the smallest number of cherries that could be in the bag?	Collaborating with peers to improve learning

Key competencies

Communication

Collaboration

Divisibility Tests

Learning Ladder Assessment Content	6N2a Number patterns: odd, even, multiples and prime numbers 6N2c Prime factors and co-prime numbers Note: The focus for this lesson is divisibility tests.
Lesson duration	40 minutes
Book reference	Mathematics Textbook for Class VI, Chapter 3

Time (mins)	**Learning Outcomes** What we want the learners to know, understand and be able to do.	**Lesson Activities**	**Assessment** How AfL strategies will be used How AOs will be embedded
5	Recognises and appreciates (through patterns) the broad classification of numbers as even, odd, prime, co-prime, etc. Note: This lesson focuses on divisibility tests. After assessing learners' prior knowledge of divisibility tests, you will need to decide which new divisibility test to introduce in this lesson and which to cover in subsequent lessons.	Write 24 on the board and ask: *What are the factors of 24? How do you know?* Building on learners' answers, establish that 24 'is divisible by' its factors, which means that 24 can be divided by each factor exactly without leaving a remainder. Explain that today we will build on our understanding of finding factors to define tests for finding out whether one number is divisible by another, and we call these 'divisibility tests'.	Making connections with prior knowledge Sharing learning intentions

10

On a large sheet of paper write a large number and a table to record whole-class answers:

Is 27648 divisible by …?	✓ or ×
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	

Ask learners to first think individually about the number given: *Is this number divisible by each number in the table? How do you know?*

Self-assessment assessment of prior knowledge

Notes:

- It is unlikely that all learners will know divisibility tests for all these numbers initially – this activity is about establishing their prior knowledge and deciding on next steps for this and subsequent lessons.
- For your reference, clear descriptions of the divisibility tests are available at: https://nrich.maths.org/1308

Next ask learners to discuss the questions in pairs.

Then ask pairs to share their ideas with the class. Ask questions, and encourage other learners to ask questions, to clarify strategies and to express them concisely as divisibility tests.

Activating learners as learning resources for one another; sharing and feeding back in discussions to develop each other's learning

Also, use questioning to make connections with prior learning and to reinforce mathematical language, e.g.

- *How do you know it's divisible by 1? (All numbers have 1 as a factor.)*

Discussion to enable teacher to assess prior understanding and inform next steps

	• *So you know it's divisible by 2 because it ends in 8. What other numbers could in end in and still be divisible by 2? … What are numbers called that end in 0, 2, 4, 6 or 8? (even numbers)* Enter ticks and crosses in the table to reflect definite decisions so far. Revisit the table later and in subsequent lessons as learners become familiar with more divisibility tests	
10 (repeat for other divisibility tests in subsequent lessons as needed)	Use a silent teacher approach to model one of the divisibility tests that learners were not familiar with. Demonstrate the process on the board without speaking. Learners observe and, once your demonstration is complete, suggest what you were doing. Use a pose, pause, pounce, bounce approach to establish your divisibility test, e.g. • [pose] What did I do first? What do think I was thinking? [pause] • [pounce on one learner] Maryam, what do you think? • [bounce to others] Do you agree? If not, what do you think? If learners are unsure, show a second demonstration (for an example or non-example) for the same divisibility test.	Using responses to questions and discussion to elicit evidence of learners' understanding
	Ask learners to use the divisibility test on other examples and non-examples, including the large number from earlier in the lesson. Ask: • *Why is _____ divisible by _____?* • *Why is _____ not divisible by _____?* • *What could the missing digit be if this number (with a digit covered by an ink blot) is divisible by _____?*	
5	Ask learners to record the divisibility tests discussed in the lesson in their book in their own words.	Self-assessment of understanding

10	Learners complete practice questions on divisibility tests discussed across the lesson. Include ‘What could the missing digit be?’ questions. Encourage learners to use the 3B B4 Me approach to promote independent learning and use of peers as a learning resource: 1. Brain 2. Book 3. Buddy before 4. Me (Teacher) Challenge questions for learners can be found here: https://donsteward.blogspot.com/2011/06/divisibility-rules.html	Activating learners as learning resources for one another

Key competencies

Collaboration

Communication

Learning to Learn

Applying Divisibility Tests

Learning Ladder Assessment Content	6N2a Number patterns: odd, even, multiples and prime numbers 6N2c Prime factors and co-prime numbers Note: The focus for this lesson is divisibility tests.
Lesson duration	40 minutes
Book reference	Mathematics Textbook for Class VI, Chapter 3

Time (mins)	**Learning Outcomes** What we want the learners to know, understand and be able to do.	**Lesson Activities**	**Assessment** How AfL strategies will be used How AOs will be embedded
10	Recognises and appreciates (through patterns) the broad classification of numbers as even, odd, prime, co-prime, etc Note: This lesson builds on earlier lessons that introduce and practise specific divisibility tests. It focuses on application of a range of divisibility tests.	Explain that in this lesson we are aiming to apply the rules of divisibility from previous lessons to find the factors of a number quickly. Display a large number, and ask*: Is this number divisible by 2, 3, 4, 5, 6, 7, 8, 9, 10, 11?* Give individuals time to recall the divisibility tests and apply them to your number. Then ask learners to discuss their answers in pairs. Select learners to share their answers and reasoning with the class: *How do you know that it is / is not divisible by …?*	Sharing learning intentions Self-assessment assessment of prior knowledge Teacher assesses prior understanding to inform next steps
		Introduce the NRICH ‘American Billions’ game to learners: https://nrich.maths.org/796 Reveal the example game and model it with 0–9 digit cards. Then choose one or two pairs of learners to take turns to demonstrate playing the game for the class. Pair up learners and provide each pair with a set of 0–9 number cards. Learners play the game a few times recording their numbers as they play.	Pairs demonstrating the game to confirm understanding

	Once the learners have played the game a few times ask the first question from the NRICH task: *Are there any good strategies to help you to win?* Give pairs of learners time to discuss. Then bring the class together and select pairs to explain their strategies. Use a pose, pause, pounce, bounce approach, e.g. • *[pose] Can you explain one of your strategies? [pause]* • *[pounce on one learner] Arjun, do you think that is a good strategy? Why?* • *[bounce to others] Do you agree? If not, can you describe a better strategy?*	Sharing of strategies to develop each other's learning Discussion of strategies so teacher can observe and develop learners' mathematical thinking.
10	Ask pairs to return to the game, but this time working collaboratively. Ask them to investigate the other questions from the NRICH task: • *What's the longest number you can make that satisfies the rules of the game?* • *Is it possible to use all ten digits to create a ten-digit number?* Tell learners that you will ask some pairs to share their work at the end. Observe pairs as they work on the task, using questioning to prompt deeper thinking as needed: • *What have you discovered?* • *How did you find that out? / Why do you think that?* • *What made you decide to do it that way?* • *Do you think you have found the best solution? Why?* (See https://nrich.maths.org/10341 for more guidance on asking questions to support a problem-solving approach.)	Activating learners as learning resources for one another Teacher questioning to promote and assess mathematical thinking

10	Ask pairs to share with the class their approach to finding the longest number and what they found. Use questioning to help learners to structure their explanations: • *How did you start?* • *Why did you decide to do that?* • *How did you know that these were the only possibilities?* • *How your knowledge of divisibility tests help you with the problem?* Encourage other learners to ask questions and give feedback on approaches.	Eliciting evidence of learners' understanding through presentations Peer assessment of approaches

Key competencies

Collaboration

Communication

Critical Thinking (evaluating approaches)

Line Symmetry

Learning Ladder Assessment Content	7G3a Identify rotational symmetry and line symmetry Note: The focus for this lesson is line symmetry.
Lesson duration	40 minutes
Book reference	Mathematics Textbook for Class VII, Chapter 14
Resources	• Real life images, some with line symmetry and some with no line symmetry (e.g. company logos, road signs, car hub caps, patterns of floor or wall tiles) • Mirrors • Large copies of the real-life images or images of them projected onto an interactive whiteboard (optional) • Large paper equilateral triangle • Paper regular polygons for each pair (triangle, square, pentagon, hexagon, heptagon, octagon) • Worksheet with table to record findings • Large images of a variety of quadrilaterals (e.g. projected into an interactive whiteboard)

Time	**Learning Outcomes**	**Lesson Activities**	**Assessment**
(mins)	What we want the learners to know, understand and be able to do.		How AfL strategies will be used How AOs will be embedded
10	This lesson focuses on developing prior learning about line symmetry using the following Class VII NCERT Pedagogical processes from the Learning Ladder: • symmetrical figures from their environment [and which shows rotational symmetry] • the symmetry through paper folding activities	Draw an example of a shape with line symmetry and an example of a shape without line symmetry on the board and ask learners: • *How are the two shapes different?* Ask learners to think back to their Class VI learning about symmetry. • *How do you know that this shape has line symmetry?* • *How do you know that this shape does not have line symmetry?*	

	Learners work in pairs or small groups. Give out sets of real-life images, some with line symmetry and some with no line symmetry (e.g. company logos, road signs, car hub caps, patterns of floor or wall tiles). Ask learners to sort the images into two groups – those with line symmetry and those without. Also ask them to identify the number of lines of symmetry for the symmetrical images. Have mirrors available for learners to use to help them if needed.	Learners collaborating to recall prior knowledge
	Ask learners from each group to show one of their images and talk about its line symmetry: • *Does the image have line symmetry? How do you know?* • *How many lines of symmetry does it have?* • *Where are the lines of symmetry?* • *Why does/doesn't the shape have line symmetry?* **Note**: It would be helpful to have large copies of the images at this point or images of them projected onto an interactive whiteboard.	Discussion to enable teacher to assess prior understanding and inform next steps
15	Ask learners to work in pairs to find 2D shapes and patterns with line symmetry in the classroom or school. Explain that 3D objects have planes of symmetry, so they are looking for 2D shapes and patterns, which includes shapes and patterns on the surfaces of objects, e.g. patterns in tiles or curtain fabric. Learners should try to find examples with no, 1, 2, 3, 4 .... lines of symmetry. If possible, learners take photographs of their shapes and patterns, but they could also draw sketches to record their findings. Then ask pairs of learners to join with another pair and share their images. Learners should check the numbers of lines of symmetry that their peers have assigned to their shapes and patterns.	Discussion in pairs and small groups to enable informal peer assessment

10	Ask learners to work in pairs to find 2D shapes and patterns with line symmetry in the classroom or school. Explain that 3D objects have planes of symmetry, so they are looking for 2D shapes and patterns, which includes shapes and patterns on the surfaces of objects, e.g. patterns in tiles or curtain fabric. Learners should try to find examples with no, 1, 2, 3, 4 .... lines of symmetry. If possible, learners take photographs of their shapes and patterns, but they could also draw sketches to record their findings. Then ask pairs of learners to join with another pair and share their images. Learners should check the numbers of lines of symmetry that their peers have assigned to their shapes and patterns.	Discussion in pairs and small groups to enable informal peer assessment
10	Demonstrate, or invite a learner to demonstrate, using folding to identify the lines of symmetry of a large paper equilateral triangle. Learners then work in pairs using folding to identify the number of lines of symmetry for other regular polygons. They record their findings in the table provided at end of this lesson plan. Ask learners questions as they work: • *Have you found all of the lines of symmetry? How do you know?* • *What patterns have you noticed?* • *How will you check your rule?* • *Will the rule always be true for regular polygons? What about irregular polygons?*	Modelling to develop understanding Collaborating with peers to improve learning Using questioning to elicit evidence of learners' understanding and move learning forwards
5	Show large images of quadrilaterals. Ask learners to indicate, by holding up fingers or on mini whiteboards, how many lines of symmetry each quadrilateral has. Sometimes ask individuals to come out and show where the lines of symmetry are. If there are differences of opinion, ask learners with different answers to explain their thinking, and then give an opportunity for learners to 're-vote'.	Self-assessment and teacher assessment of understanding

Key competencies

Collaboration

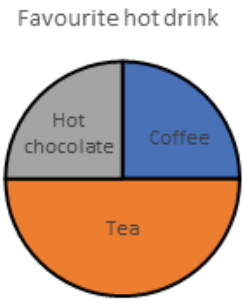
Communication

Critical Thinking

Learning to Learn

Using the Mean, Median and Mode

Learning Ladder Assessment Content	8S2a Draw and interpret bar charts and pie charts for discrete data. Note: This lesson focuses on interpreting pie charts.
Lesson duration	40 minutes
Book reference	Mathematics Textbook for Class VIII, Chapter 5

Time	Learning Outcomes	Lesson Activities	Assessment
(mins)	What we want the learners to know, understand and be able to do.		How AfL strategies will be used How AOs will be embedded
10	Draws and interprets bar charts and pie charts. Note: This lesson focuses on interpreting pie charts. Learners will not have formally learned about pie charts in earlier classes, but may have encountered them in real-life situations.	Show this pie chart to learners (it could be projected, drawn on the board or printed out). Explain that a group of people were surveyed about their favourite hot drink and the pie chart shows the results:  Ask learners to look at the pie chart and write down as many facts or pieces of information as they can about the favourite hot drinks of this group of people. Display the following words on the board to prompt learners' thinking: • fraction • percentage • ratio • equal • most	Activity to apply prior learning to a new context, in order to support new learning

- least

Collect facts about the pie chart from learners, for example:

Discussion to elicit evidence of prior learning and understanding

- ¼ of the people surveyed like hot chocolate the best.
- Equal numbers of the people like hot chocolate the best and like coffee the best.
- Tea was the most popular hot drink.
- 50% of the people surveyed said they like tea the best.
- The ratio of these people having a favourite drink of tea: coffee: hot chocolate is 2:1:1
- If there were 20 people surveyed in total, then 10 would like tea, 5 would like coffee and 5 would like hot chocolate.

Ensure learners give a reason with their statements, by asking: How do you know?

Questioning, discussion and feedback to prepare learners to access a second, more complex question

Ask learners some follow up questions. Give them time to discuss in pairs, and then select a pair to share their ideas with the class:

- *If 100 people were represented by this pie chart how many people would have said that hot chocolate is their favourite drink?*
- *If 80 people were represented by this pie chart how many people would have said that tea is their favourite drink?*
- *If 15 people said that their favourite drink was coffee how many people said their favourite drink was tea? How many people were asked altogether?*

Emphasise that pie charts show proportion and that we can only know how many people or items are in a particular category if we are given some additional information to go with the pie chart (for example, how many people were asked).

Repeat the activity with another pie chart with more sectors:

Method of travel to school for Class VIII

	Ask learners to write down as many facts as they can about the pie chart. Select learners to share their facts. Remember to ask probing questions such as: • *How do you know that?* • *What feature(s) of the pie chart show that?*	
10	Learners work in pairs on a pie chart matching activity from: https://www.tes.com/teaching-resource/maths-which-pie-chart-game-6163415. Once learners have had a chance to discuss and complete the matching in pairs, provide solutions so learners can assess their own work.	Collaborative working in pairs, using peers as a resource for learning Self- and peer-assessment of learning
10	Explain to learners that we are going to develop our interpretation of pie charts further. Show the learners an example of a pie chart which includes more challenging angles, such as the example in Task 1 from: https://variationtheory.com/2018/10/30/interpreting-pie-charts/ Pose these questions to learners: • *How can we describe the proportions shown on the pie chart?* • *How can we calculate the amount of people represented in each category on the pie chart?* Initially, ask learners to think about the questions individually for a couple of minutes. Then ask them to discuss their thinking in pairs for a couple more minutes. Finally, select learners to share their thoughts with the rest of the class. Guide learners through the worked example in Task 1 by asking questions such as: *where d*Ask learners to work in pairs to attempt the 'your turn' problem. Once learners have had time to discuss and complete this, share the solutions for learners to self-assess their understanding.	Sharing learning intentions Collaborative working using peers as a resource for learning Questioning to elicit evidence of understanding and move learning forwards Self-assessment of own understanding

10	Ask learners to answer questions on interpreting pie charts from: https://donsteward.blogspot.com/2012/06/interpreting-pie-charts.html Some problem-solving style questions, which could be used to stretch more confident learners, can be found from the links below: https://mathshko.com/2017/11/27/pie-charts/ https://www.stem.org.uk/resources/elibrary/resource/35556/pie-chart-puzzles	Activity to elicit evidence of understanding

Key competencies

Collaboration

Communication

Critical Thinking

Creative Thinking

Learning to Learn

Using the Mean, Median and Mode

Learning Ladder Assessment Content	9S1c Calculate mean, median and mode of ungrouped data
Lesson duration	40 minutes
Book reference	Mathematics Textbook for Class IX, Chapter 14

Time	**Learning Outcomes**	**Lesson Activities**	**Assessment**
(mins)	What we want the learners to know, understand and be able to do.		How AfL strategies will be used How AOs will be embedded
10	Identifies and classifies the daily life situations in which mean, median and mode can be used. Note: Calculation of mean, median and mode is introduced in Class VII. This lesson starts with consolidating this prior learning.	Ask learners to think back to their Class VII learning about mean, median and mode. Select learners to explain how to calculate the mean, median or mode. Support them by writing a set of numbers on the board and asking them to explain how to find the mean, median or mode for the set of numbers. Check all learners' understanding of calculating the mean, median and mode. Display a data set and ask all learners to calculate and show the mean, then the median, then the mode on mini whiteboards.	Sharing explanations to help learners recall the pre-requisite prior knowledge Whole-class response technique providing quick assessment of all learners' understanding to inform next steps
10		Ask learners to work in pairs on the problem-solving task 'M, M and M' from NRICH: https://nrich.maths.org/6267 For confident learners, there is also a follow up activity that could be used called 'Unequal Averages': https://nrich.maths.org/11281 Bring the class together and select pairs to share their solutions, using questioning to elicit further information from learners. • *What approach did you use?* • *How did you know you needed to use those values?*	Collaborative working to develop each other's thinking through sharing and feeding back on ideas and approaches Questioning and discussion to elicit evidence of understanding and move learning forwards

	• *Have we found all of the possibilities? How can we be sure?*	
15	Ask learners to work in pairs to discuss the advantages and disadvantages of each of the three averages – mean, median and mode. After time to discuss, select pairs to share one advantage or disadvantage of one of the averages with the rest of the class. Record learner's ideas on the board. Encourage learners to ask questions and comment on the suggestions shared by their peers. If learners are finding it challenging to identify advantages and disadvantages of the different averages, show some examples. For example, show a data set with an extreme value and ask about the appropriateness of finding the mean. There are some simple examples which can be used to prompt ideas at: https://www.tes.com/teaching-resource/which-average-is-best-6278226 Once the class has produced a summary of the advantages and disadvantages, ask learners to work in pairs on some real-life situations. They must decide which average would be most appropriate to use and why. A clear summary of advantages and disadvantages of mean, median and mode and a card sorting activity with examples can be found at: https://www.tes.com/teaching-resource/which-average-to-use-mmmr-11053824	Sharing of ideas and discussion to elicit evidence of understanding and move learning forwards Collaborative working and real-life contexts to support learners in applying their learning
5	Ask learners to share their thoughts on the use of 'average' in everyday life. • *Is it always clear which average is being used?* • *Are all the averages used appropriate? Why? Why not?* • *Why might people or companies decide to use one of mean, median or mode? Is this always based on which is most appropriate?* The following article share ideas around use of 'average' in everyday life: http://news.bbc.co.uk/2/hi/uk_news/magazine/7581120.stm	Whole class discussion encourages sharing of ideas and experiences to develop each other's learning

Key competencies

Collaboration

Communication

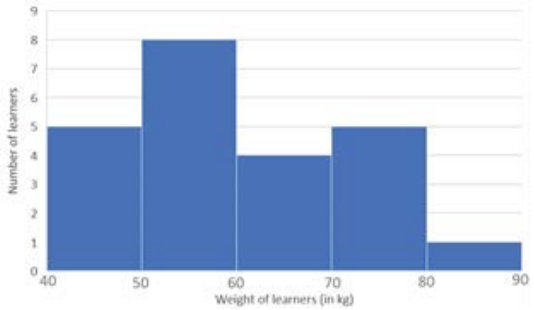
Critical Thinking

Social Responsibility

Bar Graphs and Histograms

Learning Ladder Assessment Content	9S1b Presentation of data — construct and interpret bar graphs, histograms (with varying base lengths), frequency polygons for data given in various forms: ungrouped / grouped data in list or tables. Note: The focus of this lesson is bar graphs and histograms with equal class widths.
Lesson duration	40 minutes
Book reference	Mathematics Textbook for Class IX, Chapter 14

Time	**Learning Outcomes**	**Lesson Activities**	**Assessment**
(mins)	What we want the learners to know, understand and be able to do.		How AfL strategies will be used How AOs will be embedded
5	Analyses data by representing it in different forms like, tabular form (grouped or ungrouped), bar graph, histogram (with equal and varying width and length), and frequency polygon. Note: This introductory lesson focuses on bar graphs and histograms with equal class widths. A later lesson will introduce histograms with varying class widths, frequency density and frequency polygons.	Ask learners to think back to their previous learning about bar graphs. Show an example of a bar graph with some questions on interpreting the bar graph, such as the example from https://ssddproblems.com/the-bar-chart/. Ask learners to write some other questions that they could ask based on this bar graph. Select learners to share their answers to the original questions and to share the other questions they have created. Ensure learners give reasons for their answers by asking probing questions such as: • *How do you know?* • *Why do you think that?* Further questions to ask learners include: • *What is the variable in this bar graph?* • *What type of data can be presented in a bar graph?* • *What are the important features of a bar graph?*	Assessing prior knowledge and understanding of bar graphs through answering and asking questions Eliciting evidence of their understanding through sharing of ideas as a class.

	Establish that a bar graph is a pictorial representation of discrete data and should have equal bar widths, equal gaps between bars and a consistent scale.	
10	Explain to learners that today they will be exploring representing data in a histogram. Show learners an example of a histogram with equal class widths, for example:  Ask learners: • *What does this histogram show?* • *How is it the same as a bar graph? How is it different from a bar graph?* • *What is the variable represented in this histogram?* • *What type of data is presented in this histogram?* • *How many learners weigh between 60 and 70 kilograms?* • *Is it possible to tell how many learners weigh 46kg? Why not?* Establish that a histogram is a pictorial representation of data, similar to a bar graph, but it is used for continuous class intervals (such as weight). Learners should observe that there are no gaps between the bars and the horizontal axis is a continuous scale. Explain that in a histogram, the areas of the bars are proportional to the corresponding frequencies. In this case, the bars have equal width, but that is not always the case with histograms. They will encounter histograms with varying width in a subsequent lesson.	Sharing learning intentions Sharing and feeding back on ideas to develop each other's learning Using discussion to elicit evidence of learners' understanding

20	Explain to learners that they are going to draw a histogram to represent the heights of learners in the class. Give each learner a measuring tape or ruler for them to measure their height and then collect the data by writing the height of each learner on the board. Explain to learners that they should initially present this raw data in a table and will need to decide on the class widths they will use. (They should be encouraged to use equal class widths at this point).	
	Once learners have drawn their histograms, encourage them to compare their representation of the data with other learners, especially those who have chosen to use a different class width. • *What is the same and what is different about your histograms?* • *What are the benefits of using smaller/ bigger class widths?* As an extension, you could ask learners how they would represent the data in a histogram with variable class widths. Remind learners that the area (rather than the height) of the bars should be in proportion to the frequency.	Using discussion to elicit evidence of learners' understanding
5	Ask learners to write three questions about the histogram they have drawn, e.g., How many learners were taller than 140 cm? What proportion of learners were between 160 and 175 cm tall? Learners swap their questions with a partner and answer their partner's questions.	Collaborating with peers to improve learning

Key competencies

Collaboration

Communication

Creative Thinking

Lowest Common Multiple (LCM)

Learning Ladder Assessment Content	10A3a Solve quadratic equations by factorisation and by using the quadratic formula (where roots are real). Note: This lesson focuses on solving quadratic equations, where the coefficient of x^2 is 1, by factorising.
Lesson duration	40 minutes
Book reference	Mathematics Textbook for Class X Chapter 4

Time	Learning Outcomes	Lesson Activities	Assessment
(mins)	What we want the learners to know, understand and be able to do.		How AfL strategies will be used How AOs will be embedded
10	Demonstrates strategies of finding roots and determining the nature of roots of a quadratic equation. Note: This lesson focuses on solving quadratic equations, where the coefficient of x^2 is 1, by factorising. It builds on work on factorising quadratic expressions from Class IX	Write x^2+4x+3 and x^2-4x+3 on the board. Ask learners to think back to their Class IX learning about factorising quadratic expressions. Ask them to think individually about how they would factorise these two quadratic expressions. Then ask them to discuss their approach and solution in pairs. Select pairs of learners to share their factorisations with the class, asking *how* they found them: • *What was your approach to factorising these expressions?* • *How can you check your factorisation is correct?* Learners work in pairs to investigate the problems from https://undergroundmathematics.org/quadratics/factorisable-quads. As learners are working, circulate and ask questions such as: • *What was your approach to finding the solutions?* • *How many solutions did you find?* • *Have you found all the possibilities? How do you know?*	Self-assessment of prior understanding of how to find multiples Discussion to enable teacher to assess prior understanding and to inform next steps

10	Explain to learners that we are going to use factorising quadratics to help us to solve quadratic equations. Write the following question on the board: ★ ×■=0 Ask learners to think about this question, giving them a minute or so to do so. Ask learners: • *What can you tell me about ★ or ■?* • *How do you know?* Through whole class discussion of these questions, develop the idea that either ★ or ■ (or both) must be 0. Then, pose each of the following questions in a sequence, asking learners for their suggestions at each stage and demonstrating on the board how you would set out the working as learners respond: • *If ab=0, what are the possible values of a and b?* Establish that a=0 or b=0. • *If a(b+1)=0 what are the possible values of a and b?* Establish that a=0 or b+1=0, and hence a=0 or b=-1. • *If (a+1)(b-3)=0, what are the possible values of a and b?* Establish that a+1=0 or b-3=0, and hence a=-1 or b=3. • *If (a-3)(a+2)=0, what are the possible values of a?* Establish that either a-3=0 or a+2=0, and hence a=3 or a=-2. Ask learners to practise finding solutions to factorised expressions by answering the questions from: https://variationtheory.com/2018/03/23/solve-when-factorised/	Sharing learning intentions Questioning to establish understanding and to develop learning through sharing of ideas
15	Bring learners back together as a class. Write $x^2+5x+4=0$ on the board. Ask learners to attempt this question individually using the factorising and solving skills that we have been using during the lesson. Select learners to share the steps in solving this quadratic.	Questioning and discussion elicits evidence of understanding from earlier in the lesson

	Give further examples, including examples involving negative coefficients (but ensure the coefficient of x^2 is always 1). Each time, select learners to share the steps in solving the quadratic. Ask learners to practise factorising and solving quadratics where the coefficient of x^2 is 1, for example those from https://variationtheory.com/2018/11/26/solving-quadratics-by-factorising/ As an extension to this, confident learners can work on factorising and solving quadratics where the coefficient of x^2 is not 1, from https://variationtheory.com/2019/09/28/solving-quadratics-by-factorising-when-coefficient-of-%f0%9d%91%a521/. Some learners may be able to do this with little guidance, but it is suggested that this is taught to the whole class in a subsequent lesson. A similar approach to this lesson can be taken, e.g. factorising quadratic expressions where the coefficient of x^2 is not 1, then solving $a(2b+1)=0$, solving $(2a+3)(a-4)=0$, solving $(2a+5)(a+3)=0$, and then combining the skills to solve quadratic equations such as $2a^2+3a+1=0$.	Sharing of ideas through questioning and discussion develops learning
5	Ask learners to work in pairs and give them example work with deliberate errors in it from https://www.tes.com/teaching-resource/clumsy-clive-on-solving-quadratics-11569114 Learners identify the errors in the first two questions and write the corrected working.	Collaborative working, using peers as a resource for learning

Key competencies

Collaboration

Communication

Learning to Learn

Solving Quadratic Equations using the Quadratic Formula

Learning Ladder Assessment Content	10A3a Solve quadratic equations by factorisation and by using the quadratic formula (where roots are real). Note: This lesson focuses on solving quadratic equations using the quadratic formula.
Lesson duration	40 minutes
Book reference	Mathematics Textbook for Class X, Chapter 4

Time	Learning Outcomes	Lesson Activities	Assessment
(mins)	What we want the learners to know, understand and be able to do.		How AfL strategies will be used How AOs will be embedded
5	Demonstrates strategies of finding roots and determining the nature of roots of a quadratic equation. Note: This lesson focuses on on solving quadratic equations using the quadratic formula. It assumes that learners have previously derived the quadratic formula by completing the square in a previous lesson.	Write a selection of expressions on the board and ask learners to find the value of these by using substitution, e.g. • $d+3(e-2f)$ *where* $d=5,e=8,f=3$ • $3g^2-2(5h-2i)$ *where* $g=3,h=4,i=-3$ • $3j-2(k+3l)^2$ *where* $j=-4,k=4,l=-1$ • $\frac{m+n}{3}$ *where* $m=4,n=-1$ • $p+\frac{2q-r}{p}$ *where* $p=5,q=3,r=2$ • $3s+\sqrt{(t^2-4u)}$ *where* $s=-2,t=6,u=5$ • $3s+\sqrt{(t^2-4u)}$ *where* $s=6,t=-3,u=-4$ As learners are working, circulate and ask questions such as: • When we substitute the values into this expression what does our calculation become? • How do we know what order to perform the calculations in? • Why is it important to follow the correct order of operations?	Questioning to probe understanding and mathematical thinking and inform next steps

10	Write the quadratic formula on the board: $x=\frac{b\pm\sqrt{(b^2-4ac}}{2a}$ Bring the whole class together and remind learners that they derived the quadratic formula by completing the square in a previous lesson. Explain that the quadratic formula is a way of finding the solutions (or roots) of quadratic equations. Start by demonstrating using the formula to solve a quadratic equation that can also easily be solved by factorisation and completing the square, for example $x^2+4x+3=0$. Emphasise that we need the quadratic equation to be in the form $ax^2+bx+c=0$ to find the coefficients a, b and c to substitute into the formula: • *What are the values of a, b, c for this quadratic?* • *What does the quadratic formula read when we substitute these values?* • *What order do we perform the calculations in the quadratic formula in?* Once the values of x have been found by using the quadratic formula, use factorisation and completing the square to confirm that all three methods give the same solutions: • *How can we factorise this quadratic?* • *How can we complete the square for this quadratic?* • *What are the benefits of using each method to solve a quadratic equation?*	Questioning and discussion elicits evidence of understanding Sharing of ideas through questioning and discussion develops learning
10	Explain to learners that first they are going to focus on correctly identifying the coefficients a, b and c in quadratic equations. On the board give a range of examples of quadratic equations (with a mixture of positive and negative coefficients). Ask learners to show their values of a, b and c on mini whiteboards. Learners work in pairs to collaborate on completing the following jigsaw based on identifying a, b and c: https://www.tes.com/teaching-resource/introduction-to-the-quadratic-formula-jigsaw-6124171	Sharing learning intentions Collaborative working, using peers as a resource for learning through sharing of approaches

10	Explain to learners that they are now going to use the worked example from earlier to solve quadratics by using the formula. Ask learner to work on the following structured worksheets (filling the blanks in calculation steps) by discussing in their pairs: https://www.tes.com/teaching-resource/gcse-the-quadratic-formula-revision-worksheet-6112360 https://www.tes.com/teaching-resource/the-quadratic-formula-12332649	Sharing learning intentions Collaborative working, using peers as a resource for learning through sharing of approaches
5	As a whole class ask learners to identify errors in the calculation steps for some questions where a quadratic equation has been solved using the formula: https://www.tes.com/teaching-resource/the-quadratic-formula-spot-the-mistake-6427939 Learners can indicate errors using mini whiteboards. Select learners to explain their answers, especially where learners disagree about the errors. In a following lesson, learners will need consolidation on using the quadratic formula to solve quadratic equations, including answering non-scaffolded questions.	Analysing response to identify issues and move learning forwards Whole class questioning and response to establish understanding and to develop learning through sharing of ideas

Key competencies

Collaboration

Communication

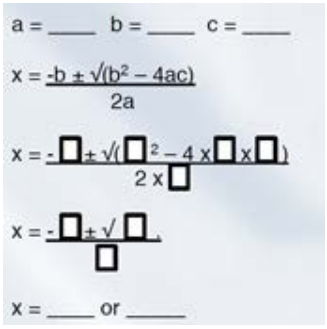
Critical Thinking

Learning to Learn

The Nature of the Roots of a Quadratic Equation

Learning Ladder Assessment Content	10A3b Know and use the relationship between the discriminant and the nature of the roots.
Lesson duration	40 minutes
Book reference	Mathematics Textbook for Class X, Chapter 4

Time	Learning Outcomes	Lesson Activities	Assessment
(mins)	What we want the learners to know, understand and be able to do.		How AfL strategies will be used How AOs will be embedded
5	Demonstrates strategies of finding roots and determining the nature of roots of a quadratic equation. Note: This lesson focuses on determining the nature of roots. It assumes that learners are already confident, from previous lessons, in finding roots of quadratic equations using factorisation and the quadratic formula, and by completing the square.	Write a quadratic equation (with two real roots) on the board, such as $4x^2-2x-5=0$. Ask learners to think back to previous lessons where they have found the roots of quadratic equations. Ask them to solve your quadratic equation using the quadratic formula. They should clearly display the steps in their working and can use the scaffolding below for support: a = ____ b = ____ c = ____ $x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$ $x = \frac{-\square \pm \sqrt{(\square^2 - 4 \times \square \times \square)}}{2 \times \square}$ $x = \frac{-\square \pm \sqrt{\square}}{\square}$ x = ____ or ____ Display the solutions for x on the board and ask learners to peer assess their work.	Self- and peer-assessment of prior learning

20	Explain that today we are going to investigate the number of real roots of quadratic equations. Learners work in small groups. Give each learner a die, a coin and some sheets of paper. Explain that they are going to individually create random quadratic equations by rolling a die (for the number) and flipping a coin (heads for positive and tails for negative) to generate the coefficients a, b and c. For example, a=-3 if the die shows 3 and the coin shows tails, b=4 the die shows 4 and the coin shows heads, and c=1 if the die shows 1 and the coin shows heads. So, the generated quadratic equation is	Sharing learning intentions
15	$-3x^2+4x+1=0$ Learners write each quadratic equation on the top of a piece of paper and solve it using the formula. They should write out clear working showing the substitution of a, b and c. They can use the scaffolding for support if needed:  As they solve each quadratic equation, learners should classify the quadratics according to the number of roots that their equation has. In their groups, learners should begin to collect a group of quadratics with two real roots, a group of quadratics with just one real root and a group of quadratics with no real roots and should begin to look for any patterns they can see. As learners are working, circulate and ask questions such as: • *How many roots can a quadratic equation have? Is that all of the possibilities?* • *What have you found out so far?*	Collaborative working in groups, using peers as a resource for learning Questioning elicits evidence of learner understanding and enables sharing of ideas between peers

	• *What is the same about this group of quadratic equations?* • *What is different between the groups of quadratic equations?* • *Can you see a pattern/rule which tells you how many roots a quadratic equation had without solving it?* • *Have you tested your rule?* • *Can you explain why your rule works?*	Encouraging learners to discuss their thinking and communicate mathematically.
5	Once learners have had some time to work on the problem, bring the whole class together to discuss their findings. Select learners or groups of learners to share what they have found. The questions above can be used again at this point to elicit understanding.	Questioning elicits evidence of learner understanding and enables sharing of ideas between peers
	Establish the connection between the value of b^2-4ac in the square root of the quadratic formula and the number of roots of the quadratic equation. Explain that b^2-4ac is called 'the discriminant'. Ask learners to write a rule using the discriminant to indicate the number of roots for a quadratic equation, e.g. $b^2-4ac<0$ means there are no real roots $b^2-4ac=0$ means there is one real root $b^2-4ac>0$ means there are two distinct real roots	
10	Give learners some further quadratics to group by number of roots using the discriminant. This can be completed in pairs as a card sort activity. Provide the solutions to allow learners to check their work. As an extension to this, confident learners can consider the following problem: $x^2+px+4=0$ *What values of p would give two real roots?*	Working collaboratively in pairs to improve learning and check understanding Providing solutions to allow for self-checking

Key competencies

Collaboration

Communication

Critical Thinking

Learning to Learn