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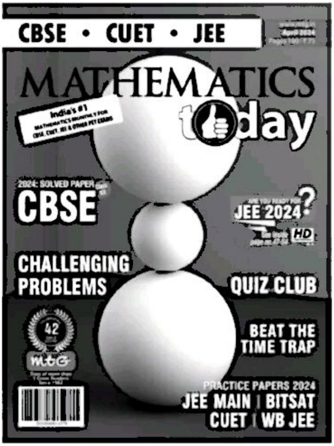

MATHEMATICS today

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Managing Editor
Mahabir Singh

Editor
Anil Ahlawat

Corporate Office:
Plot 99, Sector 44 Institutional area, Gurugram -122 003 (HR).
Tel : 0124-6601200 e-mail : info@mtg.in website : www.mtg.in

Regd. Office:
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PRACTICE PAPER

JEE Main

2024

Exam Dates
Session-2
Between 4^{th} April and 15^{th} April 2024

SECTION-A (MULTIPLE CHOICE QUESTIONS)

1. Let z be a complex number satisfying $|z + 16| = 4|z + 1|$. Then
   (a) $|z| = 4$ (b) $|z| = 5$
   (c) $|z| = 6$ (d) $3 < |z| < 68$

2. If $(1 + x - 2x^2)^8 = a_0 + a_1x + a_2x^2 + ... + a_{16}x^{16}$, then the sum $a_1 + a_3 + a_5 + ... + a_{15}$ is equal to
   (a) -2^7 (b) 2^7
   (c) 2^8 (d) None of these

3. Let $g(x) = 1 + x - [x]$, $[x]$ is the greatest integer not greater than x. If $f(x)=\begin{cases}-1, & x<0\\ 0, & x=0\\ 1, & x>0\end{cases}$, then for all x, $f(g(x))$ is equal to
   (a) x (b) 1 (c) $f(x)$ (d) $g(x)$

4. If m and M are the least and greatest values of $(\cos^{-1} x)^2 + (\sin^{-1} x)^2$, then $M - m =$
   (a) $\frac{9\pi^2}{8}$ (b) $\frac{5\pi^2}{8}$ (c) $\frac{\pi^2}{8}$ (d) $\frac{3\pi^2}{8}$

5. The domain of $f(x) = \frac{3}{4-x^2}+\log_{10}(x^3-x)$ is
   (a) $(1, 2)$
   (b) $(-1, 0) \cup (1, 2)$
   (c) $(1, 2) \cup (2, \infty)$
   (d) $(-1, 0) \cup (1, 2) \cup (2, \infty)$

6. In a triangle ABC, AD is altitude from A. If $b > c$, $C = 23°$ and $AD=\frac{abc}{b^2-c^2}$, then $B =$
   (a) 107° (b) 97° (c) 117° (d) 113°

7. The number of positive integers, which can be formed by using any number of digits 0, 1, 2, 3, 4, 5 using each digit not more than once in each number, is
   (a) 1600 (b) 1620 (c) 1630 (d) 1720

8. The sum of four numbers in A.P. is 48, and the product of the extremes is to the product of the two middle terms is 27 to 35. The largest term of the A. P. is
   (a) 10 (b) 12 (c) 14 (d) 18

9. Evaluate : $\lim_{x\to 0}(\cos x+a\sin bx)^{\frac{1}{x}}$
   (a) a (b) ab (c) $e^{\sqrt{ab}}$ (d) e^{ab}

10. The line L given by $\frac{x}{5}+\frac{y}{b}=1$ passes through the point (13, 32). The line K is parallel to L and has the equation $\frac{x}{c}+\frac{y}{3}=1$. The distance between L and K is
    (a) $\sqrt{17}$ (b) $\frac{17}{\sqrt{15}}$ (c) $\frac{23}{\sqrt{17}}$ (d) $\frac{23}{\sqrt{15}}$

11. The lines $2x - 3y = 5$ and $3x - 4y = 7$ are diameters of a circle of area 154 sq. units, then the equation of the circle is
    (a) $x^2 + y^2 + 2x - 2y = 62$
    (b) $x^2 + y^2 + 2x - 2y = 47$
    (c) $x^2 + y^2 - 2x + 2y = 47$
    (d) $x^2 + y^2 - 2x + 2y = 62$

12. The line passing through the extremity A of the major axis and extremity B of the minor axis of the ellipse $x^2 + 9y^2 = 9$ meets the auxiliary circle at the point M. The area of the triangle AMO is
    (a) $\frac{31}{10}$ (b) $\frac{29}{10}$ (c) $\frac{21}{10}$ (d) $\frac{27}{10}$

13. The mean and variance of 5 observations of an experiment are 4 and 5.2 respectively. From these observations three are 1, 2 and 6 and $\lambda = |x_1 - x_2| + 8$, where x_1 and x_2 are remaining observations. Then number of solution of equation $10 - x^2 - 2x = \lambda$ are

(a) 1 (b) 2 (c) 3 (d) 4

14. If the letters of the word NALGONDA are arranged in arbitrary order, the probability that the letters G, O, D appear in that order is

(a) $\frac{1}{6}$ (b) $\frac{1}{24}$ (c) $\frac{1}{120}$ (d) $\frac{1}{10}$

15. Let $A=\begin{bmatrix}2 & 2\\ 2 & 2\end{bmatrix}$ and $B=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$. Consider the following statements.

$(S_1) : (A + B)^2 = A^2 + 2\,AB + B^2$ for the given matrices.

$(S_2) : AB = BA$ for the above matrices.

Then

(a) Both (S_1) and (S_2) are true
(b) only (S_1) is true
(c) only (S_2) is true
(d) Both (S_1) and (S_2) are false

16. The function $f(x) = [x^2] - [x]^2, \forall x \in R$ is defined for $[-1, 0]$ also, where $[\cdot]$ denotes the greatest integer function, then

(a) $f(x)$ is continuous at the end points of the interval $[-1, 0]$
(b) $f(x)$ is not continuous at the end points of the interval $[-1, 0]$
(c) $f(x)$ is continuous only at $x = 0$
(d) None of these

17. If $f(x)=\begin{cases}3x^2+12x-1, & -1\le x\le 2\\ 37-x, & 2<x\le 3\end{cases}$, then

(a) $f(x)$ is increasing on $[-1, 2]$
(b) $f(x)$ is continuous on $[-1, 3]$
(c) $f'(2)$ does not exist
(d) All of these

18. If $\int \frac{3\cot 3x - \cot x}{\tan x - 3\tan 3x}\,dx = ax + b\log\left|\frac{\sqrt{3}-\tan x}{\sqrt{3}+\tan x}\right| + C$, then

(a) $a = 1, b = \sqrt{3}$ (b) $a = 1, b = -\sqrt{3}$
(c) $a = 1, b = \frac{1}{\sqrt{3}}$ (d) $a = \frac{1}{2}, b = \frac{5}{4\sqrt{3}}$

19. Let $A = \int_{-\pi/2}^{\pi/2} \sin^8 x \cos^4 x\,dx$ and

$B = \int_0^{\pi/2} \sin^4 x \cos^8 x\,dx$, then

(a) $A = B$ (b) $A = \frac{\pi}{1024}$
(c) $B = \frac{7\pi}{1024}$ (d) $B = \frac{7\pi}{2048}$

20. The area bounded by the curve $y = x|x|$, x-axis and the ordinates $x = 1, x = -1$ is given by

(a) 1/2 sq. unit (b) 1/3 sq. unit
(c) 2/3 sq. unit (d) 1 sq. unit

SECTION-B (NUMERICAL VALUE TYPE)

Attempt any 5 questions out of 10.

21. If $x\frac{dy}{dx} = x^2 + y - 2$, $y(1) = 1$, then $y(2)$ equals ______.

22. Let $\vec{u}, \vec{v}, \vec{w}$ be such that $\vec{u} + \vec{v} + \vec{w} = 0$. If $|\vec{u}| = 3, |\vec{v}| = 4, |\vec{w}| = 5$, then $|\vec{u}\cdot\vec{v} + \vec{v}\cdot\vec{w} + \vec{w}\cdot\vec{u}|$ is equal to ______.

23. If the shortest distance between the straight lines $3(x-1)=6(y-2)=2(z-1)$ and $4(x-2)=2(y-\lambda)=(z-3)$, $\lambda \in R$ is $\frac{1}{\sqrt{38}}$, then the integral value of λ is equal to ______.

24. Of the three independent events, E_1, E_2 and E_3 the probability that only E_1 occurs is α, only E_2 occurs is β and only E_3 occurs is γ. Let the probability p that none of the events E_1, E_2 or E_3 occurs satisfy the equations $(\alpha - 2\beta)p = \alpha\beta$ and $(\beta - 3\gamma)p = 2\beta\gamma$. All the given probabilities are assumed to lie in the interval (0, 1). Then $\frac{\text{Probability of occurrence of } E_1}{\text{Probability of occurrence of } E_3} =$ ______.

25. If $f(x) = \left(100^5 - x^{10}\right)^{\frac{1}{10}}$, then the value of $\frac{1}{2^{10}} f(f(1024))$ is ______.

26. Suppose F_1 and F_2 are the foci of the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$. P is a point on ellipse such that $PF_1 : PF_2 = 2 : 1$. The area of the triangle $PF_1\,F_2$ is ______ sq. units.

27. If $b^2 - 4ac \le 0$ (where $a \ne 0$ and $a, b, c, x, y \in R$) satisfies the system $ax^2 + x(b-3) + c + y = 0$ and $ay^2 + y(b-1) + c + 3x = 0$, then the value of $\frac{y}{x}$ is ______.

28. Let X be the set consisting of the first 2018 terms of the arithmetic progression 1, 6, 11, ... , and Y be the set consisting of the first 2018 terms of the arithmetic progression 9, 16, 23, ... Then, the number of elements in the set $X \cup Y$ is ______.

29. Let a and b respectively be the points of local maximum and local minimum of the function $f(x) = 2x^3 - 3x^2 - 12x$. If A is the total area of the region bounded by $y = f(x)$, the x-axis and the lines $x = a$ and $x = b$, then $4A$ is equal to ______.

30. Let A & B are two matrices of order 3×3 where $A = \begin{pmatrix} 1 & 3 & \lambda-3 \\ 2 & -1 & 1 \\ 3 & 2 & 1 \end{pmatrix}$ & $B = \begin{pmatrix} 3 & 2 & 4 \\ 3 & 2 & 5 \\ 2 & 1 & 4 \end{pmatrix}$ if A is a singular matrix, then trace $(A + B)$ equals ______.

SOLUTIONS

1. (a) : We have, $|z+16|^2 = 16|z+1|^2$

$\Rightarrow (z+16)(\bar{z}+16) = 16(z+1)(\bar{z}+1)$

$\Rightarrow z\bar{z} + 16z + 16\bar{z} + 256 = 16z\bar{z} + 16z + 16\bar{z} + 16$

$\Rightarrow z\bar{z} = 16 \Rightarrow |z|^2 = 16 \Rightarrow |z| = 4$

2. (a) : Putting $x = 1$, $x = -1$ in the given equation, we get

$(a_0 + a_2 + a_4 + ... + a_{16}) + (a_1 + a_3 + ... + a_{15}) = 0$ ...(i)

and $(a_0 + a_2 + a_4 + ... + a_{16}) - (a_1 + a_3 + ... + a_{15}) = 2^8$ ...(ii)

Subracting (ii) from (i), we get

$\therefore\ a_1 + a_3 + a_5 + ... + a_{15} = \frac{1}{2}(0 - 2^8) = \frac{-1}{2}(2^8) = -2^7$

3. (b) : $g(x) = 1 + x - [x] = 1 + (x - [x]) \ge 1 > 0$

$\therefore\ g(x) > 0$ for all x $\quad\therefore\ f(g(x)) = 1$ for all x.

4. (a) : $(\cos^{-1}x)^2 + (\sin^{-1}x)^2$

$= (\cos^{-1}x + \sin^{-1}x)^2 - 2\sin^{-1}x\cos^{-1}x$

$= \frac{\pi^2}{4} - 2\sin^{-1}x\left(\frac{\pi}{2} - \sin^{-1}x\right)$

$= 2\left[(\sin^{-1}x)^2 - \frac{\pi}{2}\sin^{-1}x + \frac{\pi^2}{8}\right]$

$= 2\left[\left(\sin^{-1}x - \frac{\pi}{4}\right)^2 + \frac{\pi^2}{16}\right] = f(x)$

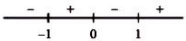
$m = f\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi^2}{8}$,

$M = f(-1) = 2\left[\left(-\frac{\pi}{2} - \frac{\pi}{4}\right)^2 + \frac{\pi^2}{16}\right] = \frac{5\pi^2}{4}$

$\therefore\ M - m = \frac{5\pi^2}{4} - \frac{\pi^2}{8} = \frac{9}{8}\pi^2$

5. (d) : The domain of the first term is $R - \{-2, 2\}$.
Domain of second term exist if $x^3 - x > 0$

$\Rightarrow x(x+1)(x-1) > 0$



$\therefore\ x \in (-1, 0) \cup (1, \infty)$

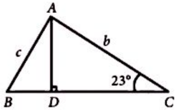
The desired domain is the intersection of the above two sets, $(-1, 0) \cup (1, 2) \cup (2, \infty)$

6. (d) : $c\sin B = AD = \frac{abc}{b^2 - c^2}$

$\Rightarrow \sin B\,(b^2 - c^2) = ab$

$\Rightarrow \sin B\,(\sin^2 B - \sin^2 C)$

$= \sin A \sin B$



$\Rightarrow \sin(B + C)\sin(B - C)$

$= \sin A = \sin(B + C)$

$\therefore\ \sin(B - C) = 1$

$\Rightarrow B - C = 90° \quad \Rightarrow \quad B = 90° + C$

$\Rightarrow B = 90° + 23° = 113°$

7. (c) : Number of 1 digit numbers is 5.

Number of 2 digit numbers is $5 \cdot 5 = 25$

Number of 3 digit numbers is $5 \cdot 5 \cdot 4 = 100$

Number of 4 digit numbers is $5 \cdot 5 \cdot 4 \cdot 3 = 300$

Number of 5 digit numbers is $5 \cdot 5 \cdot 4 \cdot 3 \cdot 2 = 600$

Number of 6 digit numbers is $5 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 600$

The desired number $= 5 + 25 + 100 + 300 + 600 + 600$
$= 1630$

8. (d) : Let the A.P. be $a - 3d, a - d, a + d, a + 3d$.

The sum of the terms $= 48 \Rightarrow a = 12$

$\frac{(12-3d)(12+3d)}{(12-d)(12+d)} = \frac{27}{35}$

$\Rightarrow 35(16 - d^2) = 3(144 - d^2)$

$\Rightarrow d^2 = 4 \Rightarrow d = \pm 2$

$\therefore$ The numbers are $12 - 6, 12 - 2, 12 + 2, 12 + 6$ or $12 + 6, 12 + 2, 12 - 2, 12 - 6 = 6, 10, 14, 18$ or $18, 14, 10, 6$

$\therefore$ Largest term is 18.

9. (d) : $\lim_{x \to 0} (\cos x + a\sin bx)^{1/x}$ $\quad$ (1^∞ form)

$= \exp \lim_{x \to 0} \frac{\cos x + a\sin bx - 1}{x}$

$$= \exp \lim_{x\to 0}\left(\frac{a\sin bx}{x} - \frac{2\sin^2\frac{x}{2}}{x}\right) = e^{ab}$$

10. (c) : The slopes of L and K are equal.

$\therefore \quad \frac{-b}{5} = \frac{-3}{c} \Rightarrow bc = 15$

L contains the point (13, 32)

$\therefore \quad \frac{13}{5} + \frac{32}{b} = 1 \Rightarrow \frac{32}{b} = \frac{-8}{5} \Rightarrow b = -20, c = \frac{-3}{4}$

L and K are $4x - y = 20$ and $4x - y = -3$

The distance between them is $\frac{23}{\sqrt{17}}$.

11. (c) : The lines $2x - 3y = 5$ and $3x - 4y = 7$ meet at the centre (1, –1).

Area $= \frac{22}{7}r^2 = 154 \Rightarrow r = 7$

$\therefore$ The circle is $(x-1)^2 + (y+1)^2 = 49$

or $x^2 + y^2 - 2x + 2y = 47$.

12. (d) : We have, $\frac{x^2}{9} + \frac{y^2}{1} = 1 \Rightarrow A \equiv (3,0), B \equiv (0,1)$

The line $AB : \frac{x}{3} + \frac{y}{1} = 1$ meets the circle $x^2 + y^2 = 9$ at M.

$\therefore \quad x^2 + \left(1 - \frac{x}{3}\right)^2 = 9 \Rightarrow \frac{10}{9}x^2 - \frac{2x}{3} - 8 = 0$

$\Rightarrow 5x^2 - 3x - 36 = 0 \Rightarrow x = -\frac{12}{5}, y = \frac{9}{5}$

$\therefore M \equiv \left(-\frac{12}{5}, \frac{9}{5}\right)$

$$\text{Area of } \Delta AMO = \frac{1}{2}\begin{vmatrix} 3 & 0 & 1 \\ -\frac{12}{5} & \frac{9}{5} & 1 \\ 0 & 0 & 1 \end{vmatrix} = \frac{27}{10} \text{ sq. units.}$$

13. (a) : Mean $(\bar{x}) = 4$, variance = 5.2

Let x_1, x_2 be the remaining values.

Mean, $\bar{x} = \frac{1+2+6+x_1+x_2}{5} \Rightarrow x_1 + x_2 = 11$ ...(i)

Variance, $\sigma^2 = 5.2 = \frac{1^2+2^2+6^2+x_1^2+x_2^2}{5} - (\bar{x})^2$

$\Rightarrow x_1^2 + x_2^2 = 65$ ...(ii)

$\Rightarrow |x_1 - x_2| = 3$

Now, $\lambda = |x_1 - x_2| + 8$

$\Rightarrow \lambda = 11 \Rightarrow 10 - x^2 - 2x = 11$

$\Rightarrow (x+1)^2 = 0$, one solution

14. (a) : We have, the letters G, O, D, N, N, A, A, L.

$\therefore$ Required probability $= \left(\binom{8}{3}\frac{5!}{2!2!}\right) \div \left(\frac{8!}{2!2!}\right)$

$$= \frac{1}{3!} = \frac{1}{6}$$

15. (a) : Here, $AB = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$$= \begin{bmatrix} 0+2 & 2+0 \\ 0+2 & 2+0 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$$

And $BA = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 0+2 & 0+2 \\ 2+0 & 2+0 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$

Thus, $AB = BA$.

Consequently, $(A+B)^2 = A^2 + AB + BA + B^2$

$= A^2 + AB + AB + B^2 = A^2 + 2AB + B^2$.

16. (b) : Checking continuity at $x = -1$

Now, $f(-1) = [(-1)^2] - [-1]^2$

$= [1] - (-1)^2 = 1 - 1 = 0$

R.H.L. $= \lim_{x\to -1^+}([x^2] - [x]^2) = 0 - 1 = -1$

So, $f(-1) \neq$ R.H.L.

Continuity at $x = 0$

$f(0) = [(0)^2] - [0]^2 = 0 - 0 = 0$

L.H.L. $= \lim_{x\to 0^-}([x^2] - [x]^2) = 0 - 1 = -1$

So, $f(0) \neq$ L.H.L.

Hence, the function is not continuous at the end points of the interval [–1, 0].

17. (d) : Since, $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) = f(2)$

So, $f(x)$ is continuous at $x = 2$.

Hence, $f(x)$ is continuous on [–1, 3].

Also, (L.H.D. at $x = 2$) $= \left\{\frac{d}{dx}(3x^2 + 12x - 1)\right\}_{\text{at } x=2} = 24$

and (R.H.D. at $x = 2$) $= \left\{\frac{d}{dx}(37 - x)\right\}_{\text{at } x=2} = -1$

Clearly, $f(x)$ is not differentiable at $x = 2$.

So, $f'(2)$ does not exist.

Now, $f'(x) = \begin{cases} 6x+12, & -1 \le x \le 2 \\ -1, & 2 < x \le 3 \end{cases}$

Clearly, $f'(x) > 0$ for all $x \in [-1, 2]$

So, $f(x)$ is increasing on [–1, 2].

18. (c) : We have, $\dfrac{3\cot 3x - \cot x}{\tan x - 3\tan 3x}$

$$= \frac{3\dfrac{\tan x}{\tan 3x} - 1}{\tan^2 x\left(1 - \dfrac{3\tan 3x}{\tan x}\right)} = \frac{1 - 3\tan^2 x}{3 - \tan^2 x} = \frac{-8}{3 - \tan^2 x} + 3$$

Thus, required integral

$$I = \int\left(\frac{-8}{3 - \tan^2 x} + 3\right)dx = \int \frac{-8}{(3 - \tan^2 x)}dx + 3x + C$$

Putting $\tan x = t \Rightarrow \sec^2 x\, dx = dt$

$$\Rightarrow dx = \frac{dt}{\sec^2 x} = \frac{dt}{1 + t^2}$$

$$\therefore\ I = \int \frac{-8dt}{(1 + t^2)(3 - t^2)} + 3x + C$$

$$\Rightarrow I = -2\int\left(\frac{1}{3 - t^2} + \frac{1}{1 + t^2}\right)dt + 3x + C$$

$$\Rightarrow I = -2 \cdot \frac{1}{2\sqrt{3}} \cdot \log\left|\frac{\sqrt{3} + t}{\sqrt{3} - t}\right| - 2\tan^{-1}(t) + 3x + C$$

$$\Rightarrow I = -\frac{1}{\sqrt{3}} \cdot \log\left|\frac{\sqrt{3} + \tan x}{\sqrt{3} - \tan x}\right| + x + C$$

$$= \frac{1}{\sqrt{3}}\log\left|\frac{\sqrt{3} - \tan x}{\sqrt{3} + \tan x}\right| + x + C$$

$$\therefore\ a = 1,\ b = \frac{1}{\sqrt{3}}$$

19. (d) : $A = \displaystyle\int_{-\pi/2}^{\pi/2} \sin^8 x \cos^4 x dx$

$$= 2\int_0^{\pi/2} \sin^8 x \cos^4 x dx$$

$$\left(\because \int_{-a}^{a} f(x)dx = 2\int_0^a f(x)dx \text{ if } f(-x) = f(x)\right)$$

$$= 2\int_0^{\pi/2} \sin^8\left(\frac{\pi}{2} - x\right)\cos^4\left(\frac{\pi}{2} - x\right)dx$$

$$\left(\because \int_0^a f(x)dx = \int_0^a f(a - x)dx\right)$$

$$\therefore\ A = 2\int_0^{\pi/2} \cos^8 x \sin^4 x dx = 2B = 2\int_0^{\pi/2} \cos^m x \sin^n x dx$$

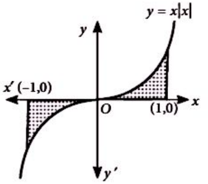
Here, $m = 8$, $n = 4$

$\therefore\ m + n = 12$

$$= 2\left[\frac{(7\cdot5\cdot3\cdot1)(3\cdot1)}{12\cdot10\cdot8\cdot6\cdot4\cdot2}\right] \times \frac{\pi}{2} = \frac{7\pi}{1024}$$

$$\therefore\ A = 2B = \frac{7\pi}{1024}$$

20. (c) : Let A denotes the required area. Then,



$$A = \left|\int_{-1}^{1} x|x|dx\right|$$

$$\Rightarrow A = \left|\int_{-1}^{0} x|x|dx\right| + \int_0^1 x|x|dx = \left|\int_{-1}^{0} -x^2 dx\right| + \int_0^1 x^2 dx$$

$$\Rightarrow A = \left|\left[-\frac{x^3}{3}\right]_{-1}^{0}\right| + \left[\frac{x^3}{3}\right]_0^1 = \frac{1}{3} + \frac{1}{3} = \frac{2}{3} \text{ sq. unit}$$

21. (2) : Given $\frac{dy}{dx}-\frac{1}{x}y=\left(x-\frac{2}{x}\right)$

$\therefore$ I.F. $= e^{-\int \frac{1}{x}dx} = e^{-\ln x} = \frac{1}{x}$

Now, general solution is given by

$$\frac{y}{x}=\int\left(x-\frac{2}{x}\right)\frac{1}{x}dx \Rightarrow \frac{y}{x}=x+\frac{2}{x}+C$$

As $y(1) = 1, C = -2$

$\therefore \quad \frac{y}{x}=x+\frac{2}{x}-2 \Rightarrow y=x^2-2x+2$

Hence, $y(2) = (2)^2 - 2(2) + 2 = 2$

22. (25) : $0 = |\vec{u}+\vec{v}+\vec{w}|^2 = (\vec{u}+\vec{v}+\vec{w})\cdot(\vec{u}+\vec{v}+\vec{w})$

$= |\vec{u}|^2 + |\vec{v}|^2 + |\vec{w}|^2 + 2(\vec{u}\cdot\vec{v}+\vec{v}\cdot\vec{w}+\vec{w}\cdot\vec{u})$

$\Rightarrow \quad |\vec{u}\cdot\vec{v}+\vec{v}\cdot\vec{w}+\vec{w}\cdot\vec{u}| = \left|-\frac{1}{2}(3^2+4^2+5^2)\right|$

$= |-25| = 25$

23. (3) : Let $L_1 : \frac{(x-1)}{2}=\frac{(y-2)}{1}=\frac{(z-1)}{3}; \ \vec{r}_1 = 2\hat{i}+\hat{j}+3\hat{k}$

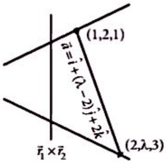
$L_2 : \frac{(x-2)}{1}=\frac{(y-\lambda)}{2}=\frac{(z-3)}{4}; \ \vec{r}_2 = \hat{i}+2\hat{j}+4\hat{k}$

Now, shortest distance = Projection of $\vec{a}$ on $\vec{r}_1\times\vec{r}_2$

$$=\frac{|a\cdot(\vec{r}_1\times\vec{r}_2)|}{|\vec{r}_1\times\vec{r}_2|}$$

Now, $|\vec{a}\cdot(\vec{r}_1\times\vec{r}_2)| = \begin{vmatrix}1 & \lambda-2 & 2\\ 2 & 1 & 3\\ 1 & 2 & 4\end{vmatrix} = |14-5\lambda|$

and $|\vec{r}_1\times\vec{r}_2| = \sqrt{38}$



$\Rightarrow \quad \frac{1}{\sqrt{38}}=\frac{|14-5\lambda|}{\sqrt{38}}$

$\Rightarrow |14-5\lambda| = 1 \Rightarrow 14-5\lambda = 1$

or $\quad 14-5\lambda = -1 \ \therefore \ \lambda = \frac{13}{5}$ or $\lambda = 3$

$\therefore \quad$ Integral value of $\lambda = 3$

24. (6) : Let x, y and z be the probabilities of E_1, E_2 and E_3. Then

$x(1-y)(1-z) = \alpha$; $y(1-x)(1-z) = \beta$; $z(1-x)(1-y) = \gamma$

Also, $(1-x)(1-y)(1-z) = p$

On dividing, we get, $\frac{x}{1-x}=\frac{\alpha}{p} \ \therefore \ x=\frac{\alpha}{\alpha+p}$,

$y=\frac{\beta}{\beta+p}$ and $z=\frac{\gamma}{\gamma+p}$

$$\frac{P(E_1)}{P(E_3)}=\frac{\frac{\alpha}{\alpha+p}}{\frac{\gamma}{\gamma+p}}=\frac{1+\frac{p}{\gamma}}{1+\frac{p}{\alpha}}$$

Again, $(\alpha-2\beta)p = \alpha\beta$; $(\beta-3\gamma)p = 2\beta\gamma$

We have, $\alpha p = \beta(\alpha+2p)$ and $3\gamma p = \beta(p-2\gamma)$

From above, $\frac{\alpha}{3\gamma}=\frac{\alpha+2p}{p-2\gamma}$

$\Rightarrow \quad \frac{\alpha+2p}{\alpha}=\frac{p-2\gamma}{3\gamma} \quad \Rightarrow \quad 1+\frac{2p}{\alpha}=\frac{p}{3\gamma}-\frac{2}{3}$

$\Rightarrow \quad \frac{5}{3}=\frac{p}{3\gamma}-\frac{2p}{\alpha} \quad \Rightarrow \quad 5=\frac{p}{\gamma}-\frac{6p}{\alpha}$

$\Rightarrow \quad 6\left(1+\frac{p}{\alpha}\right)=1+\frac{p}{\gamma} \ \therefore \ \frac{1+\frac{p}{\gamma}}{1+\frac{p}{\alpha}}=6 \Rightarrow \frac{P(E_1)}{P(E_3)}=6$

25. (1) : $f(f(x)) = (100^5 - (f(x))^{10})^{1/10}$

$= [100^5 - (100^5 - x^{10})]^{1/10} = x$

$f(f(1024)) = 1024 \Rightarrow \frac{1}{2^{10}}f(f(1024))=1$

26. (4) : $e^2 = 1-\frac{b^2}{a^2}=1-\frac{4}{9}=\frac{5}{9}\Rightarrow e=\frac{\sqrt{5}}{3}$

$F_1F_2 = 2\sqrt{5}$

also $\quad PF_1 + PF_2 = 6$ and $PF_1 = 2(PF_2)$ (given)

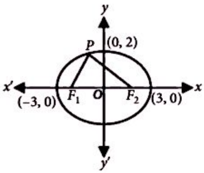
$\therefore \quad 3PF_2 = 6$

$\Rightarrow PF_2 = 2$ and $PF_1 = 4$

Since $(PF_2)^2 + (PF_1)^2$

$= (F_1F_2)^2$

$\Rightarrow \angle P = 90°$



Area $= \frac{4\cdot 2}{2} = 4$ sq. units

27. (3) : Given equations are

$ax^2 + bx + c = 3x - y$ ...(i)

$ay^2 + by + c = y - 3x$ ...(ii)

Now, $b^2 - 4ac \le 0$

$\therefore \quad ax^2 + bx + c \le 0$ and $ay^2 + by + c \le 0$

$\therefore \quad 3x - y \le 0$ and $y - 3x \le 0$

$\Rightarrow \quad 3x = y$

$\Rightarrow \quad \frac{y}{x} = 3$

28. (3748) : Here X = {1, 6, 11, ..., 10086}

and $Y = \{9, 16, 23, ..., 14128\}$

The intersection of X and Y is an A.P. with 16 as first term and 35 as common difference.

The series becomes 16, 51, 86, ...

Now, k^{th} term $= 16 + (k - 1)\,35 \le 10086$

$$i.e.\ k \le \frac{10105}{35}$$

$\therefore\ k \le 288$ (as k is to be an integer)

$n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$

$n(X \cup Y) = 2018 + 2018 - 288 = 3748$

29. (114) : We have, $f(x) = 2x^3 - 3x^2 - 12x$

$\Rightarrow\ f'(x) = 6x^2 - 6x - 12 = 6(x - 2)(x + 1)$

Now, $f'(x) = 0 \Rightarrow x = -1, 2$

and $f''(x) = 12x - 6$

$\Rightarrow f''(2) > 0 \Rightarrow b = 2$

and $f''(-1) < 0 \Rightarrow a = -1$

Consider, $f(x) = 0$

$$\Rightarrow\ x(2x^2 - 3x - 12) = 0$$

$$\Rightarrow\ x = 0, \frac{3 \pm \sqrt{105}}{4}$$

Now, $\dfrac{3-\sqrt{105}}{4} < -1$ and $\dfrac{3+\sqrt{105}}{4} > 2$

The only root lying between -1 & 2 is $x = 0$.

$$\text{Let, } A_1 = \left|\int_{-1}^{0}(2x^3 - 3x^2 - 12x)\,dx\right|$$

$$= \left|\left[\frac{x^4}{2} - x^3 - 6x^2\right]_{-1}^{0}\right| = \left|0 - \left(\frac{1}{2} + 1 - 6\right)\right|$$

$$= 6 - \frac{3}{2} = \frac{9}{2} \text{ sq. units and}$$

$$A_2 = \left|\int_{0}^{2}(2x^3 - 3x^2 - 12x)\,dx\right| = \left|\left[\frac{x^4}{2} - x^3 - 6x^2\right]_{0}^{2}\right|$$

$$= |8 - 8 - 24| = 24$$

$$\therefore\ A = A_1 + A_2 = 24 + \frac{9}{2} = \frac{57}{2} \text{ sq. units}$$

$\Rightarrow\ 4A = 57 \times 2 = 114$ sq. units

30. (10) : $\because A = \begin{pmatrix} 1 & 3 & \lambda-3 \\ 2 & -1 & 1 \\ 3 & 2 & 1 \end{pmatrix}$ is a singular matrix

$\therefore\ |A| = 0$

$$\Rightarrow\ |A| = \begin{vmatrix} 1 & 3 & \lambda-3 \\ 2 & -1 & 1 \\ 3 & 2 & 1 \end{vmatrix} = 0$$

$$= 1(-3) - 3(-1) + (\lambda - 3)(7) = 0 \ \Rightarrow\ \lambda = 3$$

$$\therefore\ A + B = \begin{pmatrix} 1 & 3 & 0 \\ 2 & -1 & 1 \\ 3 & 2 & 1 \end{pmatrix} + \begin{pmatrix} 3 & 2 & 4 \\ 3 & 2 & 5 \\ 2 & 1 & 4 \end{pmatrix} = \begin{pmatrix} 4 & 5 & 4 \\ 5 & 1 & 6 \\ 5 & 3 & 5 \end{pmatrix}$$

$\therefore$ trace $(A + B) = 4 + 1 + 5 = 10$

◈◈

SAMURAI SUDOKU

Samurai Sudoku puzzle consists of five overlapping sudoku grids. The standard sudoku rules apply to each 9 × 9 grid. Place digits from 1 to 9 in each empty cell. Every row, every column and every 3 × 3 box should contain one of each digit.

The puzzle has a unique answer.

6				9	8	3		7				4				5				6
			6											9	6		3			
			2					4							8					
	6					8								1			8	5	9	
7							6					9			5	1				3
1	8	5	9	3								5					9		6	7
	2		3							5	2						1			8
							4				7		6			8				
9				8	4				9		4							4		9
						6	3	7				5								
						4								9						
								8				7	4	2						
1		8							2		1				2	1				3
				3			1		7				2							
9			7						8	4							1		6	
5	6		4					9								2	5	7	3	4
8				9	3			6					8			7				
	1	9	8		5	2								2	1		9		8	
					1							1					2			
			6		7	4									6		3			
2				4	9			7				3		8	4	5				2

Readers can send their responses at editor@mtg.in or post us with complete address. Winners' name with their valuable feedback will be published in next issue.

PRACTICE PAPER

BITSAT

Exam Dates
Session-I
Between 19th May to 24th May
Session-II
Between 22nd June to 26th June

1. The minimum value of $|z + 1| + |z - 2|$ is equal to
(a) 1 (b) 2 (c) 3 (d) 4

2. Three unequal positive numbers a, b, c are such that a, b, c are in G.P. while $\log\left(\frac{5c}{2a}\right), \log\left(\frac{7b}{5c}\right), \log\left(\frac{2a}{7b}\right)$ are in A.P. Then a, b, c are the lengths of the sides of
(a) an isosceles triangle
(b) an equilateral triangle
(c) a scalene triangle
(d) a right-angled triangle

3. The number of 3-digit numbers of which at least one digit is 2, is
(a) 251 (b) 252 (c) 270 (d) 271

4. Let $(x + 10)^{50} + (x - 10)^{50} = a_0 + a_1x + a_2x^2 + \ldots + a_{50}x^{50}$, for all $x \in R$; then $\frac{a_2}{a_0}$ is equal to
(a) 12.00 (b) 12.75 (c) 12.25 (d) 12.50

5. A card is drawn from a well-shuffled standard deck of cards. What is the probability that the card drawn is either an ace or a jack or a king?
(a) $\frac{4}{13}$ (b) $\frac{3}{13}$ (c) $\frac{3}{52}$ (d) $\frac{469}{2197}$

6. Find the domain of the function,
$$f(x) = \frac{x^2+1}{x^2-3x+3}$$
(a) $R-\{1, 2\}$ (b) $R-\{1, 4\}$
(c) R (d) $R-\{1\}$

7. If $49^n + 16n + \lambda$ is divisible by 64 for all $n \in N$, then the least negative integral value of λ is
(a) -1 (b) -2 (c) -3 (d) -4

8. The maximum value of $5\cos\theta + 3\cos\left(\theta + \frac{\pi}{3}\right) + 3$ is
(a) 5 (b) 11 (c) 10 (d) -1

9. If $A = \begin{bmatrix} 1 & 1 \\ 0 & i \end{bmatrix}$ and $A^{2018} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then $(a + d)$ equals
(a) $1 + i$ (b) 0 (c) 2 (d) 2018

10. $\dfrac{\tan^{-1}(\sqrt{3}) - \sec^{-1}(-2)}{\operatorname{cosec}^{-1}(-\sqrt{2}) + \cos^{-1}\left(-\frac{1}{2}\right)}$ is equal to
(a) $\frac{4}{5}$ (b) $-\frac{4}{5}$ (c) $\frac{3}{5}$ (d) 0

11. If $A = \begin{bmatrix} 3 & 4 \\ 5 & 7 \end{bmatrix}$, then $|A \cdot (\text{adj } A)|$ is equal to
(a) A (b) $|A|$
(c) $2|A|$ (d) None of these

12. If function $f(x) = \begin{cases} x - \frac{|x|}{x}, & x < 0 \\ x + \frac{|x|}{x}, & x > 0 \\ 1, & x = 0 \end{cases}$, then
(a) $\lim_{x \to 0^-} f(x)$ does not exist
(b) $\lim_{x \to 0^+} f(x)$ does not exist
(c) $f(x)$ is continuous at $x = 0$
(d) $\lim_{x \to 0^-} f(x) \neq \lim_{x \to 0^+} f(x)$

13. If the lines $\frac{2x-1}{2} = \frac{3-y}{1} = \frac{z-1}{3}$ and $\frac{x+3}{2} = \frac{z+1}{p} = \frac{y+2}{5}$ are perpendicular to each other, then p is equal to
(a) 1 (b) -1 (c) 10 (d) $-7/5$

14. The equation of tangents to the circle $x^2 + y^2 = 4$ which are parallel to $x + 2y + 3 = 0$ are

(a) $x-2y=\pm 2\sqrt{5}$ (b) $x-2y=\pm 2$
(c) $x+2y=\pm 2\sqrt{3}$ (d) $x+2y=\pm 2\sqrt{5}$

15. The mean of 6 distinct observations is 6.5 and their variance is 10.25. If 4 out of 6 observations are 2, 4, 5 and 7, then the remaining two observations are
(a) 8, 13 (b) 1, 20 (c) 10, 11 (d) 3, 18

16. 4 consecutive years are taken such that exactly one of them is a leap year. One of these four years is randomly chosen. What is the probability that it will have 53 Wednesdays ?
(a) 1/7 (b) 3/14 (c) 1/4 (d) 5/28

17. Consider a Linear Programming Problem :
Minimize $Z = 5x + 3y$,
Subject to : $3x + y \geq 10$, $2x + 2y \geq 14$ and $x + 2y \geq 9$.
Which one of the following points lies outside the feasible region?
(a) (1, 9) (b) (4, 2) (c) (6, 2) (d) (12, 2)

18. If $\log_{16} x + \log_4 x + \log_2 x = 14$, then x is equal to
(a) 16 (b) 32 (c) 64 (d) 256

19. The standard deviation of 25 numbers is 40. If each of the numbers is increased by 5, then the new standard deviation will be
(a) 40 (b) 25 (c) $\sqrt{40}$ (d) 1600

20. Solve the differential equation $\frac{dy}{dx} = \log(x+1)$.
(a) $y = (x - 1) \log (x + 1) + x + C$
(b) $y = (x + 1) \log (x + 1) - x + C$
(c) $y = (x + 1) \log (x + 1) + x + C$
(d) $y = (x - 1) \log (x + 1) - x + C$

21. $\lim_{n\to\infty}\left(\frac{1}{5}\right)^{\log_{\sqrt{5}}\left(\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\ldots\ldots n \text{ terms}\right)}$ equals
(a) 2 (b) 4 (c) 8 (d) 0

22. The distance between the point (1, 1) and the tangent to the curve $y = e^{2x} + x^2$ drawn from the point $x = 0$, is
(a) $\frac{1}{\sqrt{5}}$ (b) $\frac{-1}{\sqrt{5}}$ (c) $\frac{2}{\sqrt{5}}$ (d) $\frac{-2}{\sqrt{5}}$

23. If the rate of increase of the radius of a circle is 5 cm/sec, then the rate of increase of its area, when the radius is 20 cm, will be
(a) 10π cm^2/sec (b) 20π cm^2/sec
(c) 200π cm^2/sec (d) 400π cm^2/sec

24. If $f(x)$ is a function such that $f'(x) = (x - 1)^2 (4 - x)$, then
(a) $f(0) = 0$
(b) $f(x)$ is decreasing in (0, 3)
(c) $x = 4$ is a critical point of $f(x)$
(d) $f(x)$ is decreasing in (3, 5)

25. If $y = m\log x + nx^2 + x$ has its extreme values at $x = 2$ and $x = 1$, then $2m + 10n =$
(a) −3 (b) −4 (c) −2 (d) 1

26. If $\int \frac{dx}{\sqrt{\sin^3 x \cos x}} = g(x)+c$, then $g(x)=$
(a) $\frac{-2}{\sqrt{\tan x}}$ (b) $\frac{2}{\sqrt{\cot x}}$
(c) $\frac{2}{\sqrt{\tan x}}$ (d) $\frac{-2}{\sqrt{\cot x}}$

27. $\int \frac{xe^{2x}}{(1+2x)^2}dx$ is equal to
(a) $\frac{e^{2x}}{1+2x}+C$ (b) $\frac{e^{2x}}{2(1+2x)}+C$
(c) $\frac{4e^{2x}}{1+2x}+C$ (d) $\frac{e^{2x}}{4(1+2x)}+C$

28. If $\int \frac{dx}{\sqrt{16-9x^2}} = A \sin^{-1}(Bx) + C$, then $A + B =$
(a) 9/4 (b) 19/4 (c) 3/4 (d) 13/12

29. $\int_0^1 x(1-x)^5 dx$ is equal to
(a) $\frac{1}{5}$ (b) $\frac{1}{42}$ (c) $\frac{1}{13}$ (d) $\frac{13}{42}$

30. The area bounded by the curves $y = \cos x$ and $y = \sin x$ between the ordinates $x = 0$ and $x = \frac{3\pi}{2}$ is
(a) $4\sqrt{2}+2$ (b) $4\sqrt{2}-2$
(c) $4\sqrt{2}+1$ (d) $4\sqrt{2}-1$

31. If the lines $2x - 3y + 5 = 0$, $9x - 5y + 14 = 0$ and $3x - 7y + \lambda = 0$ are concurrent, then the value of λ is equal to
(a) 7 (b) 8 (c) 10 (d) 9

32. If the slopes of the lines $Kx^2 - 4xy + y^2 = 0$ differ by 2, then $K =$
(a) 3 (b) 4 (c) $\frac{21}{5}$ (d) $\frac{4}{5}$

33. If $|\vec{a}|=2, |\vec{b}|=3$ and $\vec{a}\cdot\vec{b}=4$, then $|\vec{a}-\vec{b}|$ is equal to
(a) $\sqrt{5}$ (b) $\sqrt{7}$ (c) $\sqrt{6}$ (d) 5

34. If $\vec{a}=\hat{i}+\hat{j}-2\hat{k}, \vec{b}=2\hat{i}-\hat{j}+\hat{k}$ and $\vec{c}=3\hat{i}-\hat{k}$ and $\vec{c}=m\vec{a}+n\vec{b}$, then $m+n=$
(a) 0 (b) 1 (c) 2 (d) -1

35. Let the position vectors of the points A, B and C be $\vec{a}, \vec{b}$ and $\vec{c}$ respectively. Let Q be the point of intersection of the medians of the triangle ABC. Then $\overline{QA}+\overline{QB}+\overline{QC}=$
(a) $\frac{\vec{a}+\vec{b}+\vec{c}}{2}$ (b) $2\vec{a}+\vec{b}+\vec{c}$
(c) $\vec{0}$ (d) $\frac{\vec{a}+\vec{b}+\vec{c}}{3}$

36. A straight line makes an angle of 30°, 45° and 60° with the positive direction of X-axis, Y-axis and Z-axis respectively. What are the direction cosines of the straight line?
(a) $\frac{1}{2}, \frac{1}{\sqrt{2}}, \frac{\sqrt{3}}{2}$ (b) $0, \frac{1}{\sqrt{2}}, \frac{1}{2}$
(c) $\frac{\sqrt{3}}{2}, \frac{1}{\sqrt{2}}, \frac{1}{2}$ (d) $\frac{1}{\sqrt{2}}, \frac{\sqrt{3}}{2}, 1$

37. If the line $\vec{r}=2\hat{i}+\hat{j}+t(3\hat{i}+\hat{j}-2\hat{k})$ is parallel to the plane $2x+4y+az=8$, then the value of a is equal to
(a) 2 (b) 3 (c) 4 (d) 5

38. The solution set of the inequality
$5(4x+6) < 25x+10$ is
(a) $(4, \infty)$ (b) $(-\infty, 4)$ (c) $(-\infty, 5)$ (d) $(5, \infty)$

39. The contrapositive of the statement "If you will work then you will earn money", is
(a) You will earn money, if you will not work
(b) If you will earn money, you will work
(c) If you will not earn money, you will not work
(d) To earn money, you need to work

40. $\lim_{x\to 0}\frac{\log(1+x)+1-e^x}{4x^2-9x}$ is equal to
(a) 0 (b) $\frac{1}{9}$ (c) $\frac{-1}{18}$ (d) $\frac{1}{18}$

SOLUTIONS

1. **(c)** : We have, $|z+1|+|z-2| = |z+1|+|2-z|$
$\geq |z+1+2-z| = 3$

2. **(c)** : We have, $a, b, c \in$ G.P. ...(i)

and $\log\left(\frac{5c}{2a}\right), \log\left(\frac{7b}{5c}\right), \log\left(\frac{2a}{7b}\right) \in$ A.P.

$\Rightarrow 2\log\left(\frac{7b}{5c}\right)=\log\left(\frac{5c}{2a}\right)+\log\left(\frac{2a}{7b}\right)$

$\Rightarrow \left(\frac{7b}{5c}\right)^2=\left(\frac{5c}{7b}\right) \Rightarrow c=\frac{7}{5}b$

Also, $b^2=ac \Rightarrow b^2=a\cdot\frac{7}{5}b \Rightarrow a=\frac{5}{7}b$

So, the ratio of sides is $\frac{5}{7}:1:\frac{7}{5}$

$\therefore$ ΔABC is scalene triangle.

3. **(b)** : Total number of 3-digit numbers
$=9\times 10\times 10=900$
Number of 3-digit numbers in which 2 does not appear
$=8\times 9\times 9=648$
$\therefore$ Number of 3-digit numbers in which at least one digit is $2=900-648=252$

4. **(c)** : Here, $(x+10)^{50}+(x-10)^{50}$
$=a_0+a_1x+a_2x^2+\ldots+a_{50}x^{50}$
$\Rightarrow 2[{}^{50}C_0x^{50}+{}^{50}C_2x^{48}10^2+\ldots+{}^{50}C_{48}x^2 10^{48}$
$+{}^{50}C_{50}10^{50}]$
$=a_0+a_1x+a_2x^2+\ldots+a_{50}x^{50}$
$\Rightarrow a_0=2\times 10^{50}, a_2=2\times {}^{50}C_{48}\,10^{48}$
$\therefore \frac{a_2}{a_0}=\frac{{}^{50}C_{48}}{10^2}=12.25$

5. **(b)**

6. **(c)** : Given function is $f(x)=\frac{x^2+1}{x^2-3x+3}$
Since, $x^2-3x+3>0\ \forall\ x\in R$
Also, $x^2+1>0\ \forall\ x\in R$
$\therefore$ Domain of $f(x)=R$

7. **(a)** : Let $p(n): 49^n+16n+\lambda$ is divisible by 64 for all $n\in N$
$\therefore$ $p(1): 49+16+\lambda$ is divisible by 64
$=65+\lambda$ is divisible by 64
We observe that for $65+\lambda$ is divisible by 64, if λ would be -1 *i.e.*, $65+(-1)$ is divisible by 64
So, the least negative integral value of λ is -1.

8. **(c)** : Let $f(x)=5\cos\theta+3\cos\left(\theta+\frac{\pi}{3}\right)+3$

$\Rightarrow f(x)=5\cos\theta+3\cos\theta\cos\frac{\pi}{3}-3\sin\theta\sin\frac{\pi}{3}+3$

$$= \left(5\cos\theta + \frac{3}{2}\cos\theta\right) - \frac{3\sqrt{3}}{2}\sin\theta + 3$$

$$= \frac{13}{2}\cos\theta - \frac{3\sqrt{3}}{2}\sin\theta + 3$$

Now, $\frac{13}{2}\cos\theta - \frac{3\sqrt{3}}{2}\sin\theta \le \sqrt{\left(\frac{13}{2}\right)^2 + \left(\frac{-3\sqrt{3}}{2}\right)^2} = 7$

$\therefore \quad \frac{13}{2}\cos\theta - \frac{3\sqrt{3}}{2}\sin\theta + 3 \le 7 + 3 = 10$

So, maximum value of $f(x)$ is 10.

9. (b) : Given, $A = \begin{bmatrix} 1 & 1 \\ 0 & i \end{bmatrix}$

$\Rightarrow \quad A^2 = \begin{bmatrix} 1 & 1+i \\ 0 & -1 \end{bmatrix}$ ...(i)

$\Rightarrow \quad A^3 = \begin{bmatrix} 1 & i \\ 0 & -i \end{bmatrix} \Rightarrow A^4 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Also, $A^{2018} = A^{4 \times 504 + 2} = A^2 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$\therefore$ Using (i), we get $a + d = 1 + (-1) = 0$

10. (b) : $\dfrac{\tan^{-1}(\sqrt{3}) - \sec^{-1}(-2)}{\operatorname{cosec}^{-1}(-\sqrt{2}) + \cos^{-1}\left(\frac{-1}{2}\right)}$

$$= \frac{(\pi/3) - (2\pi/3)}{(-\pi/4) + (2\pi/3)} = \frac{\frac{-\pi}{3}}{\frac{-3\pi + 8\pi}{12}} = \frac{-\pi/3}{5\pi/12} = \frac{-4}{5}$$

11. (b) : Given, $A = \begin{bmatrix} 3 & 4 \\ 5 & 7 \end{bmatrix}$

$\therefore \quad |A| = 21 - 20 = 1$

$\therefore \quad A\,(\text{adj } A) = \begin{bmatrix} 3 & 4 \\ 5 & 7 \end{bmatrix}\begin{bmatrix} 7 & -4 \\ -5 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \quad |A\,(\text{adj } A)| = |A|$

12. (c) : We have, $f(0) = 1$

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0}\left(x - \frac{|x|}{x}\right) = \lim_{x\to 0}\left(x + \frac{x}{x}\right)$$
$$= \lim_{x\to 0}(x+1) = 1$$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0}\left(x + \frac{|x|}{x}\right) = \lim_{x\to 0}\left(x + \frac{x}{x}\right)$$
$$= \lim_{x\to 0}(x+1) = 1$$

Thus, $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = f(0)$

$\therefore \quad f(x)$ is continuous at $x = 0$.

13. (a) : The lines are $\frac{2x-1}{2} = \frac{3-y}{1} = \frac{z-1}{3}$

i.e., $\quad \frac{x - \frac{1}{2}}{1} = \frac{y-3}{-1} = \frac{z-1}{3}$ and $\frac{x+3}{2} = \frac{y+2}{5} = \frac{z+1}{p}$

The lines are perpendicular to each other.

So, $1 \times 2 + (-1) \times 5 + 3 \times p = 0$

$\Rightarrow \quad 2 - 5 + 3p = 0 \Rightarrow p = 1$

14. (d) : Given, equation of the circle is $x^2 + y^2 = 4$ and equation of the line is $x + 2y + 3 = 0$

Slope of the line $= -1/2$

Since, the required tangents are parallel to the given line.

$\therefore \quad$ Slope of tangents $(m) = -\frac{1}{2}$

We know that the equation of the tangents to the circle $x^2 + y^2 = a^2$ with slope m are

$$y = mx \pm \sqrt{a^2(1+m^2)}$$

$\therefore \quad$ The required equation of tangents are

$$y = -\frac{1}{2}x \pm \sqrt{(2)^2\left(1 + \left(\frac{-1}{2}\right)^2\right)}$$

$\Rightarrow \quad y = -\frac{1}{2}x \pm \sqrt{5} \Rightarrow 2y + x = \pm 2\sqrt{5}$

15. (c) : Let the remaining two observations be a and b.

Then, mean $= \frac{2+4+5+7+a+b}{6} = 6.5$ (Given)

$\Rightarrow \quad a + b = 21$ ...(i)

Also, variance = 10.25 (Given)

$\Rightarrow \quad \frac{1}{6}(2^2 + 4^2 + 5^2 + 7^2 + a^2 + b^2) - (6.5)^2 = 10.25$

$\Rightarrow \quad a^2 + b^2 = 315 - 94 = 221$ ...(ii)

Now, $(a+b)^2 + (a-b)^2 = 2(a^2 + b^2)$

$\Rightarrow \quad (a-b)^2 = 2 \times 221 - (21)^2$ [From (i) and (ii)]

$\Rightarrow \quad (a-b)^2 = 1 \Rightarrow a - b = \pm 1$

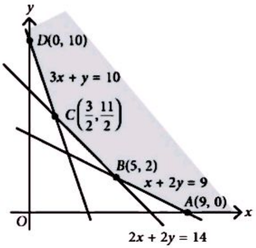
If $a - b = 1$, then $a = 11$, $b = 10$

If $a - b = -1$, then $a = 10$, $b = 11$

16. (d)

17. (b) : On plotting the constraints, we get $A(9, 0)$, $B(5, 2)$, $C\left(\frac{3}{2}, \frac{11}{2}\right)$ and $D(0, 10)$ as corner points of feasible region which is unbounded.

Now, (1, 9), (6, 2) and (12, 2) lies inside the feasible region and point (4, 2) lies outside the feasible region.



18. (d) : We have, $\log_{2^4} x + \log_{2^2} x + \log_2 x = 14$

$\Rightarrow \frac{1}{4}\log_2 x + \frac{1}{2}\log_2 x + \log_2 x = 14$

$\Rightarrow \left[\frac{1}{4} + \frac{1}{2} + 1\right] \log_2 x = 14$

$\Rightarrow \frac{7}{4} \log_2 x = 14 \Rightarrow \log_2 x = 8$

$\therefore\ x = 2^8 = 256$

19. (a) : We know that, if each item of a data is increased or decreased by the same constant, the standard deviation of the data remains unchanged.

20. (b) : We have, $\frac{dy}{dx} = \log(x+1)$

$\Rightarrow dy = \log(x+1)dx$

$\Rightarrow \int dy = \int \log(x+1)dx$

$\Rightarrow y = \int 1\cdot\log(x+1)dx$

$y = x\log(x+1) - \int \frac{1}{(x+1)}\cdot x\,dx$

$y = x\log(x+1) - \int \frac{(x+1)-1}{(x+1)}dx$

$y = x\log(x+1) - \int\left(1 - \frac{1}{x+1}\right)dx$

$y = x\log(x+1) - x + \log(x+1) + C$

$y = (x+1)\log(x+1) - x + C$

21. (b) : We have, $\lim_{n\to\infty}\left(\frac{1}{5}\right)^{\log_{\sqrt{5}}\left(\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\ldots n \text{ terms}\right)}$

$= \lim_{n\to\infty} 5^{-2\log_5 \frac{1}{4}\frac{\left(1-\left(\frac{1}{2}\right)^n\right)}{1-1/2}}$

[$\because$ Sum of n terms of G.P. $= \frac{a(1-r^n)}{1-r}$ if $r < 1$]

$= \lim_{n\to\infty} 5^{\log_5\left[\frac{1}{4}\left(1-\left(\frac{1}{2}\right)^n\right)\times\frac{2}{1}\right]^{-2}} = \left(\frac{1}{4}\times\frac{2}{1}\right)^{-2}$

$\left[\because \lim_{n\to\infty}\left(\frac{1}{2}\right)^n = 0 \text{ and } a^{\log_a x} = x\right]$

$= (2^{-1})^{-2} = 2^2 = 4$

22. (c) : Putting $x = 0$ in $y = e^{2x} + x^2$, ...(i)

we get $y = 1$

$\therefore$ The given point is $P(0, 1)$.

From (i), we get $\left[\frac{dy}{dx}\right]_{x=0} = 2$

$\therefore$ Equation of tangent to (i) from P is

$y - 1 = 2(x - 0) \Rightarrow 2x - y + 1 = 0$ ...(ii)

$\therefore$ Required distance = Length of $\perp$ from (1, 1) to (ii)

$= \frac{2-1+1}{\sqrt{4+1}} = \frac{2}{\sqrt{5}}$

23. (c) : Let radius is r.

$\therefore \frac{dr}{dt} = 5$ cm/sec (Given)

Area of circle $(A) = \pi r^2$

$\therefore \frac{dA}{dt} = \frac{d(\pi r^2)}{dt} = \pi(2r)\frac{dr}{dt}$

When $r = 20$, then $\frac{dA}{dt} = \pi\cdot 2\cdot 20\cdot 5 = 200\pi$ cm^2/sec

24. (c) : We have, $f'(x) = (x-1)^2(4-x)$

$= -x^3 + 6x^2 - 9x + 4$

On integrating, $f(x) = -\frac{x^4}{4} + 2x^3 - \frac{9x^2}{2} + 4x + C$

Putting $x = 0$, $f(0) = C$ (may or may not be zero)

$\because$ $f'(x) > 0$ in (0, 3) $\therefore$ $f(x)$ is increasing in (0, 3)

$\because$ $f'(4) = 0$ $\therefore$ $x = 4$ is a critical point of $f(x)$.

$\because$ $f'(x) > 0$ in (3, 4) and $f'(x) < 0$ in (4, 5)

$\therefore$ We can't say that $f(x)$ is decreasing in (3, 5).

25. (a) : We have, $y = m\log x + nx^2 + x$

$\Rightarrow \frac{dy}{dx} = \frac{m}{x} + 2nx + 1 \Rightarrow \frac{m}{2} + 4n + 1 = 0$ (at $x = 2$) ...(i)

and $m + 2n + 1 = 0$ (at $x = 1$) ...(ii)

From (i) and (ii), we get $6n + 1 = 0$

$\Rightarrow n = -\frac{1}{6}$ & $m = -\frac{2}{3}$ $\Rightarrow 2m + 10n = \frac{-4}{3} - \frac{5}{3} = -3$

26. (a) : Let $I = \int \frac{dx}{\sqrt{\sin^3 x\cos x}} = \int \frac{\sec^2 x}{\sqrt{\frac{\sin^3 x\cos x}{\cos^4 x}}}dx$

$= \int \frac{\sec^2 x}{\sqrt{\tan^3 x}}dx$

Put $\tan x = \theta \Rightarrow \sec^2 x\, dx = d\theta$

$$\therefore\ I = \int \frac{d\theta}{\theta^{3/2}} = \frac{-2}{\sqrt{\theta}} + c$$

$$\Rightarrow\ I = \frac{-2}{\sqrt{\tan x}} + c \Rightarrow g(x) = \frac{-2}{\sqrt{\tan x}}$$

27. (d) : Let $I = \int \frac{xe^{2x}}{(1+2x)^2}\, dx$

Using partial fraction, we have

$$\frac{x}{(1+2x)^2} = \frac{A}{1+2x} + \frac{B}{(1+2x)^2}$$

$\Rightarrow\ x = A(1 + 2x) + B$

For $x = -1/2$, $B = -1/2$

Comparing the coefficient of constant value, we get $A + B = 0$

$\therefore\ A = 1/2$

$$\therefore\ \frac{x}{(1+2x)^2} = \frac{1}{2(1+2x)} - \frac{1}{2(1+2x)^2}$$

$$\Rightarrow\ I = \int \frac{xe^{2x}}{(1+2x)^2}\, dx = \int e^{2x}\left[\frac{1}{2(1+2x)} - \frac{1}{2(1+2x)^2}\right] dx$$

$$= \frac{1}{2}\int \frac{e^{2x}}{1+2x}\, dx - \frac{1}{2}\int \frac{e^{2x}}{(1+2x)^2}\, dx$$

$$= \frac{1}{2}\left[\frac{1}{1+2x}\int e^{2x} dx - \int\left[\frac{d}{dx}\left(\frac{1}{1+2x}\right)\int e^{2x}\, dx\right] dx - \int \frac{e^{2x}}{(1+2x)^2}\, dx\right]$$

$$= \frac{1}{2}\left[\frac{e^{2x}}{2(1+2x)} + \int \frac{e^{2x}}{(1+2x)^2}\, dx - \int \frac{e^{2x}}{(1+2x)^2}\, dx\right]$$

$= \frac{e^{2x}}{4(2x+1)} + C$, C is integrating constant.

28. (d) : Let $I = \int \frac{dx}{\sqrt{16 - 9x^2}} = \frac{1}{3}\int \frac{dx}{\sqrt{(4/3)^2 - x^2}}$

$$= \frac{1}{3}\sin^{-1}\frac{x}{(4/3)} + C = \frac{1}{3}\sin^{-1}\left(\frac{3x}{4}\right) + C$$

On comparing above equation with $A\sin^{-1}(Bx) + C$,

we get $A = \frac{1}{3}$, $B = \frac{3}{4} \Rightarrow A + B = \frac{1}{3} + \frac{3}{4} = \frac{13}{12}$

29. (b) : Let $I = \int_0^1 x(1-x)^5\, dx$

$$= -\int_0^1 (1-x-1)(1-x)^5\, dx$$

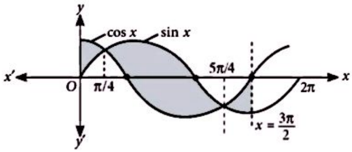
$$= -\left[\int_0^1 (1-x)^6\, dx - \int_0^1 (1-x)^5\, dx\right]$$

$$= \int_0^1 (1-x)^5\, dx - \int_0^1 (1-x)^6\, dx$$

$$= \left[\frac{(1-x)^6}{-6}\right]_0^1 - \left[\frac{(1-x)^7}{-7}\right]_0^1 = \frac{1}{6} - \frac{1}{7} = \frac{1}{42}$$

30. (b) : Given curves are $y = \cos x$, $y = \sin x$;

$x = 0$ and $x = \frac{3\pi}{2}$



$$\therefore \text{ Required area} = \int_0^{\pi/4} (\cos x - \sin x)\, dx + \int_{\pi/4}^{5\pi/4} (\sin x - \cos x) + \int_{5\pi/4}^{3\pi/2} (\cos x - \sin x)\, dx$$

$$= 2\int_0^{\pi/4} (\cos x - \sin x)\, dx + \int_{\pi/4}^{5\pi/4} (\sin x - \cos x)\, dx$$

$$= 2\left[\sin x + \cos x\right]_0^{\pi/4} + (-1)\left[\cos x + \sin x\right]_{\pi/4}^{5\pi/4}$$

$$= 2\left[\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - 1\right] - \left[\left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right) - \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)\right]$$

$= (4\sqrt{2} - 2)$ sq. units

31. (c) : $\because$ The given lines are concurrent

$$\therefore \begin{vmatrix} 2 & -3 & 5 \\ 9 & -5 & 14 \\ 3 & -7 & \lambda \end{vmatrix} = 0$$

$\Rightarrow\ 2(-5\lambda + 98) + 3(9\lambda - 42) + 5(-63 + 15) = 0$

$\Rightarrow\ 17\lambda = 170 \Rightarrow \lambda = 10$

32. (a) : Given, equation of lines is $Kx^2 - 4xy + y^2 = 0$

Let m_1 and m_2 be the slope of lines.

If m_1 and m_2 are slopes of two lines represented by $ax^2 + 2hxy + by^2 = 0$, then

$$m_1 + m_2 = \frac{-2h}{b} \text{ and } m_1 m_2 = \frac{a}{b}$$

$$\therefore \quad m_1 + m_2 = \frac{4}{1}, \quad m_1 m_2 = \frac{K}{1}$$

Also, $m_1 - m_2 = 2$ [Given]

Now, $(m_1 - m_2)^2 = (m_1 + m_2)^2 - 4m_1 m_2$

$\Rightarrow (2)^2 = (4)^2 - 4K$

$\Rightarrow 4K = 12 \Rightarrow K = 3$

33. (a) : $|\vec{a} - \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2(\vec{a} \cdot \vec{b})$

$= (2)^2 + (3)^2 - 2 \times 4 = 5$

$\Rightarrow |\vec{a} - \vec{b}| = \sqrt{5}$

34. (c) : $\vec{a} = \hat{i} + \hat{j} - 2\hat{k}, \vec{b} = 2\hat{i} - \hat{j} + \hat{k}, \vec{c} = 3\hat{i} - \hat{k}$

Also, $\vec{c} = m\vec{a} + n\vec{b}$

$3\hat{i} - \hat{k} = m(\hat{i} + \hat{j} - 2\hat{k}) + n(2\hat{i} - \hat{j} + \hat{k})$

$= (m + 2n)\hat{i} + (m - n)\hat{j} + (-2m + n)\hat{k}$

$\Rightarrow m + 2n = 3, m - n = 0, -2m + n = -1$

$\therefore m - n = 0 \Rightarrow m = n$

and $m + 2n = m + 2m = 3 \Rightarrow m = 1$

$\therefore m = n = 1 \Rightarrow m + n = 2$

35. (c) : $\vec{Q} = \dfrac{\vec{a} + \vec{b} + \vec{c}}{3}$

$$\therefore \quad \overline{QA} = \vec{a} - \frac{\vec{a} + \vec{b} + \vec{c}}{3} = \frac{2\vec{a} - \vec{b} - \vec{c}}{3}$$

Similarly, $\overline{QB} = \dfrac{-\vec{a} + 2\vec{b} - \vec{c}}{3}, \overline{QC} = \dfrac{-\vec{a} - \vec{b} + 2\vec{c}}{3}$

$\therefore \overline{QA} + \overline{QB} + \overline{QC} = 0$

36. (c) : Let direction cosines of a line making angle α with X-axis, β with Y-axis and γ with Z-axis are l, m, n.

$\therefore l = \cos\alpha, m = \cos\beta, n = \cos\gamma$

Given, $\alpha = 30°, \beta = 45°$ and $\gamma = 60°$

$\therefore$ Direction cosines of the line are $l = \cos 30°$, $m = \cos 45°$ and $n = \cos 60°$

$$\Rightarrow l = \frac{\sqrt{3}}{2}, m = \frac{1}{\sqrt{2}} \text{ and } n = \frac{1}{2}$$

37. (d) : Since, the line $\vec{r} = 2\hat{i} + \hat{j} + t(3\hat{i} + \hat{j} - 2\hat{k})$ is parallel to the plane $2x + 4y + az = 8$.

$\therefore 3(2) + 4(1) + a(-2) = 0 \Rightarrow 2a = 10 \Rightarrow a = 5$

38. (a) : We have, $5(4x + 6) < 25x + 10$

$\Rightarrow 20x + 30 < 25x + 10$

$\Rightarrow 5x > 20 \Rightarrow x > 4$

$\Rightarrow x \in (4, \infty)$

39. (c) : Let p = you will work

q : you will earn money.

We have, $p \to q$

$\Rightarrow$ Contrapositive of this will be $\sim q \to \sim p$

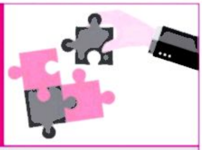
i.e. "If you will not earn money, you will not work".

40. (a) : $\displaystyle\lim_{x \to 0} \frac{\log(1+x) + 1 - e^x}{4x^2 - 9x}$

$$= \lim_{x \to 0} \left(\frac{\frac{1}{1+x} - e^x}{8x - 9} \right) \quad \text{[By L' Hospital's Rule]}$$

$= 0$

PUZZLE CORNER



MATHDOKU

Introducing MATHDOKU, a mixture of ken-ken, sudoku and Mathematics.

In this puzzle 6 × 6 grid is given, your objective is to fill the digits 1-6 so that each appear exactly once in each row and each column.

Notice that most boxes are part of a cluster. In the upper-left corner of each multibox cluster is a value that is combined using a specified operation on its numbers. For example, if that value is 3 for a two-box cluster and operation is multiply, you know that only 1 and 3 can go in there. But it is your job to determine which number goes where! A few cluster may have just one box and that is the number that fills that box.

11+	2÷		20×	6×	
	3–			3÷	
240×		6×			
		6×	7+	30×	
6×					9+
8+			2÷		

Readers can send their responses at editor@mtg.in or post us with complete address. Winners' name with their valuable feedback will be published in next issue.

CUET (UG)

Exam between 15^{th} to 31^{st} May 2024

PRACTICE PAPER 2024

General Instructions

This practice paper contains two sections i.e. Section A and Section B [B1 and B2].
Section A has 15 questions covering both i.e. Mathematics/Applied Mathematics which is compulsory for all candidates.
Section B1 has 35 questions from Mathematics out of which 25 questions need to be attempted.
Section B2 has 35 questions purely from Applied Mathematics out of which 25 question need to be attempted.

SECTION A

1. If $P=\begin{bmatrix} i & 0 & -i \\ 0 & -i & i \\ -i & i & 0 \end{bmatrix}$ and $Q=\begin{bmatrix} -i & i \\ 0 & 0 \\ i & -i \end{bmatrix}$, then PQ is equal to

(a) $\begin{bmatrix} -2 & 2 \\ 1 & -1 \\ 1 & -1 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & -2 \\ -1 & 1 \\ -1 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & -2 \\ -1 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

2. Solve the system of equations $x + 2y + z = 4$, $-x + y + z = 0$ and $x - 3y + z = 4$.
(a) $x = 2, y = 0, z = 2$ (b) $x = 2, y = 0, z = -2$
(c) $x = -2, y = 2, z = 0$ (d) $x = -2, y = 0, z = 2$

3. Let A be a 2×2 matrix.

Statement-I : $\text{adj}(\text{adj } A) = A$.

Statement-II : $|\text{adj } A| = |A|$.
In the light of above statements, choose the correct answer from the options given below.
(a) Both statement I and statement II are true
(b) Both statement I and statement II are false
(c) Statement I is true but statement II is false
(d) Statement I is false but statement II is true

4. If $y = A \sin x + B \cos x$, then $\frac{d^2x}{dy^2}$ is equal to
(a) $-y$ (b) y (c) xy (d) $x + y$

5. The function $f(x) = 1 - x^3 - x^5$ is decreasing for
(a) $1 \le x \le 5$ (b) $x \le 1$
(c) $x \ge 1$ (d) all values of x

6. Find the least value of a so that the function $f(x) = x^2 + ax + 1$ is strictly increasing on $[1, 2]$.
(a) -2 (b) 2 (c) 3 (d) -3

7. Consider the following statements and choose the correct option.

I. $\int_a^b f(x)dx$, if it exists, is a uniquely determined real number.

II. $\int_0^{2\pi} \sin^2 x\, dx = 4\int_0^{\pi/2} \sin^2 x\, dx$

III. $\int \frac{1}{1-2x}dx = -\frac{1}{2}\log|2x-1| + C$

IV. $\int \frac{x-\sin x}{1-\cos x}dx = \log|(1-\cos x)| + C$

(a) Only I (b) Only IV
(c) I, II and III (d) All are correct

8. Evaluate : $\int_0^{\pi/4} \sqrt{1-\sin 2x}\, dx$
(a) $\sqrt{2}-1$ (b) $\sqrt{2}+1$
(c) $\sqrt{2}$ (d) None of these

9. Consider the following statements and choose the correct option.
I. Area of region bounded by $y^2 = 16x$ and $x^2 = 12y$ is 64 sq. units.

II. Area of region bounded by $y^2 = 4ax$ and its latus rectum is $\frac{8a^2}{3}$ sq. units.

III. Area of region bounded by an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is ab sq. units.

(a) Only I (b) Only III
(c) I and II (d) None of the above

10. The area bounded by the curve $y = x^2 - 1$ and the straight line $x + y = 3$ is

(a) $\frac{9}{2}$ sq. units (b) 4 sq. units
(c) $\frac{7\sqrt{17}}{2}$ sq. units (d) $\frac{17\sqrt{17}}{6}$ sq. units

11. The solution of differential equation $(e^y + 1)\cos x\, dx + e^y \sin x\, dy = 0$ is

(a) $(e^y + 1)\sin x = c$ (b) $e^x \sin x = c$
(c) $(e^x + 1)\cos x = c$ (d) None of these

12. $\int \frac{a}{(1+x^2)\tan^{-1}x}dx$ is equal to

(a) $a \log|\tan^{-1}x| + C$ (b) $\frac{a}{2}(\tan^{-1}x)^2 + C$
(c) $a\log(1 + x^2) + C$ (d) None of these

13. Match the Column-I with Column-II and select the correct answer using the options given below :

Column-I		Column-II	
(P)	$Z = 7x - y$, subject to $5x + y \geq 5$, $x + y \geq 3$, $x \geq 0$, $y \geq 0$. The minimum value of Z occurs at	(1)	(1.5, 4)
(Q)	$Z = 8x + 10y$, subject to $2x + y \geq 7$, $2x + 3y \geq 15$, $y \geq 2$, $x \geq 0$, $y \geq 0$. The minimum value of Z occurs at	(2)	(3.5, 0)
(R)	$Z = 6x_1 + 2x_2$, subject to $5x_1 + 9x_2 \leq 90$, $x_1 + x_2 \geq 4$, $x_2 \leq 8$, $x_1 \geq 0$, $x_2 \geq 0$. The minimum value of Z occurs at	(3)	(0, 4)
(S)	$Z = 4x_1 + 5x_2$, subject to $2x_1 + x_2 \geq 7$, $2x_1 + 3x_2 \leq 15$, $x_2 \leq 3$, $x_1, x_2 \geq 0$. The minimum value of Z occurs at	(4)	(0, 5)

(a) (P) → (1), (Q) → (2), (R) → (3), (S) → (4)
(b) (P) → (1), (Q) → (3), (R) → (4), (S) → (2)
(c) (P) → (2), (Q) → (1), (R) → (3), (S) → (4)
(d) (P) → (4), (Q) → (1), (R) → (3), (S) → (2)

14. **Statement-I :** Addition of matrices is an example of binary operation on the set of matrices of the same order.

Statement-II : Addition of matrix is commutative.

In the light of above statements, choose the correct answer from the options given below.

(a) Both statement I and statement II are true
(b) Both statement I and statement II are false
(c) Statement I is true but statement II is false
(d) Statement I is false but statement II is true

15. Match the Column-I with Column-II and select the correct answer using the options given below :

Column-I		Column-II	
(P)	$\int_0^{\pi/2} \log(\tan x)\,dx$ is equal to	(1)	$\int_0^{\pi/2} \cos^2 x \sin x\, dx$
(Q)	$\int_0^{\pi/2} \sin^2 x\, dx$ is equal to	(2)	$\int_0^{\pi/2} \log(\cot x)dx$
(R)	$\int_0^{\pi/2} \sin^3 x\, dx$ is equal to	(3)	$\int_0^{\pi/2} \cos^2 x\, dx$
(S)	$\int_0^{\pi/2} \sin^2 x \cos x\, dx$ is equal to	(4)	$\int_0^{\pi/2} \sin x \sin 2x\, dx$

(a) (P) → (1), (Q) → (2), (R) → (3), (S) → (4)
(b) (P) → (2), (Q) → (3), (R) → (4), (S) → (1)
(c) (P) → (2), (Q) → (3), (R) → (1), (S) → (4)
(d) (P) → (2), (Q) → (1), (R) → (3), (S) → (4)

SECTION B1 (MATHEMATICS)

16. Let R be a relation on the set N be defined by $\{(x, y) : x, y \in N, 2x + y = 41\}$. Then, R is

(a) Reflexive (b) Symmetric
(c) Transitive (d) None of these

17. Let f and g be functions from R to R defined as

$$f(x)=\begin{cases}7x^2+x-8, & x\leq 1\\ 4x+5, & 1<x\leq 7\\ 8x+3, & x>7\end{cases} \text{ and } g(x)=\begin{cases}|x|, & x<-3\\ 0, & -3\leq x<2\\ x^2+4, & x\geq 2\end{cases}$$

Then,

(a) $(fog)(-3) = 8$ (b) $(fog)(9) = 683$
(c) $(gof)(0) = -8$ (d) $(gof)(6) = 427$

18. $4\tan^{-1}\frac{1}{5}-\tan^{-1}\frac{1}{239}$ is equal to

(a) π (b) $\pi/2$ (c) $\pi/3$ (d) $\pi/4$

19. If m and M are the least and the greatest values of $(\cos^{-1}x)^2+(\sin^{-1}x)^2$, then $\frac{M}{m}=$

(a) 10 (b) 5 (c) 4 (d) 2

20. The function $f(x)=x^2-2x$ is strictly decreasing in the interval

(a) $(-\infty, 1)$ (b) $(1, \infty)$
(c) R (d) None of these

21. Statement-I : If $y=\log_{10}x+\log_e x$, then $\frac{dy}{dx}=\frac{\log_{10}e}{x}+\frac{1}{x}$

Statement-II : $\frac{d}{dx}(\log_{10}x)=\frac{\log x}{\log 10}$ and $\frac{d}{dx}(\log_e x)=\frac{\log x}{\log e}$

In the light of above statements, choose the correct answer from the options given below.

(a) Both statement I and statement II are true
(b) Both statement I and statement II are false
(c) Statement I is true but statement II is false
(d) Statement I is false but statement II is true

22. Consider the following statements and choose the correct option.

(I) Matrix multiplication is always commutative.
(II) Any square matrix can be expressed as the sum of a symmetric and a skew-symmetric matrix.
(III) If A and B are invertible matrix of same order, then $(AB)^{-1}=A^{-1}B^{-1}$.

(a) Only I (b) Only I and II
(c) Only II and III (d) Only II

23. If x is a complex root of the equation

$$\begin{vmatrix}1 & x & x\\ x & 1 & x\\ x & x & 1\end{vmatrix}+\begin{vmatrix}1-x & 1 & 1\\ 1 & 1-x & 1\\ 1 & 1 & 1-x\end{vmatrix}=0,$$

then $x^{2007}+x^{-2007}=$

(a) 1 (b) -1 (c) -2 (d) 2

24. If the points $(3, -2)$, $(x, 2)$, $(8, 8)$ are collinear, then find the value of x.

(a) 2 (b) 3 (c) 4 (d) 5

25. If the equations $x+ay-z=0$, $2x-y+az=0$, $ax+y+2z=0$ have non-trivial solutions, then $a=$

(a) 2 (b) -2 (c) $\sqrt{3}$ (d) $-\sqrt{3}$

26. Consider the following statements and choose the correct option.

(I) The exponential function a^x is continuous everywhere, where a is a positive real number other than unity.
(II) The function $f(x)=\sin|x|$ is continuous for all $x\in R$.
(iii) The function $f(x)=\sqrt{x-2}$ is not continuous.

(a) Only I (b) Only III
(c) I and II (d) None of the above

27. **Statement-I :** Consider the experiment of drawing a card from a deck of 52 playing cards, in which the elementary events are assumed to be equally likely. If E and F denote the events the card drawn is a spade and the card drawn is an ace respectively, then $P(E|F)=\frac{1}{4}$ and $P(F|E)=\frac{1}{13}$.

Statement-II : E and F are two events such that the probability of occurrence of one of them is not affected by occurrence of the other. Such events are called independent events.

In the light of above statements, choose the correct answer from the options given below.

(a) Both statement I and statement II are true
(b) Both statement I and statement II are false
(c) Statement I is true but statement II is false
(d) Statement I is false but statement II is true

28. The derivative of $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ with respect to $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ is

(a) -1 (b) 1 (c) 2 (d) 4

29. If $y=\log\left[e^x\left(\frac{x-1}{x+2}\right)^{1/2}\right]$, then $\frac{dy}{dx}$ is equal to

(a) 7 (b) $\frac{3}{x-2}$
(c) $\frac{3}{(x-1)}$ (d) None of these

30. A spherical iron ball 10 cm in radius is coated with a layer of ice of uniform thickness that melts at a rate of 50 cm^3/min. When the thickness of ice is 15 cm, then the rate at which the thickness of ice decreases, is

(a) $\frac{5}{6\pi}$ cm/min (b) $\frac{1}{54\pi}$ cm/min
(c) $\frac{1}{18\pi}$ cm/min (d) $\frac{1}{36\pi}$ cm/min

31. An aeroplane can carry a maximum of 250 passengers. A profit of ₹ 1500 is made on each executive class ticket and a profit of ₹ 900 is made on each economy class ticket. The airline reserves at least 30 seats of executive class. However, at least 4 times as many passengers prefer to travel by economy class than by executive class. Formulate L.P.P in order to maximize the profit for the airline.

(a) Maximize $Z = 1500x + 900y$
Subject to $x + y \le 250, x \le 30, y \ge 4x$
(b) Maximize $Z = 1500x + 900y$
Subject to $x + y \le 250, x \ge 30, y \le 4x$
(c) Maximize $Z = 1500x + 900y$
Subject to $x + y \le 250, x \le 30, y \le 4x$
(d) Maximize $Z = 1500x + 900y$
Subject to $x + y \le 250, x \ge 30, y \ge 4x$

32. Match the Column-I with Column-II and select the correct answer using the options given below :

	Column-I		Column-II
(P)	If $f(x) = [4 - (x - 7)^3]$, then $f^{-1}(x)$ is	(1)	$\frac{\sqrt{x-6}-3}{2}$
(Q)	Let $f: \left[\frac{\pi}{2}, \frac{2\pi}{3}\right] \to [0, 4]$ be a function defined as $f(x) = \sqrt{3}\sin x - \cos x + 2$. Then, $f^{-1}(x)$ is	(2)	$\sin^{-1}\left(\frac{x-2}{2}\right) + \frac{\pi}{6}$
(R)	The inverse of $f(x) = 4x^2 + 12x + 15$ is	(3)	$7 + (4 - x)^{1/3}$

(a) (P) → (1), (Q) → (3), (R) → (2)
(b) (P) → (3), (Q) → (1), (R) → (2)
(c) (P) → (2), (Q) → (3), (R) → (1)
(d) (P) → (3), (Q) → (2), (R) → (1)

33. **Statement-I :** 'x' is not an integrating factor for the differential equation $x\frac{dy}{dx} + 2y = e^x$.

Statement-II : $x\left(x\frac{dy}{dx}+2y\right)=\frac{d}{dx}(x^2y)$.

In the light of above statements, choose the correct answer from the options given below.
(a) Both statement I and statement II are true
(b) Both statement I and statement II are false
(c) Statement I is true but statement II is false
(d) Statement I is false but statement II is true

34. Evaluate : $\int\frac{x+2}{\sqrt{x^2+5x+6}}dx$

(a) $\sqrt{x^2+5x+6}+\frac{1}{2}\log\left|\left(x+\frac{5}{2}\right)+\sqrt{x^2+5x+6}\right|+C$

(b) $\sqrt{x^2+5x+6}-\frac{1}{2}\log\left|\left(x+\frac{5}{2}\right)+\sqrt{x^2+5x+6}\right|+C$

(c) $\sqrt{x^2+5x+6}-\frac{1}{2}\log\left|\left(x+\frac{5}{2}\right)-\sqrt{x^2+5x+6}\right|+C$

(d) None of these

35. Five bad eggs are mixed with 10 good ones. If three eggs are drawn one by one with replacement, then find the probability distribution of the number of good eggs drawn.

(a)

X	0	1	2	3
P(X)	4/9	5/9	7/9	1/9

(b)

X	0	1	2	3
P(X)	5/54	7/54	2/27	7/27

(c)

X	0	1	2	3
P(X)	1/13	2/13	9/26	3/26

(d)

X	0	1	2	3
P(X)	1/27	2/9	4/9	8/27

36. The value of $\int_0^{2\pi}\frac{dx}{e^{\sin x}+1}$ is

(a) π (b) 0 (c) 3π (d) $\frac{\pi}{2}$

37. Match the Column-I with Column-II and select the correct answer using the options given below :

	Column-I		Column-II
(P)	If $\vec{a}, \vec{b}, \vec{c}$ are three non-coplanar vectors and $\vec{p}, \vec{q}, \vec{r}$ are vectors defined by the relations $\vec{p}=\frac{\vec{b}\times\vec{c}}{[\vec{a}\ \vec{b}\ \vec{c}]}$, $\vec{q}=\frac{\vec{c}\times\vec{a}}{[\vec{a}\ \vec{b}\ \vec{c}]}$, $\vec{r}=\frac{\vec{a}\times\vec{b}}{[\vec{a}\ \vec{b}\ \vec{c}]}$, then $(\vec{a}+\vec{b})\cdot\vec{p}+(\vec{b}+\vec{c})\cdot\vec{q}+(\vec{c}+\vec{a})\cdot\vec{r}=$	(1)	2
(Q)	If the vectors $\vec{a}=\hat{i}-2\hat{j}+\hat{k}$, $\vec{b}=p\hat{i}-5\hat{j}+3\hat{k}$, $\vec{c}=5\hat{i}-9\hat{j}+4\hat{k}$ are coplanar, then $p=$	(2)	41

(R)	If A (4, 2, 1), B (2, 1, 0), C (3, 1, −1) and D (1, −1, 2), then the volume of the parallelepiped with segments AB, AC and AD as concurrent edges is	(3)	3
(S)	The volume of the parallelepiped with segments AB, AC and AD as concurrent edges, where the position vectors of A, B, C, D are $\hat{i}+\hat{j}+\hat{k}$, $2\hat{i}-\hat{j}+3\hat{k}$, $3\hat{i}-2\hat{j}-2\hat{k}$ and $3\hat{i}+3\hat{j}+4\hat{k}$ respectively is	(4)	7

(a) (P) → (3), (Q) → (1), (R) → (2), (S) → (4)
(b) (P) → (3), (Q) → (1), (R) → (4), (S) → (2)
(c) (P) → (1), (Q) → (3), (R) → (4), (S) → (2)
(d) (P) → (2), (Q) → (1), (R) → (3), (S) → (4)

38. The area bounded by the curves $y = \sin x$, $y = \cos x$ and $x = 0$ is
(a) $(\sqrt{2}-1)$ sq. units (b) 1 sq. unit
(c) $\sqrt{2}$ sq. units (d) $(1+\sqrt{2})$ sq. units

39. The solution curve of $\frac{dy}{dx} = \frac{y^2-2xy-x^2}{y^2+2xy-x^2}$, $y(-1) = 1$ is
(a) a straight line (b) a parabola
(c) a circle (d) an ellipse

40. If $\vec{a}=2\hat{i}+\hat{j}+\hat{k}, \vec{b}=\hat{i}-2\hat{j}-\hat{k}, \vec{c}=\hat{i}+\hat{j}+\hat{k}$, then $\vec{a}\times(\vec{b}\times\vec{c})$ equals
(a) $5\hat{i}-7\hat{j}-3\hat{k}$ (b) $5\hat{i}+7\hat{j}-3\hat{k}$
(c) $5\hat{i}-7\hat{j}+3\hat{k}$ (d) zero vector

41. If the vectors $a\hat{i}+a\hat{j}+c\hat{k}$, $\hat{i}+\hat{k}$ and $c\hat{i}+c\hat{j}+b\hat{k}$ are coplanar, then c^2 =
(a) a^2b (b) ab (c) a^2b^2 (d) $\sqrt{ab}$

42. The shortest distance between the lines $x = y + 2 = 6z - 6$ and $x + 1 = 2y = -12z$ is
(a) 1/2 (b) 2 (c) 1 (d) 3/2

Case Based MCQs

Case I : Read the following passage and answer the questions from 43 to 46.

Two cars are moving with acceleration $(3t + 4)^3$ m/s^2 and $5t$ m/s^2 respectively. The formula for finding velocity when acceleration is known, is given by $\int a(t)dt = v(t) + c$, where $a(t)$ = acceleration at time 't', $v(t)$ = velocity at time 't' and c = constant of integration.

43. The velocity of first car is
(a) $\left(\frac{(3t+4)^3}{12}+C\right)$ m/s (b) $\left(\frac{(3t+4)^5}{12}+C\right)$ m/s
(c) $\left(\frac{(3t+4)^4}{12}+C\right)$ m/s (d) $\left(\frac{(3t+4)^2}{12}+C\right)$ m/s

44. At t = 10 seconds, the velocity of second car will be
(a) (250 + C) m/s (b) (225 + C) m/s
(c) (500 + C) m/s (d) (200 + C) m/s

45. If velocity of second car is $\frac{5t^2+4}{2}$, then $\int\frac{2dt}{5t^2+4}$ equals
(a) $\frac{2}{\sqrt{5}}\tan^{-1}\left(\frac{\sqrt{5}t}{2}\right)+C$ (b) $\frac{1}{\sqrt{5}}\tan^{-1}\left(\frac{\sqrt{5}t}{2}\right)+C$
(c) $\frac{1}{\sqrt{5}}\tan^{-1}\left(\sqrt{5}t\right)+C$ (d) $\frac{1}{\sqrt{5}}\tan^{-1}\left(\frac{t}{2}\right)+C$

46. If acceleration is $\sin^2 t$, then the velocity is
(a) $\frac{t}{2}+\frac{\sin 2t}{2}+C$ (b) $\frac{t}{2}+\sin 2t+C$
(c) $t - \sin 2t + C$ (d) $\frac{t}{2}-\frac{\sin 2t}{4}+C$

Case II : Read the following passage and answer the questions from 47 to 50.

A factory has three machines A, B and C to manufacture LED bulbs. Machine A manufactures 40%, machine B manufactures 30% and machine C manufactures 30% of the LED bulbs respectively. Out of their respective outputs, 11%, 8% and 4% are defective. A bulb is drawn at random from total production and it is found to be defective.

47. Probability that defective bulb is manufactured by machine A is
(a) $\frac{1}{2}$ (b) $\frac{11}{20}$ (c) $\frac{12}{25}$ (d) $\frac{15}{23}$

48. Probability that defective bulb is manufactured by machine B is
(a) $\frac{1}{2}$ (b) $\frac{3}{10}$ (c) $\frac{19}{25}$ (d) $\frac{7}{20}$

49. Probability that defective bulb is manufactured by machine C is
(a) $\frac{1}{4}$ (b) $\frac{3}{20}$ (c) $\frac{15}{61}$ (d) $\frac{7}{20}$

50. Probability that defective bulb is not manufactured by machine A is
(a) $\frac{17}{25}$ (b) $\frac{9}{20}$ (c) $\frac{2}{2}$ (d) 0

SECTION B2 (APPLIED MATHEMATICS)

16. A mixture of Hydrochloric Acid (HCl) and Sulphuric Acid is taken in the ratio 1 : 2 and another mixture of the same is taken in the ratio 2 : 3. How many parts of the two mixture must be taken to attain a new mixture consisting of HCl and sulphuric acid in the ratio 3 : 5?
(a) 2 : 3 (b) 5 : 3 (c) 3 : 5 (d) 3 : 2

17. Find the local maximum and local minimum values of the function $f(x) = \sin x - \cos x, 0 < x < 2\pi$.
(a) Local maximum value = $\sqrt{2}$,
Local minimum value = $-\sqrt{2}$
(b) Local maximum value = $2\sqrt{2}$,
Local minimum value = $-\sqrt{3}$
(c) Local maximum value = 1,
Local minimum value = $\frac{2}{3}$
(d) Local maximum value = 2,
Local minimum value = $-\sqrt{2}$

18. For what values of x and y are the matrices $A = \begin{bmatrix} 2x+1 & 3y \\ 0 & y^2-5y \end{bmatrix}, B = \begin{bmatrix} x+3 & y^2+2 \\ 0 & -6 \end{bmatrix}$ equal?
(a) 2, 3 (b) 3, 4
(c) 2, 2 (d) 3, 3

19. Mr. Bharti wishes to purchase a flat for ₹ 6000000 with a down payment of ₹ 1000000 and balance in equal monthly payments for 20 years. If bank charges 7.5% p.a. compounded monthly, calculate the EMI. (Given $(1.00625)^{240} = 4.4608$)
(a) ₹ 40279.70 (b) ₹ 2527.50
(c) ₹ 3027.30 (d) None of these

20. The random variable X can take only the values 0, 1, 2, 3. Given that $P(X = 0) = P(X = 1) = p$ and $P(X = 2) = P(X = 3)$ such that $\Sigma p_i x_i^2 = 2\Sigma p_i x_i$, find the value of p.
(a) $\frac{4}{5}$ (b) $\frac{3}{8}$ (c) $\frac{1}{2}$ (d) $\frac{1}{5}$

21. Find the last three digits of the product 1234 × 5678.
(a) 352 (b) 458 (c) 652 (d) 423

22. If $\frac{5-2x}{3} \leq \frac{x}{6} - 5$, then $x \in$
(a) $[2, \infty)$ (b) $[-8, 8]$ (c) $[4, \infty)$ (d) $[8, \infty)$

23. If $C(x) = 3 + 2x - \frac{1}{4}x^2$, then find the critical point of $C(x)$.
(a) 3 (b) 2
(c) 4 (d) None of these

24. Match the Column-I with Column-II and select the correct answer using the options given below

Column-I		Column-II	
(P)	A is a real skew-symmetric matrix such that $A^2 + I = 0$, then AA' is equal to	(1)	A
(Q)	A is a matrix such that $A^2 = A$ if $(I + A)^n = I + \lambda A$, then λ equals	(2)	0
(R)	If for a matrix A, $A^2 = A$ and $B = I - A$, then $AB + BA + I - (I - A)^2$ equals	(3)	$2^n - 1$
(S)	If $A' = A$ and $B' = B$, then $(AB - BA)' - (BA - AB)$ equals	(4)	I

(a) (P) → (1), (Q) → (2), (R) → (4), (S) → (3)
(b) (P) → (2), (Q) → (1), (R) → (3), (S) → (4)
(c) (P) → (3), (Q) → (4), (R) → (2), (S) → (1)
(d) (P) → (4), (Q) → (3), (R) → (1), (S) → (2)

25. **Statement-I** : If $\begin{bmatrix} xy & 4 \\ z+5 & x+y \end{bmatrix} = \begin{bmatrix} 4 & w \\ 0 & 4 \end{bmatrix}$, then $x = 2, y = 2, z = -5$ and $w = 4$.
Statement-II : Two matrices are equal, if their orders are same and their corresponding elements are equal.
In the light of above statements, choose the correct answer from the option given below.
(a) Both statement I and statement II are true
(b) Both statement I and statement II are false
(c) Statement I is true but statement II is false
(d) Statement I is false but statement II is true

26. A bond that matures in 5 years has coupon rate of 10% per annum and has a face value of ₹ 10000. Find the fair value of bond if the yield to maturity is 8%.
(a) ₹ 10798 (b) ₹ 11798
(c) ₹ 998 (d) None of these

27. A boat goes 32 km upstream and 36 km downstream in 7 hrs. Again it goes to 40 km upstream and 48 km downstream in 9 hrs. The speed of boat in still water is
(a) 2 km/h (b) 10 km/h (c) 5 km/h (d) 4 km/h

28. The minimum value of $Z = 4x + 3y$ subject to the constraints $3x + 2y \geq 160, 5x + 2y \geq 200, x + 2y \geq 80$; $x, y \geq 0$ is

(a) 220 (b) 300
(c) 230 (d) none of these

29. If a matrix A is both symmetric and skew-symmetric, then
(a) A is a diagonal matrix (b) A is a zero matrix
(c) A is a scalar matrix (d) A is a square matrix

30. Consider the following statements and choose the correct option.
I. In a L.P.P. the objective function is always quadratic.
II. The feasible region for a L.P.P. is always a convex polygon.
III. A feasible region of a system of linear inequalities is said to be an unbounded if it can be enclosed within a circle.
(a) Only I (b) I & III Only
(c) Only III (d) II Only

31. Consider the following statements and choose the correct option.
I. Maximum value of $f(x) = 3x^2 + 6x + 8, x \in R$ is 2.
II. Critical points of $f(x) = (x - 1)^3(x + 1)^2$ are 1, −1 and −1/5.
III. The function $f(x)=x+\frac{4}{x}$ has local maxima at $x = 2$.
IV. Critical points of $f(x) = x^2(x + 1)$ are 0 and 1.
(a) Only I (b) Only II
(c) Only I and III (d) None of the above

32. A and B started a business and invested ₹ 60 and ₹ 50 respectively. At the end of 4 months A withdrew half of his capital and B withdrew the half of his capital after 8 months, if C entered in the business after 6 months with a capital of ₹ 90. At the end of a year, in what ratio will the profit be distributed?
(a) 24 : 25 : 27 (b) 25 : 24 : 26
(c) 27 : 25 : 24 (d) 24 : 26 : 25

33. If $y = \sin x + e^x$, then $\frac{d^2x}{dy^2}$ is equal to
(a) $\frac{\sin x - e^x}{(\cos x + e^x)^2}$ (b) $\frac{\sin x - e^x}{(\cos x + e^x)^3}$
(c) $\frac{\sin x - e^x}{(\cos x + e^x)}$ (d) $(-\sin x + e^x)^{-1}$

34. Suppose that two cards are drawn at random from a deck of cards. Let X be the number of kings obtained. Then, the value of $E(X)$ is
(a) 37/221 (b) 5/13
(c) 1/13 (d) 2/13

35. If A and B are symmetric matrices of the same order, then
(a) AB is a symmetric matrix
(b) $A - B$ is a skew-symmetric matrix
(c) $AB + BA$ is a symmetric matrix
(d) $AB - BA$ is a symmetric matrix

36. A tank can be filled by two taps A and B in 12 hours and 15 hours respectively. The full tank can be emptied by tap C in 8 hours. If all the taps opened at the same time, in how much time will the empty tank be filled completely?
(a) $\frac{120}{13}$ hours (b) $\frac{120}{7}$ hours
(c) $\frac{48}{3}$ hours (d) $\frac{40}{3}$ hours

37. Find the present value of a sequence of payments of ₹ 10000 made at the end of every 3 months and continuing forever, if money is worth 8% per annum compounded quarterly.
(a) 2,00,000 (b) 5,00,000
(c) 7,00,000 (d) 1,00,000

38. Find the stationary points of the function $f(x) = 3x^4 - 8x^3 + 6x^2$.
(a) 0 and 1 (b) 2 and 4 (c) 1 and 2 (d) 1 and 2

39. A random variable X takes the values 0, 1, 2, 3 and its mean is 1.3. If $P(X = 3) = 2P(X = 1)$ and $P(X = 2) = 0.3$. Then match the Column-I with Column-II and select the correct answer using the options given below :

	Column-I		Column-II
(P)	P(X = 0) is	(1)	0.1
(Q)	Variance of X is	(2)	0.4
(R)	Standard deviation of X is	(3)	1.41
(S)	P(X = 1) is	(4)	1.19

(a) (P) → (2), (Q) → (3), (R) → (4), (S) → (1)
(b) (P) → (1), (Q) → (2), (R) → (3), (S) → (4)
(c) (P) → (3), (Q) → (2), (R) → (1), (S) → (4)
(d) (P) → (4), (Q) → (3), (R) → (2), (S) → (1)

40. The revenue function is given by $R(x) = 100x - x^2 - x^3$. Find the marginal revenue at $x = 2$
(a) ₹ 84 (b) ₹ 80 (c) ₹ 70 (d) ₹ 65

41. Find (186×93)mod 7.
(a) 1 (b) 2 (c) 3 (d) 4

42. In a race of 200 m, A beats B by 31 m and C by 18 m, then by how many metres C will defeat B in a 420 metres race?
(a) 20 m (b) 22 m (c) 25 m (d) 30 m

43. Rajesh and Ramesh started business with investment in the ratio of 12 : 11 and their annual profits were in the ratio 3 : 1. If Rajesh invested the money for 11 months; then for what time Ramesh invested the money?

(a) 4 months (b) 3 months
(c) 6 months (d) 8 months

44. Find (486 + 729) mod 12.

(a) 1 (b) 2 (c) 3 (d) 4

45. The speed of a boat in still water is 12 km/h. It takes twice as long as to go upstream to a point as to return downstream to the starting point. What is the speed of the stream?

(a) 3 km/h (b) 2 km/h
(c) 4 km/h (d) None of these

46. A machine costing ₹ 50000 has a useful life of 4 years. The estimated scrap value is ₹ 10000. Find the annual depreciation.

(a) ₹ 8000 (b) ₹ 10000 (c) ₹ 50000 (d) ₹ 15000

47. A factory owner wants to purchase two types of machines, A and B, for his factory. The machine A requires an area of 1000 m^2 and 12 skilled men for running it and its daily output is 50 units, whereas the machine B required 1200 m^2 area and 8 skilled men, and its daily output is 40 units. If an area of 7600 m^2 and 72 skilled men be available to operate the machine, how many machines A and B respectively should be purchased to maximize the daily output?

(a) 4, 3 (b) 2, 6 (c) 6, 2 (d) 3, 4

48. A machine costing ₹ 30000 is expected to have a useful life of 13 years and a final scrap value of ₹ 4000. Find the annual depreciation charge.

(a) ₹ 2000 (b) ₹ 1000
(c) ₹ 5000 (d) None of these

49. **Statement-I :** A measurable characteristics of population is known as parameter.

Statement-II : A measurable characteristic of a sample is known as statistic.

In the light of above statements, choose the correct answer from the options given below.

(a) Both statement I and statement II are true
(b) Both statement I and statement II are false
(c) Statement I is true but statement II is false
(d) Statement I is false but statement II is true

50. The rise and fall of share market is an example of

(a) Seasonal trend (b) Cyclical trend
(c) Secular trend (d) Irregular trend

SOLUTIONS

1. **(b) :** Since, $P=\begin{bmatrix} i & 0 & -i \\ 0 & -i & i \\ -i & i & 0 \end{bmatrix}$ and $Q=\begin{bmatrix} -i & i \\ 0 & 0 \\ i & -i \end{bmatrix}$

$$\therefore \quad PQ=\begin{bmatrix} i & 0 & -i \\ 0 & -i & i \\ -i & i & 0 \end{bmatrix}\begin{bmatrix} -i & i \\ 0 & 0 \\ i & -i \end{bmatrix}$$

$$=\begin{bmatrix} -i^2-i^2 & i^2+i^2 \\ i^2 & -i^2 \\ i^2 & -i^2 \end{bmatrix}=\begin{bmatrix} 1+1 & -1-1 \\ -1 & 1 \\ -1 & 1 \end{bmatrix}=\begin{bmatrix} 2 & -2 \\ -1 & 1 \\ -1 & 1 \end{bmatrix}$$

2. **(a) :** Given system of equations is $x + 2y + z = 4$, $-x + y + z = 0$ and $x - 3y + z = 4$

In matrix form, it can be written as

$$\begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & 1 \\ 1 & -3 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 4 \\ 0 \\ 4 \end{bmatrix}$$

or $AX = B$

where, $A=\begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & 1 \\ 1 & -3 & 1 \end{bmatrix}$, $B=\begin{bmatrix} 4 \\ 0 \\ 4 \end{bmatrix}$ and $X=\begin{bmatrix} x \\ y \\ z \end{bmatrix}$

Now, $|A| = 1(1 + 3) - 2(-1 - 1) + 1(3 - 1)$
$= 4 + 4 + 2 = 10$

$\because \quad |A| \neq 0$, hence unique solution exists.

Cofactor matrix of $A=\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$

$$\therefore \quad \text{adj}(A)=\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}^T=\begin{bmatrix} 4 & -5 & 1 \\ 2 & 0 & -2 \\ 2 & 5 & 3 \end{bmatrix}$$

$$A^{-1}=\frac{1}{|A|}\text{adj}(A)=\frac{1}{10}\begin{bmatrix} 4 & -5 & 1 \\ 2 & 0 & -2 \\ 2 & 5 & 3 \end{bmatrix}$$

Now, $X=A^{-1}B=\frac{1}{10}\begin{bmatrix} 4 & -5 & 1 \\ 2 & 0 & -2 \\ 2 & 5 & 3 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \\ 4 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix}=\frac{1}{10}\begin{bmatrix} 16+0+4 \\ 8+0-8 \\ 8+0+12 \end{bmatrix}=\frac{1}{10}\begin{bmatrix} 20 \\ 0 \\ 20 \end{bmatrix}=\begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}$$

On comparing the corresponding elements, we get

$x = 2, y = 0$ and $z = 2$

3. (a) : Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, adj $A = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$\Rightarrow$ adj (adj A) $= \begin{bmatrix} a & b \\ c & d \end{bmatrix} = A$

adj $A = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \Rightarrow |\text{adj } A| = ad - bc = |A|$.

4. (a) : $y = A \sin x + B \cos x$

$$\frac{dy}{dx} = \cos x + e^x$$

$$\frac{d^2y}{dx^2} = -A\sin x - B\cos x = -(A\sin x + B\cos x) = -y$$

5. (d): Given, $f(x) = 1 - x^3 - x^5$

Differentiating w.r.t. x, we get

$f'(x) = -3x^2 - 5x^4$

$\Rightarrow f'(x) = -(3x^2 + 5x^4) \Rightarrow f'(x) < 0$ for all values of x.

6. (a) : Given, $f(x) = x^2 + ax + 1, x \in [1, 2]$

Differentiating w.r.t. x, we get $f'(x) = 2x + a$

Now, $x \in [1, 2]$

$\Rightarrow 1 \le x \le 2 \Rightarrow 2 \le 2x \le 4 \Rightarrow 2 + a \le 2x + a \le 4 + a$

$\Rightarrow 2 + a \le f'(x) \le 4 + a$

For $f(x)$ to be strictly increasing on [1, 2]

$\therefore 2 + a \ge 0 \Rightarrow a \ge -2$

$\therefore$ Least value of $a = -2$

7. (c) : I. $\int_a^b f(x)\,dx$, if exists, gives a uniquely determined real number.

II. We have, $\int_0^{2\pi} \sin^2 x\,dx = \int_0^{2\pi} (\sin(2\pi - x))^2 dx = 2\int_0^{\pi} \sin^2 x dx$

$= 2\int_0^{\pi} (\sin(\pi - x))^2 dx = 4\int_0^{\pi/2} \sin^2 x\ dx$, which is correct.

III. $\int \frac{1}{1-2x}dx = \frac{-1}{2}\int \frac{-2}{1-2x}dx = \frac{-1}{2}\log|2x-1| + C$,

which is correct.

IV. Let $I = \int \frac{x - \sin x}{1 - \cos x}dx = \int \frac{x}{1-\cos x}dx - \int \frac{\sin x}{1 - \cos x}dx$

$$= \int \frac{x}{2\sin^2 \frac{x}{2}}dx - \int \frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{2\sin^2\frac{x}{2}}dx$$

$$= \frac{1}{2}\int x \operatorname{cosec}^2 \frac{x}{2} dx - \int \cot\frac{x}{2}\,dx$$

$$= \frac{1}{2}\left[x(-2)\cot\frac{x}{2} - \int\left(-2\cot\frac{x}{2}dx\right)\right] - \int \cot\frac{x}{2}dx + C$$

$$= -x\cot\frac{x}{2} + \int\cot\frac{x}{2}\,dx - \int\cot\frac{x}{2}\,dx = -x\cot\frac{x}{2} + C,$$

which is incorrect.

8. (a)

9. (c) : I. We have, $y^2 = 16x$ ...(i) and $x^2 = 12y$ ...(ii)

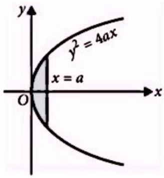
We know that area of the region bounded by $y^2 = 4ax$ ..(iii)

and $x^2 = 4by$ ...(iv) is $\frac{16ab}{3}$ sq. units

Here, on comparing we get $a = 4$ and $b = 3$

$\therefore$ Required area $= \frac{16 \times 4 \times 3}{3} = 64$ sq. units.

II. The equation $y^2 = 4ax$ represents a right hand parabola. The equation of its latus rectum is $x = a$.



$\therefore$ Required area $= 2\int_0^a |y|\,dx$

$= 2\int_0^a 2\sqrt{ax}\ dx$

$(y^2 = 4ax \Rightarrow |y| = 2\sqrt{ax})$

$= 4\sqrt{a}\int_0^a x^{1/2}dx = 4\sqrt{a}\left[\frac{x^{3/2}}{3/2}\right]_0^a$

$= \frac{8}{3}\sqrt{a}\ (a^{3/2} - 0) = \frac{8}{3}a^2 = \frac{8}{3}a^2$ sq. units.

III. We know that the area of region bounded by an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is πab sq. units.

10. (d): We have, $y = x^2 - 1$ ...(i)

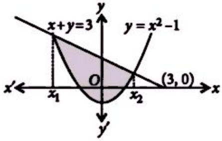
and $x + y = 3$ ...(ii)

Solving (i) and (ii), we get

$3 - x = x^2 - 1 \Rightarrow x^2 + x - 4 = 0$

$\Rightarrow x_1 + x_2 = -1$ and $x_1x_2 = -4$

$\Rightarrow x_2 - x_1 = \sqrt{17}$ ...(iii)



$\therefore$ Required area

$$= \int_{x_1}^{x_2} [(3-x)-(x^2-1)]dx = \int_{x_1}^{x_2} (4 - x - x^2)dx$$

$$=\left[4x-\frac{x^2}{2}-\frac{x^3}{3}\right]_{x_1}^{x_2}$$

$$=4(x_2-x_1)-\frac{1}{2}(x_2^2-x_1^2)-\frac{1}{3}\left(x_2^3-x_1^3\right)$$

$$=4(\sqrt{17})-\frac{1}{2}(-1)(\sqrt{17})-\frac{5\sqrt{17}}{3}=\frac{17\sqrt{17}}{6}\text{ sq. units}$$

11. (a) : Given, $(e^y+1)\cos x\,dx+e^y\sin x\,dy=0$

$$\Rightarrow \frac{e^y}{1+e^y}dy=-\frac{\cos x}{\sin x}dx$$

$$\Rightarrow \left(1-\frac{1}{1+e^y}\right)dy=-\cot x\,dx$$

$$\Rightarrow \left(1+\frac{(-e^{-y})}{1+e^{-y}}\right)dy=-\cot x\,dx$$

On integrating, we get $y+\log\left(\frac{1+e^y}{e^y}\right)=\log\left(\frac{c}{\sin x}\right)$

$$\Rightarrow y=\log\left(\frac{c}{\sin x}\right)+\log\left(\frac{e^y}{1+e^y}\right)$$

$$\Rightarrow y=\log\left(\frac{ce^y}{\sin x(1+e^y)}\right)$$

$$\Rightarrow e^y=\frac{ce^y}{\sin x(1+e^y)}\Rightarrow c=\sin x(1+e^y)$$

12. (a) **13. (d)**

14. (a) : Addition of matrices is an example of binary operation on the set of matrices of the same order.
And Statement-II is true.

15. (b): (P) We have, $I=\int_0^{\pi/2}\log(\tan x)\,dx$

$$=\int_0^{\pi/2}\log\left\{\tan\left(\frac{\pi}{2}-x\right)\right\}dx=\int_0^{\pi/2}\log(\cot)dx$$

(Q) $I=\int_0^{\pi/2}\sin^2 x\,dx=\int_0^{\pi/2}\left[\sin\left(\frac{\pi}{2}-x\right)\right]^2 dx=\int_0^{\pi/2}\cos^2 x\,dx$

(R) Let $I=\int_0^{\pi/2}\sin^3 x\,dx=\int_0^{\pi/2}\left(\frac{3}{4}\sin x-\frac{1}{4}\sin 3x\right)dx$

$$=\left[\frac{-3}{4}\cos x\right]_0^{\pi/2}+\left[\frac{1}{4\cdot 3}\cos 3x\right]_0^{\pi/2}$$

$$=\frac{-3}{4}(0-1)+\frac{1}{12}\left(\cos\frac{3\pi}{2}-\cos 0\right)=\frac{3}{4}-\frac{1}{12}=\frac{2}{3}$$

Let, $I'=\int_0^{\pi/2}\frac{2\sin x\sin 2x}{2}dx=\frac{1}{2}\int_0^{\pi/2}(\cos x-\cos 3x)dx$

$$=\frac{1}{2}\left[[\sin x]_0^{\pi/2}-\left[\frac{\sin 3x}{3}\right]_0^{\pi/2}\right]=\frac{1}{2}\left[(1-0)-\frac{1}{3}(-1-0)\right]$$

$$=\frac{1}{2}\left[1+\frac{1}{3}\right]=\frac{2}{3}$$

$$\therefore \int_0^{\pi/2}\sin^3 x\,dx=\int_0^{\pi/2}\sin x\sin 2x\,dx$$

(S) $I=\int_0^{\pi/2}\sin^2 x\cos x\,dx=\int_0^{\pi/2}(1-\cos^2 x)\cos x\,dx$

$$=\int_0^{\pi/2}\cos x\,dx-\int_0^{\pi/2}\cos^3 x\,dx$$

$$=\int_0^{\pi/2}\cos\left(\frac{\pi}{2}-x\right)dx-\int_0^{\pi/2}\left\{\cos\left(\frac{\pi}{2}-x\right)\right\}^3dx$$

$$=\int_0^{\pi/2}\sin x\,dx-\int_0^{\pi/2}\sin^3 x\,dx=\int_0^{\pi/2}(\sin x-\sin^3 x)dx$$

$$=\int_0^{\pi/2}(1-\sin^2 x)\sin x\,dx=\int_0^{\pi/2}\cos^2 x\sin x\,dx$$

SECTION B1 (MATHEMATICS)

16. (d): $R=\{(x,y):x,y\in N, 2x+y=41\}$
Reflexive : $(1,1)\notin R$ as $2\cdot 1+1=3\neq 41$. So, R is not reflexive.
Symmetric : $(1,39)\in R$ but $(39,1)\notin R$. So R is not symmetric.
Transitive : $(20,1)\in R$ and $(1,39)\in R$. But $(20,39)\notin R$, so R is not transitive.

17. (b): We have, $g(-3)=0$
$\Rightarrow f(g(-3))=f(0)=7(0)^2+0-8=-8$
Now, $g(9)=9^2+4=85$
$\Rightarrow f(g(9))=f(85)=8\times 85+3=683$
Also, $f(0)=7(0^2)+0-8=-8$
$\Rightarrow g(f(0))=g(-8)=|-8|=8$
Also, $f(6)=4\times 6+5=29$
$\Rightarrow g(f(6))=g(29)=(29)^2+4=845$

18. (d)

19. (a) : Given expression is $\left(\frac{\pi}{2}-\sin^{-1}x\right)^2+(\sin^{-1}x)^2$

$=\frac{\pi^2}{8}+2\left[\sin^{-1}x-\frac{\pi}{4}\right]^2$

$m=\frac{\pi^2}{8},\ M=\frac{5\pi^2}{4}\Rightarrow\frac{M}{m}=10$

20. (a) : $f'(x) = 2x - 2 = 2(x - 1) < 0$ if $x < 1$

i.e., $x\in(-\infty, 1)$. Hence, f is strictly decreasing in $(-\infty, 1)$

21. (c) : We have, $y = \log_{10}x + \log_e x$

$\therefore\ \frac{dy}{dx}=\frac{1}{x}\log_{10}e+\frac{1}{x}$

But Statement II is false.

22. (d)

23. (c) : Expanding the two determinants, we get

$(1 - 3x^2 + 2x^3) + (3x^2 - x^3) = 0$

$\Rightarrow\ x^3 + 1 = 0 \Rightarrow x = -\omega, -\omega^2, -1$

$x^{2007} + x^{-2007} = -1 - 1 = -2.$

24. (d): Since, the given points are collinear.

$\therefore\ \frac{1}{2}\begin{vmatrix}3 & -2 & 1\\ x & 2 & 1\\ 8 & 8 & 1\end{vmatrix}=0$

$\Rightarrow\ 3(2 - 8) + 2(x - 8) + 1(8x - 16) = 0$

$\Rightarrow\ -18 + 2x - 16 + 8x - 16 = 0 \Rightarrow 10x = 50 \Rightarrow x = 5$

25. (b) : $\begin{vmatrix}1 & a & -1\\ 2 & -1 & a\\ a & 1 & 2\end{vmatrix}=0$

Applying $R_2\to R_2-2R_1$ and $R_3\to R_3-aR_1$, we get

$$\begin{vmatrix}1 & a & -1\\ 0 & -1-2a & a+2\\ 0 & 1-a^2 & 2+a\end{vmatrix}=0$$

Expanding along C_1, we get

$(a+2)(a^2-2a-2)=0\ \Rightarrow\ a=-2,\ 1\pm\sqrt{3}$

26. (c)

27. (a)

28. (b): Let $p=\sin^{-1}\left[\frac{2x}{1+x^2}\right]=2\tan^{-1}x$

and $q=\cos^{-1}\left[\frac{1-x^2}{1+x^2}\right]=2\tan^{-1}x$

$\Rightarrow\ \frac{dp}{dx}=\frac{2}{1+x^2},\frac{dq}{dx}=\frac{2}{1+x^2}$

$\therefore\ \frac{dp}{dq}=\frac{\frac{dp}{dx}}{\frac{dq}{dx}}=\frac{\frac{2}{(1+x^2)}}{\frac{2}{(1+x^2)}}=1$

29. (d): We have, $y=\log\left[e^x\left(\frac{x-1}{x+2}\right)^{1/2}\right]$

$=\log e^x+\frac{1}{2}[\log(x-1)-\log(x+2)]$

$=x+\frac{1}{2}[\log(x-1)-\log(x+2)]$

$\therefore\ \frac{dy}{dx}=1+\frac{1}{2}\left[\frac{1}{x-1}-\frac{1}{x+2}\right]=1+\frac{3}{2(x-1)(x+2)}$

30. (c) : Given that, $\frac{dV}{dt}=50\text{ cm}^3/\text{min}$

$\Rightarrow\ \frac{d}{dt}\left(\frac{4}{3}\pi r^3\right)=50$

$\Rightarrow\ 3r^2\frac{dr}{dt}=\frac{150}{4\pi}\ \Rightarrow\ \frac{dr}{dt}=\frac{50}{4\pi r^2}$

$\Rightarrow\ \left(\frac{dr}{dt}\right)_{r=15}=\frac{50}{4\pi\times 225}=\frac{1}{18\pi}$ cm/min

31. (d): Let x be the number of passengers travelling by executive class and y be the number of passengers travelling by economy class.

Thus, $x\ge 0,\ y\ge 0$.

According to given condition, $x + y \le 250$

The airline reserves at least 30 seats for executive class.

$\therefore\ x\ge 30$

Also, $y\ge 4x$

The profit on each executive class ticket is ₹ 1500 and on each economy class ticket is ₹ 900.

$\therefore$ Total profit = ₹ $(1500x + 900y)$

$\therefore$ Given problem can be formulated as

Maximize $Z = 1500x + 900y$,

Subject to $x + y \le 250,\ x \ge 30,\ y \ge 4x$.

32. (d): (P) Given, $f(x) = [4 - (x - 7)^3]$

Consider, $y = [4 - (x - 7)^3]\ \Rightarrow\ (x - 7)^3 = 4 - y$

$\Rightarrow\ (x - 7) = (4 - y)^{1/3}\ \Rightarrow\ x = 7 + (4 - y)^{1/3}$

$\Rightarrow\ f^{-1}(x) = 7 + (4 - x)^{1/3}$

(Q) $f(x)=\sqrt{3}\sin x-\cos x+2=2\sin\left(x-\frac{\pi}{6}\right)+2$

Since, $f(x)$ is one-one and onto, f is invertible.

Now, $fof^{-1}(x)=x\ \Rightarrow\ 2\sin\left(f^{-1}(x)-\frac{\pi}{6}\right)+2=x$

$\Rightarrow \sin\left(f^{-1}(x)-\frac{\pi}{6}\right)=\frac{x}{2}-1$

$\Rightarrow f^{-1}(x)=\sin^{-1}\left(\frac{x}{2}-1\right)+\frac{\pi}{6}$

$(\because |(x/2-1)| \le 1 \ \forall\ x \in [0, 4])$

(R) Let $f(x) = y \Rightarrow 4x^2 + 12x + 15 = y$

$\Rightarrow (2x+3)^2+6=y \Rightarrow 2x+3=\sqrt{y-6}$

$\Rightarrow x=\frac{\sqrt{y-6}-3}{2} \Rightarrow f^{-1}(x)=\frac{\sqrt{x-6}-3}{2}$

33. (a)

34. (b): Let $I=\int \frac{x+2}{\sqrt{x^2+5x+6}}dx$

Put $x+2=\lambda\cdot\frac{d}{dx}(x^2+5x+6)+\mu$

$\Rightarrow x+2=\lambda(2x+5)+\mu$

Comparing the coefficients of like powers of x, we get

$1 = 2\lambda$ and $5\lambda+\mu=2 \Rightarrow \lambda=\frac{1}{2}$ and $\mu=-\frac{1}{2}$

$\therefore I=\int \frac{\frac{1}{2}(2x+5)-\frac{1}{2}}{\sqrt{x^2+5x+6}}dx$

$=\frac{1}{2}\int\frac{2x+5}{\sqrt{x^2+5x+6}}dx-\frac{1}{2}\int\frac{1}{\sqrt{x^2+5x+6}}dx$

$=\frac{1}{2}\int\frac{dt}{\sqrt{t}}-\frac{1}{2}\int\frac{1}{\sqrt{\left(x+\frac{5}{2}\right)^2-\left(\frac{1}{2}\right)^2}}dx,$

where $t = x^2 + 5x + 6$

$\therefore I=\sqrt{t}-\frac{1}{2}\log\left|\left(x+\frac{5}{2}\right)+\sqrt{x^2+5x+6}\right|+C$

$=\sqrt{x^2+5x+6}-\frac{1}{2}\log\left|\left(x+\frac{5}{2}\right)+\sqrt{x^2+5x+6}\right|+C$

35. (d): Since, the eggs are drawn one by one with replacement, the events are independent, therefore, it is a problem of binomial distribution.

Total number of eggs = 5 + 10 = 15, out of which 10 are good.

If p = probability of drawing a good egg, then

$p=\frac{10}{15}=\frac{2}{3} \quad \therefore q=1-\frac{2}{3}=\frac{1}{3}$

Thus, we have a binomial distribution with $p=\frac{2}{3}, q=\frac{1}{3}$ and $n = 3$.

If X denotes the number of good eggs drawn, then X can take values 0, 1, 2, 3.

$P(0)={}^3C_0q^3=1\times\left(\frac{1}{3}\right)^3=\frac{1}{27},$

$P(1)={}^3C_1pq^2=3\times\frac{2}{3}\times\left(\frac{1}{3}\right)^2=\frac{2}{9},$

$P(2)={}^3C_2p^2q=3\times\left(\frac{2}{3}\right)^2\times\frac{1}{3}=\frac{4}{9}$ and

$P(3)={}^3C_3p^3=1\times\left(\frac{2}{3}\right)^3=\frac{8}{27}$

$\therefore$ The required probability distribution is

X	0	1	2	3
$P(X)$	$\frac{1}{27}$	$\frac{2}{9}$	$\frac{4}{9}$	$\frac{8}{27}$

36. (a): Let $I=\int_0^{2\pi}\frac{dx}{e^{\sin x}+1}$ ...(i)

$\Rightarrow I=\int_0^{2\pi}\frac{dx}{e^{\sin(2\pi-x)}+1}$ (by property)

$\Rightarrow I=\int_0^{2\pi}\frac{dx}{e^{-\sin x}+1} \Rightarrow I=\int_0^{2\pi}\frac{e^{\sin x}}{e^{\sin x}+1}dx$ ...(ii)

Adding (i) and (ii), we get

$2I=\int_0^{2\pi}1\cdot dx=2\pi$

$\therefore I=\pi$

37. (b): (P) $(\vec{a}+\vec{b})\cdot\vec{p}=\vec{a}\cdot\vec{p}+\vec{b}\cdot\vec{p}$

$=\frac{\vec{a}\cdot(\vec{b}\times\vec{c})}{[\vec{a}\ \vec{b}\ \vec{c}]}+\frac{\vec{b}\cdot(\vec{b}\times\vec{c})}{[\vec{a}\ \vec{b}\ \vec{c}]}=1+0=1$

Similarly $(\vec{b}+\vec{c})\cdot\vec{q}=(\vec{c}+\vec{a})\cdot\vec{r}=1$

Now $(\vec{a}+\vec{b})\cdot\vec{p}+(\vec{b}+\vec{c})\cdot\vec{q}+(\vec{c}+\vec{a})\cdot\vec{r}=1+1+1=3$

(Q) Since $\bar{a},\bar{b},\bar{c}$ are coplanar.

$\therefore [\vec{a}\ \vec{b}\ \vec{c}]=0\Rightarrow\begin{vmatrix}1 & -2 & 1\\ p & -5 & 3\\ 5 & -9 & 4\end{vmatrix}=0$

$\Rightarrow (-20+27)+2(4p-15)+(-9p+25)=0$

$\Rightarrow 7+8p-30-9p+25=0\Rightarrow p=2$

(R) Here $\vec{a}=4\hat{i}+2\hat{j}+\hat{k},\ \vec{b}=2\hat{i}+\hat{j},\ \vec{c}=3\hat{i}+\hat{j}-\hat{k},$

$\vec{d}=\hat{i}-\hat{j}+2\hat{k}$

Now $\overline{AB}=\vec{b}-\vec{a}=-2\hat{i}-\hat{j}-\hat{k};\ \overline{AC}=\vec{c}-\vec{a}=-\hat{i}-\hat{j}-2\hat{k}$

and $\overrightarrow{AD}=\vec{d}-\vec{a}=-3\hat{i}-3\hat{j}+\hat{k}$

$\therefore$ Volume $=\left[\overrightarrow{AB}\ \overrightarrow{AC}\ \overrightarrow{AD}\right]=\begin{vmatrix}-2 & -1 & -1\\ -1 & -1 & -2\\ -3 & -3 & 1\end{vmatrix}$

$= -2(-1-6)+(-1-6)-(3-3) = 7$ cu. units

(S) $\overrightarrow{AB}=\hat{i}-2\hat{j}+2\hat{k}$, $\overrightarrow{AC}=2\hat{i}-3\hat{j}-3\hat{k}$,

$\overrightarrow{AD}=2\hat{i}+2\hat{j}+3\hat{k}$

$\therefore$ Volume of parallelopiped $= [\overrightarrow{AB}\ \ \overrightarrow{AC}\ \ \overrightarrow{AD}]$

$=\begin{vmatrix}1 & -2 & 2\\ 2 & -3 & -3\\ 2 & 2 & 3\end{vmatrix} = 1(-9+6)+2(6+6)+2(4+6)$

$= 41$ cu. units

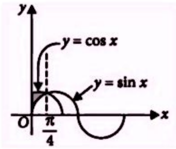
38. (a) : The given equation of curves are

$y = \sin x$ ...(i) and $y = \cos x$ ...(ii)

From equation (i) and (ii), we get

$\sin x = \cos x \Rightarrow x = \frac{\pi}{4}$

$\therefore$ Required area $= \int_0^{\pi/4} (\cos x - \sin x)dx$



$= [\sin x + \cos x]_0^{\pi/4}$

$= \left(\sin\frac{\pi}{4}+\cos\frac{\pi}{4}-\sin 0-\cos 0\right)=\left[\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}-1\right]$

$= \frac{2}{\sqrt{2}}-1=(\sqrt{2}-1)$ sq. units

39. (a) : Substitute $y = vx \Rightarrow \frac{dy}{dx}=\frac{xdv}{dx}+v$

Now, given equation becomes

$\frac{xdv}{dx}+v = \frac{v^2-2v-1}{v^2+2v-1}$

$\Rightarrow \frac{xdv}{dx} = \frac{v^2-2v-1}{v^2+2v-1} - v = -\frac{(v^2+1)(v+1)}{v^2+2v-1}$

$\Rightarrow \frac{dx}{x} = \frac{-(v^2+2v-1)dv}{(v^2+1)(v+1)} = -\left(\frac{2v}{v^2+1}-\frac{1}{v+1}\right)dv$

$\Rightarrow \ln x + \ln c = \ln\left(\frac{v+1}{v^2+1}\right) \Rightarrow xc = \frac{v+1}{v^2+1}$

$\Rightarrow xc = \frac{x(x+y)}{x^2+y^2} \Rightarrow c(x^2+y^2) = x+y$

$\because x=-1, y=1 \Rightarrow c=0 \therefore x+y=0.$

40. (a) : $\vec{b}\times\vec{c}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -2 & -1\\ 1 & 1 & 1\end{vmatrix} = -\hat{i}-2\hat{j}+3\hat{k}$

$\therefore \vec{a}\times(\vec{b}\times\vec{c})=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 1 & 1\\ -1 & -2 & 3\end{vmatrix} = 5\hat{i}-7\hat{j}-3\hat{k}$

41. (b) : Since the given vectors are coplanar

$\therefore \begin{vmatrix}a & a & c\\ 1 & 0 & 1\\ c & c & b\end{vmatrix}=0$

Applying $C_1 \to C_1 - C_2$, we get

$\begin{vmatrix}0 & a & c\\ 1 & 0 & 1\\ 0 & c & b\end{vmatrix}=0 \Rightarrow -1(ab-c^2)=0 \Rightarrow c^2=ab$

42. (b) : The given equation of lines can be written as

$\frac{x}{6}=\frac{y+2}{6}=\frac{z-1}{1}$ and $\frac{x+1}{12}=\frac{y}{6}=\frac{z}{-1}$

Here,

$\vec{a}_1=-2\hat{j}+\hat{k}, \vec{b}_1=6\hat{i}+6\hat{j}+\hat{k}, \vec{a}_2=-\hat{i}$ and $\vec{b}_2=12\hat{i}+6\hat{j}-\hat{k}$

$\therefore \vec{b}_1\times\vec{b}_2=-12\hat{i}+18\hat{j}-36\hat{k}$

Now, shortest distance

$=\frac{|(\vec{a}_2-\vec{a}_1)\cdot(\vec{b}_1\times\vec{b}_2)|}{|\vec{b}_1\times\vec{b}_2|}=\frac{|(-\hat{i}+2\hat{j}-\hat{k})\cdot(-12\hat{i}+18\hat{j}-36\hat{k})|}{\sqrt{(-12)^2+(18)^2+(-36)^2}}$

$=\frac{|12+36+36|}{\sqrt{1764}}=\frac{84}{42}=2$

43. (c) : The acceleration of first car $= (3t+4)^3$ m/s^2

Since, $v(t)=\int(3t+4)^3dt+C$

$\Rightarrow v(t)=\frac{(3t+4)^4}{4\cdot 3}+C=\left(\frac{(3t+4)^4}{12}+C\right)$ m/s

44. (a) : The acceleration of second car $= 5t$ m/s^2

$\therefore$ The velocity of second car, $v(t)=\int 5t\,dt+C$

$\Rightarrow v(t)=\frac{5t^2}{2}+C$; At $t = 10s$, $v(10)=\frac{5\times(10)^2}{2}+C$

$\Rightarrow v(10)=5\times 50+C=(250+C)$ m/s

45. (b): Now, velocity of second car, $v(t)=\frac{5t^2+4}{2}$

$$\therefore \int\frac{2dt}{5t^2+4}=2\int\frac{dt}{5t^2+(2)^2}=\frac{2}{5}\int\frac{dt}{t^2+\left(\frac{2}{\sqrt{5}}\right)^2}$$

$$=\frac{2}{5}\times\frac{\sqrt{5}}{2}\tan^{-1}\left(\frac{\sqrt{5}t}{2}\right)+C=\frac{1}{\sqrt{5}}\tan^{-1}\left(\frac{\sqrt{5}t}{2}\right)+C$$

46. (d): The velocity is, $\int\sin^2 t\,dt=\int\frac{1-\cos 2t}{2}dt$

$$=\frac{1}{2}t-\frac{\sin 2t}{2\times 2}+C=\frac{t}{2}-\frac{\sin 2t}{4}+C$$

47. (b): Let E_1, E_2 and E_3 are the events that LED bulb is manufactured by three machines A, B, and C respectively.

Let E be the event that the bulb is defective.

$$P(E_1)=\frac{40}{100}=\frac{4}{10},\ P(E_2)=\frac{30}{100}=\frac{3}{10},\ P(E_3)=\frac{30}{100}=\frac{3}{10}$$

$$P(E/E_1)=\frac{11}{100},\ P(E/E_2)=\frac{8}{100}\ \text{and}\ P(E/E_2)=\frac{4}{100}$$

Probability that defective bulb is manufactured by machine $A=P(E_1/E)$

$$=\frac{P(E/E_1)\cdot P(E_1)}{P(E/E_1)\cdot P(E_1)+P(E/E_2)\cdot P(E_2)+P(E/E_3)\cdot P(E_3)}$$

$$=\frac{\frac{11}{100}\times\frac{4}{10}}{\frac{11}{100}\times\frac{4}{10}+\frac{2}{25}\times\frac{3}{10}+\frac{1}{25}\times\frac{3}{10}}=\frac{\frac{11}{250}}{\frac{11}{250}+\frac{6}{250}+\frac{3}{250}}$$

$$=\frac{11}{20}$$

48. (b): Probability that defective bulb is manufactured by machine $B=P(E_2/E)$

$$=\frac{P(E/E_2)\cdot P(E_2)}{P(E/E_1)\cdot P(E_1)+P(E/E_2)\cdot P(E_2)+P(E/E_3)\cdot P(E_3)}$$

$$=\frac{\frac{2}{25}\times\frac{3}{10}}{\frac{11}{100}\times\frac{4}{10}+\frac{2}{25}\times\frac{3}{10}+\frac{1}{25}\times\frac{3}{10}}=\frac{\frac{6}{250}}{\frac{11}{250}+\frac{6}{250}+\frac{3}{250}}$$

$$=\frac{3}{10}$$

49. (b): Probability that defective bulb is manufactured by machine $C=P(E_3/E)$

$$=\frac{P(E/E_3)\cdot P(E_3)}{P(E/E_1)\cdot P(E_1)+P(E/E_2)\cdot P(E_2)+P(E/E_3)\cdot P(E_3)}$$

$$=\frac{\frac{1}{25}\times\frac{3}{10}}{\frac{11}{100}\times\frac{4}{10}+\frac{2}{25}\times\frac{3}{10}+\frac{1}{25}\times\frac{3}{10}}=\frac{\frac{3}{250}}{\frac{11}{250}+\frac{6}{250}+\frac{3}{250}}$$

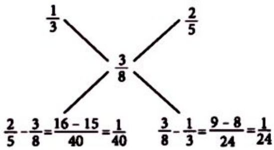
$$=\frac{3}{20}$$

50. (b): Probability that the defective bulb is not manufactured by machine A is $=1-\frac{11}{20}=\frac{9}{20}$

SECTION B2 (APPLIED MATHEMATICS)

16. (c) : In type A, mixture HCl : sulphuric acid = 1 : 2 and in type B, mixture HCl : sulphuric acid = 2 : 3

Now, we can consider HCl as sulphuric acid while using alligation method, here we are using HCl.



$\therefore$ Required ratio $=\frac{1}{40}:\frac{1}{24}=3:5$

17. (a) : Given $f(x)=\sin x-\cos x,\ 0<x<2\pi$.

i.e., $f(x)=\sqrt{2}\left(\frac{1}{\sqrt{2}}\sin x-\frac{1}{\sqrt{2}}\cos x\right)=\sqrt{2}\sin\left(x-\frac{\pi}{4}\right)$

$\Rightarrow f'(x)=\sqrt{2}\cos\left(x-\frac{\pi}{4}\right), 0<x<2\pi.$

For critical points, let $f'(x)=0\Rightarrow\sqrt{2}\cos\left(x-\frac{\pi}{4}\right)=0$

$$\Rightarrow \cos\left(x-\frac{\pi}{4}\right)=0 \Rightarrow x-\frac{\pi}{4}=\frac{\pi}{2}\ \text{or}\ \frac{3\pi}{2}$$

$$\left(\because\ 0<x<2\pi\Leftrightarrow-\frac{\pi}{4}<x-\frac{\pi}{4}<\frac{7\pi}{4}\right)$$

$$\Rightarrow x=\frac{\pi}{4}+\frac{\pi}{2}\ \text{or}\ \frac{\pi}{4}+\frac{3\pi}{2}\Rightarrow x=\frac{3\pi}{4},\frac{7\pi}{4}.$$

Now, $f''(x)=-\sqrt{2}\sin\left(x-\frac{\pi}{4}\right)$

$$f''\left(\frac{3\pi}{4}\right)=-\sqrt{2}\sin\left(\frac{3\pi}{4}-\frac{\pi}{4}\right)=-\sqrt{2}$$

$\Rightarrow f''\left(\frac{3\pi}{4}\right)<0\Rightarrow f$ has a local maximum at $x=\frac{3\pi}{4}$

Also, local maximum value $=f\left(\frac{3\pi}{4}\right)=\sqrt{2}\sin\left(\frac{3\pi}{4}-\frac{\pi}{4}\right)$

$$=\sqrt{2}\sin\frac{\pi}{2}=\sqrt{2}$$

Now, $f''\left(\frac{7\pi}{4}\right) = -\sqrt{2}\sin\left(\frac{7\pi}{4} - \frac{\pi}{4}\right) = -\sqrt{2}\sin\frac{3\pi}{2} = \sqrt{2}$

$\Rightarrow \quad f''\left(\frac{7\pi}{4}\right) > 0 \Rightarrow f$ has a local minimum at $x = \frac{7\pi}{4}$.

Also, local minimum value $= f\left(\frac{7\pi}{4}\right)$

$$= \sqrt{2}\sin\left(\frac{7\pi}{4} - \frac{\pi}{4}\right) = \sqrt{2}\sin\frac{3\pi}{2} = -\sqrt{2}$$

18. (c) **19. (a)**

20. (b): Given $P(X = 0) = P(X = 1) = p$ and $P(X = 2) = P(X = 3) = k$ (say)

The probability distribution of the random variable X is

X	0	1	2	3
$P(X)$	p	p	k	k

We know that $\Sigma p_i = 1$

$\Rightarrow \quad p + p + k + k = 1$

$\Rightarrow \quad 2p + 2k = 1$

$\Rightarrow \quad p + k = \frac{1}{2} \Rightarrow k = \frac{1}{2} - p.$ ..(i)

$\Sigma p_i x_i^2 = 2\,\Sigma p_i x_i$ (Given)

$\Rightarrow \quad 0 + p + 4k + 9k + = 2(0 + p + 2k + 3k) \Rightarrow p = 3k$

$\Rightarrow \quad p = 3\left(\frac{1}{2} - p\right)$ [From (i)]

$\Rightarrow \quad p = \frac{3}{8}$

21. (c): To find the last three digits of the product $1234 \times 5678 \pmod{1000}$.

Since $1234 \equiv 234 \pmod{1000}$

and $5678 \equiv 678 \pmod{1000}$.

So, $1234 \times 5678 \equiv 234 \times 678 \pmod{1000}$

$\equiv 158652 \pmod{1000} \equiv 652 \pmod{1000}$

Hence, the last three digits of the product 1234×5678 are 652.

22. (d): We have, $\frac{5-2x}{3} \le \frac{x}{6} - 5$

or $2(5 - 2x) \le x - 30$ or $10 - 4x \le x - 30$

or $-5x \le -40$ or $x \ge 8$

Thus, all real numbers which are greater than or equal to 8 are the solutions of the given inequality *i.e.*, $x \in [8, \infty)$.

23. (c): $C(x) = 3 + 2x - \frac{1}{4}x^2$,

$C'(x) = 2 - \frac{1}{2}x$

To find critical points of $C(x)$, put $C'(x) = 0$

$2 - \frac{1}{2}x = 0 \quad \Rightarrow x = 4$

24. (d): **(P)** Given, A is a skew-symmetric matrix.

$\therefore \quad A' = -A$

Also, $A^2 + I = 0 \Rightarrow A^2 = -I$

$\Rightarrow \quad A \cdot A = -I \Rightarrow A \cdot A \cdot A' = -I A' = I(-A') = IA = A$

$\Rightarrow \quad (A^{-1}A)AA' = A^{-1}A \Rightarrow AA' = I$

(Q) $(I + A)^n = C_0 I^n + C_1 I^{n-1} A + C_2 I^{n-2} A^2 + \dots + C_n I^{n-n} A^n$.

$= C_0 I + C_1 A + C_2 A + \dots + C_n A = I + (C_1 + C_2 + \dots + C_n)A$

$= I + (2^n - 1)A$ $\quad [\because \ (C_1 + C_2 + \dots + C_n) = 2^n - 1]$

$\Rightarrow \quad \lambda = 2^n - 1$

(R) We have, $A^2 = A$ and $B = I - A$

Now, $AB + BA + I - (I - A)^2$

$= AB + BA + I - (I + A^2 - 2A)$

$= AB + BA - A + 2A \quad (\because A^2 = A)$

$= AB + BA + A = A(I - A) + (I - A)A + A$

$= A - A^2 + A - A^2 + A = A - A + A - A + A = A$

(S) We have, $A' = A$, $B' = B$

Now, $(AB - BA)' = B'A' - A'B' = BA - AB$

$\therefore \quad (AB - BA)' - (BA - AB) = 0$

25. (a): $\begin{bmatrix} xy & 4 \\ z+5 & x+y \end{bmatrix} = \begin{bmatrix} 4 & w \\ 0 & 4 \end{bmatrix}$

On equating the corresponding elements, we get

$xy = 4$, $w = 4$, $z + 5 = 0$ and $x + y = 4$

On solving these equations, we get $x = 2$, $y = 2$, $z = -5$ and $w = 4$.

Also, the two matrices are equal, if their orders are same and their corresponding elements are equal.

26. (a): Given, $F =$ ₹ 10000, $r = 10\%$ per annum, $N = 5$ years, $d = 8\%$

$\Rightarrow \quad i = \frac{8}{100} = 0.08$

So, coupon payment $C =$ ₹ $10000 \times \frac{10}{100} =$ ₹ 1000

Pi Day : Answer to Quiz on the Mathematical Constant Pi

1. Pi Day is observed on March 14 (the third month) since 3, 1, and 4 are the first three significant figures of π
2. It was founded in 1988 by Larry Shaw, an American physicist
3. International Day of Mathematics
4. July 22 (22/7 in the day/month format, an approximation of π) and June 28 (6.28, an approximation of 2π)
5. The date is written as 3/14/15 in month/day/year format. At 9:26:53, the date and time together represented the first ten digits of p, and later that second Pi Instant represented all π's digits
6. Pie eating competitions, recalling Pi to the highest number of decimal places
7. Albert Einstein

Visual Question

Answer : Tau. Tau is double the value of Pi. Tau Time is at 6:28

$$\frac{C[1-(1+i)^{-N}]}{i}+F(1+i)^{-N}$$

$$=\frac{1000[1-(1.08)^{-5}]}{0.08}+1000(1.08)^{-5}$$

$$\therefore \quad \text{P.V.}=\frac{1000[1-0.6808]}{0.08}+10000\times 0.6808$$

$$= 3990 + 6808 = 10798$$

Hence, the fair value of bond is ₹ 10798.

27. (b) : Let speed of boat in still water be x km/h and the speed of current be y km/h.

$\therefore$ Speed of boat in downstream = $(x + y)$ km/h and that in upstream = $(x - y)$ km/h

According to problem, we have

$$\frac{32}{x-y}+\frac{36}{x+y}=7 \text{ or } 32A+36B=7 \quad \text{...(i)}$$

where $A=\frac{1}{x-y},\ B=\frac{1}{x+y}$

Similarly $\frac{40}{x-y}+\frac{48}{x+y}=9$

or $\quad 40A + 48B = 9$ ...(ii)

Solving (i) and (ii), we get $A=\frac{1}{8}, B=\frac{1}{12}$

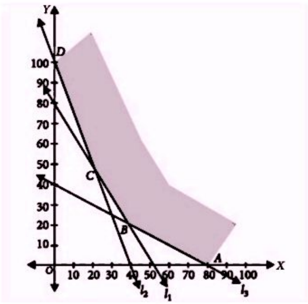
$\therefore \quad x - y = 8$ and $x + y = 12$ ...(*)

Solving the equations (*), we get

$x = 10$ and $y = 2$

$\therefore$ Speed of boat in still water = 10 km/h

28. (a) : Let $l_1 : 3x + 2y = 160$, $l_2 : 5x + 2y = 200$, $l_3 : x + 2y = 80$, $l_4 : x = 0$, $l_5 : y = 0$



For B : Solving l_1 and l_3, we get $B(40, 20)$

For C : Solving l_1 and l_2, we get $C(20, 50)$

Shaded region is the feasible region,

where $A(80, 0)$, $B(40, 20)$, $C(20, 50)$, $D(0, 100)$

Now minimize $Z = 4x + 3y$

Z at $A(80, 0) = 4 \times 80 + 3 \times 0 = 320$

Z at $B(40, 20) = 40 \times 4 + 20 \times 3 = 220$

Z at $C(20, 50) = 20 \times 4 + 50 \times 3 = 230$

Z at $D(0, 100) = 300$

Minimum value of Z is 220.

29. (b) : A is a symmetric matrix

$\therefore \quad A^T = A$ ...(i)

A is also a skew-symmetric matrix

$\therefore \quad A^T = -A$ ...(ii)

From (i) and (ii), we get $A = -A \Rightarrow A = O$

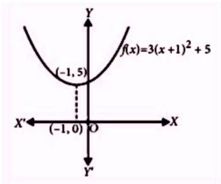
Hence, A is zero matrix.

30. (d) : I. In a LPP, objective function is always linear.

II. The feasible region for a LPP is always a convex polygon.

III. A feasible region of a system of linear inequalities is said to be bounded, if it can be enclosed within a circle.

31. (b) :



I. We have, $f(x) = 3x^2 + 6x + 8$

$\Rightarrow \quad f(x) = 3(x^2 + 2x + 1) + 5 = 3(x + 1)^2 + 5$

Now, $3(x + 1)^2 \geq 0$ for all $x \in R$

$\Rightarrow \quad 3(x + 1)^2 + 5 \geq 5$ for all $x \in R$

$\Rightarrow \quad f(x) \geq f(-1)$ for all $x \in R$.

Thus, 5 is the minimum value of $f(x)$ which it attains at $x = -1$.

Since, $f(x)$ can be made as large as we please. Therefore, the maximum value does not exist which can be observed from above figure.

II. Let $y = f(x) = (x - 1)^3(x + 1)^2$

$$\Rightarrow \quad \frac{dy}{dx}=3(x-1)^2(x+1)^2+2(x+1)(x-1)^3$$

$$= (x - 1)^2(x + 1)\{3(x + 1) + 2(x - 1)\}$$

$$= (x - 1)^2(x + 1)(5x + 1)$$

For local maximum or local minimum, we have

$$\frac{dy}{dx}=0 \Rightarrow (x-1)^2(x+1)(5x+1)=0$$

$$\Rightarrow \quad x=1 \text{ or } x=-1 \text{ or } x=-\frac{1}{5}$$

III. Given, $f(x)=x+\frac{4}{x}$

$f'(x)=1-\frac{4}{x^2} \quad \therefore f'(x)=0 \Rightarrow x=\pm 2$

and $f''(x)=\frac{8}{x^3}>0$ which is > 0 for $x = 2$ and < 0 for $x = -2$

$\therefore$ $f(x)$ has local minima at $x = 2$ and local maxima at $x = -2$.

IV. $f'(x) = 2x(x + 1) + x^2 = 2x^2 + 2x + x^2 = 3x^2 + 2x = 0$

$\Rightarrow$ $x(3x + 2) = 0 \Rightarrow x = 0$ or $x = -2/3$

32. (a) : A's share : B's share : C's share

$$=60\times 4+\frac{60}{2}\times 8:50\times 8+\frac{50}{2}\times 4:90\times 6$$

$= 480 : 500 : 540 = 48 : 50 : 54 = 24 : 25 : 27$

$\therefore$ The profit will be distributed in the ratio 24 : 25 : 27 among A, B and C respectively.

33. (b) : $y = \sin x + e^x$

$$\Rightarrow \frac{dy}{dx}=\cos x+e^x \Rightarrow \frac{dx}{dy}=\frac{1}{\cos x+e^x}$$

$$\therefore \frac{d^2x}{dy^2}=-\frac{1}{(\cos x+e^x)^2}[-\sin x+e^x]\frac{dx}{dy}$$

$$=-\frac{(e^x-\sin x)}{(\cos x+e^x)^2}\times\frac{1}{\cos x+e^x}=\frac{\sin x-e^x}{(\cos x+e^x)^3}$$

34. (d) : $P(\text{King}) = \frac{4}{52}$ and $P(\text{not a king}) = \frac{48}{52}$

$P(2 \text{ Kings}) = P(X = 2) = \frac{4}{52}\times\frac{3}{51}=\frac{1}{221}$

$P(\text{Exactly 1 king}) = P(X = 1) = \frac{4}{52}\times\frac{48}{51}+\frac{48}{52}\times\frac{4}{51}=\frac{32}{221}$

$P(\text{No king}) = P(X = 0) = \frac{48}{52}\times\frac{47}{51}=\frac{188}{221}$

The probability distribution of X is as follows:

X	0	1	2
$P(X)$	$\frac{188}{221}$	$\frac{32}{221}$	$\frac{1}{221}$

$$\therefore E(X)=\Sigma x_i p_i=0\times\frac{188}{221}+1\times\frac{32}{221}+2\times\frac{1}{221}=\frac{34}{221}=\frac{2}{13}$$

35. (c) : $(AB + BA)^T = (AB)^T + (BA)^T$

$= B^TA^T + A^TB^T = BA + AB = AB + BA$ ($\because A^T = A$ and $B^T = B$)

Hence, $AB + BA$ is a symmetric matrix.

36. (d) : Time taken by taps A and B to fill the tank is 12 hours and 15 hours respectively and time taken by tap C to empty the tank is 8 hours.

Thus, tap A and B fill up $\frac{1}{12}$ th and $\frac{1}{15}$ th part of the tank in 1 hour and C empties $\frac{1}{8}$ th part in 1 hour.

The part of tank filled in 1 hour by A, B and C *i.e.* work done by taps A, B and C in 1 hour is given by

$$=\frac{1}{12}+\frac{1}{15}-\frac{1}{8}=\frac{10+8-15}{120}=\frac{3}{120}=\frac{1}{40}$$

$\therefore$ The tank will be filled completely in 40 hours when taps A, B and C working simultaneously.

37. (b) : The given annuity is a perpetuity of first type in which R = ₹ 10000 and $r=\frac{8}{4}\%=2\%$ per quarter

So, $i=\frac{2}{100}=0.02$

Present value, $P=\frac{R}{i}=\frac{10000}{0.02}=5{,}00{,}000$

Hence, the present value is ₹ 5,00,000

38. (a) : Given $f(x) = 3x^4 - 8x^3 + 6x^2$.

It being a polynomial function is differentiable for all $x \in R$.

Differentiate it w.r.t. x, we get

$f'(x) = 3\cdot 4x^3 - 8\cdot 3x^2 + 6\cdot 2x = 12(x^3 - 2x^2 + x)$ and

$f''(x) = 12(3x^2 - 4x + 1)$.

EXAM ALERT 2024

Exam	Date
JEE Main Session 2	Between 4th April and 15th April
KARNATAKA CET MATHS/BIOLOGY	18th April
KARNATAKA CET PHYSICS/CHEMISTRY	19th April
WB JEE	28th April
COMEDK (Engg.)	12th May
CUET	Between 15th May and 31st May
JEE Advanced	26th May
BITSAT Session 1	Between 19th May to 24th May
BITSAT Session 2	Between 22nd June to 26th June

For stationary points $f'(x) = 0$

$\Rightarrow \quad 12(x^3 - 2x^2 + x) \Rightarrow x(x^2 - 2x + 1) = 0$

$\Rightarrow \quad x(x-1)^2 = 0$

$\Rightarrow \quad x = 0, 1.$

39. (a) : We have, $P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) = 1$

And, $0 \times P(X = 0) + 1 \times P(X = 1) + 2 \times P(X = 2) + 3 \times P(X = 3) = 1.3$

$\Rightarrow \quad P(X = 0) + 3P(X = 1) = 0.7$ and $7P(X = 1) = 0.7$

$\Rightarrow \quad P(X = 0) + 3P(X = 1) = 0.7$ and $P(X = 1) = 0.1$

$\Rightarrow \quad P(X = 0) = 0.4$

Variance $\text{Var}(X) = \Sigma X^2 P(X) - (\text{Mean})^2$

$= 0 \times P(X = 0) + 1 \times P(X = 1) + 4 \times P(X = 2) + 9 \times P(X = 3) - 1.69 = 1 \times 0.1 + 4 \times 0.3 + 9 \times 0.2 - 1.69$

$= 3.1 - 1.69 = 1.41$

Standard deviation, $\text{S.D}(X) = \sqrt{\text{Var } X} = \sqrt{1.41} = 1.19$

40. (a) : We have $R(x) = 100x - x^2 - x^3$

The marginal revenue function $= MR = \frac{d}{dx}(R(x))$

$= \frac{d}{dx}(100x - x^2 - x^3) = 100 - 2x - 3x^2.$

Marginal revenue at $x = 2$

$= 100 - 2(2) - 3(2)^2 = 100 - 4 - 12 = ₹84$

41. (a) : We know that

$a \cdot b \pmod n \equiv a \pmod n \cdot b \pmod n$

So, $(186 \times 93) \bmod 7 \equiv 186 \pmod 7 \cdot 93 \pmod 7$

$\equiv 4 \pmod 7 \cdot 2 \pmod 7 \equiv 4 \cdot 2 \pmod 7 \equiv 8 \pmod 7 \equiv 1 \pmod 7$

$\therefore \quad (186 \times 93) \pmod 7 \equiv 1.$

42. (d): Distance of race = 200 m, A beats B by 31 m and C by 18 m

$\therefore \quad A : B = 200 : (200 - 31) = 200 : 169$

and $A : C = 200 : (200 - 18) = 200 : 182$

$\therefore \quad C : B = \frac{C}{B} = \frac{C}{A} \times \frac{A}{B} = \frac{182}{200} \times \frac{200}{169} = 182 : 169$

When C runs 182 m, B runs 169 m

When C runs 420 m, then B will run

$= \frac{169}{182} \times 420 = 390 \text{ m}$

$\therefore \quad C$ will defeat to B by $(420 - 390) = 30$ m.

43. (a) : Let the investments of Rajesh = ₹ $12x$

and the investments of Ramesh = ₹ $11x$

Let the Ramesh invested the money for t months, then

Rajesh investment : Ramesh investment

$= (12x \times 11) : (11x \times t) = 132x : 11xt$

$\therefore \quad \frac{132x}{11xt} = \frac{3}{1} \Rightarrow \frac{12}{t} = \frac{3}{1} \Rightarrow t = 4$ months

44. (c) : $(486 + 729) \pmod{12} \equiv 486 \pmod{12} + 729 \pmod{12}$

$\equiv 6 \pmod{12} + 9 \pmod{12}$

$\equiv (6 + 9) \pmod{12} \equiv 15 \pmod{12}$

$\equiv 3 \pmod{12}$

$\therefore \quad (486 + 729) \pmod{12} \equiv 3$

45. (c) : Given, the speed of boat in still water $(x) = 12$ km/h and time taken in upstream = 2 × time taken in downstream

Let the speed of stram be y km/h.

$\therefore \quad \frac{\text{time taken in upstream}}{\text{time taken in downstream}} = \frac{2}{1}$

We know that,

$\frac{\text{time taken in upstream}}{\text{time taken in downstream}} = \frac{x+y}{x-y} \Rightarrow \frac{2}{1} = \frac{12+y}{12-y}$

$\Rightarrow \quad 12 + y = 24 - 2y \Rightarrow 3y = 12 \Rightarrow y = 4$

Hence, the speed of stream = 4 km/h

46. (b): Given original cost of machine = ₹ 50000

Scrap value of machine = ₹10000

Useful life = 4 years

$\therefore$ Annual depreciation

$= \frac{50000 - 10000}{4} = \frac{40000}{4} = 10000$

Hence, annual depreciation is ₹ 10000.

47. (a) : Let the number of machine A be x and number of machine B be y. Let Z be the daily output.

Now given information can be summarized as :

	A (x)	B (y)	Maximum available capacity
Area (m^2)	1000	1200	7600
Man power	12	8	72
Output	50	40	

According to question, x and y must satisfy the following conditions :

(Area) $\quad 1000x + 1200y \le 7600$

$\Rightarrow \quad 5x + 6y \le 38$

(Man power) $12x + 8y \le 72$

$\Rightarrow \quad 3x + 2y \le 18, x \ge 0, y \ge 0$

Mathematical formulation of the LPP is

Maximize $Z = 50x + 40y$

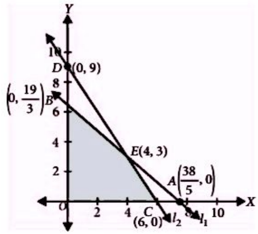
Subject to constraints :

$5x + 6y \le 38, 3x + 2y \le 18, x \ge 0, y \ge 0$

Now, we draw the lines

$l_1 : 5x + 6y = 38, l_2 : 3x + 2y = 18, l_3 : x = 0$ and $l_4 : y = 0$

Lines l_1 and l_2 meet at $E(4, 3)$.



The shaded region *OCEB* is the feasible region, which is bounded.

Vertices of the feasible region are

$O(0, 0)$, $C(6, 0)$, $E(4, 3)$ and $B\left(0, \frac{19}{3}\right)$

Maximize $Z = 50x + 40y$

At O, $Z = 50 \times 0 + 40 \times 0 = 0$

At C, $Z = 50 \times 6 + 40 \times 0 = 300$

At E, $Z = 50 \times 4 + 40 \times 3 = 320$

At B, $Z = 50 \times 0 + 40 \times \frac{19}{3} = 253.33$

Clearly, the maximum output = 320 is at $E(4, 3)$, *i.e.*, when 4 machines *A* and 3 machines *B* are purchased.

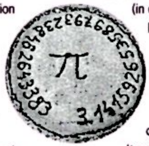
48. (a)

49. (a) : Both statements are true.

50. (b)

Pi Day : Understanding the World's Most Famous Mathematical Constant

MARCH 14, OR 3/14 as per the American convention, is celebrated as Pi Day worldwide as an ode to the most well-known approximation (3.14) of the mathematical constant Pi.



The tradition was started by physicist Larry Shaw of the Exploratorium museum in San Francisco in 1988, and has since seen global popularity.

On the day, mathematicians try to raise awareness on their subject among lay persons, through lectures, museum exhibitions and pie-eating competitions.

In 2019, UNESCO's 40th General Conference designated Pi Day as the International Day of Mathematics.

What is Pi?

Pi, often represented by the Greek letter pi, is the most famous of all mathematical constants. It represents the ratio of a circle's circumference (boundary) to its diameter (a straight line between two points on the circle's boundary, passing through its centre). Regardless of the circle's size, this ratio always remains constant.

Pi is an irrational number — it is a decimal with no end and no repeating pattern — which is most often approximated to the 3.14, or the fraction 22/7.

How is Pi calculated?

The importance of Pi has been recognized for at least 4,000 years. Academic Petr Beckman in his classic *A History of Pi* (1970) wrote that "by 2,000 BC, men had grasped the significance of the constant that is today denoted by pi, and that they had found a rough approximation of its value."

Both ancient Babylonians and ancient Egyptians came up with their own measurements, probably by drawing a circle of some diameter, and then measuring its circumference using a rope of said diameter in length. Babylonians settled at 25/8 (3.125) as the value of Pi, while ancient Egyptians settled at (16/9)2 (approximately 3.16).

It was Greek polymath Archimedes (circa 287-212 BCE) who came up with the method to calculate Pi that remained in use till the 17th century. He realised that the perimeter of a regular polygon of 'n' sides inscribed in a circle is smaller than the circumference of the circle, whereas the perimeter of a similar polygon circumscribed about the circle is greater than its circumference. This can be used this to calculate the limits within which the value of Pi must lie.

Now, as one keeps adding more and more sides to this polygon, it gets closer and closer to the shape of a circle. Having reached 96-sided polygons, Archimedes proved that 223/71 < Pi < 22/7 (in decimal notation, this is 3.14084 < p < 3.142858).

Following Archimedes, this method was used by mathematicians constantly increasing the number of sides of the polygon to calculate Pi to ever greater decimal places.

The problem with this method, however, is that it is extremely labour intensive. For instance, it took Dutch mathematician Ludolph van Ceulen (1540-1610) a staggering three decades to calculate Pi to 35 decimal points.

It would be Isaac Newton (1643-1727) who significantly simplified the process of calculating Pi. In 1666, he calculated Pi up to 16 decimal places using calculus, which he discovered along with mathematician Gottfried Wilhelm Leibniz (1646-1713). What had taken mathematicians years to calculate now could be done in a matter of days.

But why does all this matter?

Circles are everywhere in the world. So are three-dimensional shapes like cylinders, spheres, and cones, all of which carry the proportion of Pi. Knowing Pi's value, thus, has some crucial practical benefits in the fields of architecture, design, and engineering. From constructing water storage tanks to fashioning hi-tech equipment for satellites, the value of Pi is indispensable in all sorts of areas.

Moreover, Pi seems to be woven into descriptions of the deepest workings of the universe — from calculating the vastness of space or understanding the spiral of DNA.

A quiz on the mathematical constant Pi.

Question 1 : Let's start with a simple one. Why is March 14 observed as Pi Day?

Question 2 : Who founded this day and in which year?

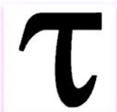
Question 3 : In 2019, UNESCO designated Pi Day to be celebrated as which day?

Question 4 : People also celebrate Pi on other dates in the year. Which are they?

Question 5 : In the year 2015, March 14 was celebrated as Super Pi Day. Why?

Question 6 : Name two common ways of celebrating Pi Day.

Question 7 : Which famous scientist's birthday falls on Pi Day?



Visual question :

What is the symbol present in the image? How is it related to Pi? There is a specific time associated with the symbol. What is it?

Courtesy : The Hindu

beat the TIME TRAP

Duration : 30 minutes

SECTION-I

Single Option Correct Type

1. The domain of the function

$f(x) = \sin^{-1}\left(\frac{|x|+5}{x^2+1}\right)$ is $(-\infty, -a] \cup [a, \infty)$.

Then a is equal to

(a) $\frac{\sqrt{17}}{2}$ (b) $\frac{\sqrt{17}-1}{2}$

(c) $\frac{1+\sqrt{17}}{2}$ (d) $\frac{\sqrt{17}}{2}+1$

2. If $3x + 2y = 1$ be tangent to $y = f(x)$ at $x = 1/2$ and

$p = \lim_{x\to 0} \frac{x(x-1)}{f\left(\frac{e^{2x}}{2}\right) - f\left(\frac{e^{-2x}}{2}\right)}$, then $\sum_{r=1}^{\infty} p^r$ equals

(a) $\frac{1}{3}$ (b) $\frac{1}{6}$

(c) $\frac{1}{2}$ (d) can't be found

3. The complex number $z = \frac{5}{(1-i)(2-i)(3-i)}$ is

(a) zero

(b) purely real number

(c) purely imaginary number

(d) None of these

4. If $a_1, a_2, ..., a_{50}$ are in G.P. where a and r are the first term and common ratio respectively, then the value of $\frac{a_1 - a_3 + a_5 - ... + a_{49}}{a_2 - a_4 + a_6 - ... + a_{50}}$ is equal to

(a) 0 (b) 1 (c) $\frac{1}{r}$ (d) r

5. If $\int f(x)dx = \psi(x)$, then $\int x^5 f(x^3)\,dx$ is equal to

(a) $\frac{1}{3}x^3\psi(x^3) - 3\int x^3\psi(x^3)dx + C$

(b) $\frac{1}{3}x^3\psi(x^3) - \int x^2\psi(x^3)dx + C$

(c) $\frac{1}{3}\left[x^3\psi(x^3) - \int x^3\psi(x^3)dx\right] + C$

(d) $\frac{1}{3}\left[x^3\psi(x^3) - \int x^2\psi(x^3)dx\right] + C$

6. The equation of the circle passing through the focii of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ and having centre at (0, 3) is

(a) $x^2 + y^2 - 6y + 7 = 0$ (b) $x^2 + y^2 - 6y - 5 = 0$

(c) $x^2 + y^2 - 6y + 5 = 0$ (d) $x^2 + y^2 - 6y - 7 = 0$

7. Water is dropped at the rate of 2 m^3/s into a cone of semi vertical angle 45°. The rate at which periphery of water surface changes when the height of the water in the cone is 2 m, is

(a) 3 m/s (b) 1 m/s (c) 2 m/s (d) 4 m/s

8. If $f(x) = \begin{cases} \frac{\sin(\cos x) - \cos x}{(\pi - 2x)^3}, & x \neq \pi/2 \\ k, & x = \pi/2 \end{cases}$

is continuous at $x = \frac{\pi}{2}$, then $k =$

(a) 0 (b) $-\frac{1}{6}$ (c) $-\frac{1}{24}$ (d) $-\frac{1}{48}$

9. Find the area of the region bounded by two parabolas $y^2 = 8x$ and $x^2 = 8y$.

(a) $\frac{14}{3}$ sq. units (b) $\frac{15}{14}$ sq. units

(c) $\frac{74}{3}$ sq. units (d) $\frac{64}{3}$ sq. units

10. The maximum value of

$(5 \sin x - 12 \cos x)(5 \cos x + 12 \sin x)$ is

(a) $\frac{13}{4}$ (b) $\frac{169}{4}$ (c) $\frac{13}{2}$ (d) $\frac{169}{2}$

SECTION-II

Numerical Answer Type

11. The papers of 4 students can be checked by any one of the 7 teachers. If the probability that all the 4 papers are checked by exactly 2 teachers is A, then the value of $49A$ must be______.

12. If $A=\begin{bmatrix}1 & -1 & 1\\ 0 & 2 & -3\\ 2 & 1 & 0\end{bmatrix}$, B = (adj A) and $C = 5A$, then the value of $\frac{|\text{adj } B|}{|C|}$ is ______.

13. The distance of the point (3, 0, 5) from the line $x - 2y + 2z - 4 = 0 = x + 3z - 11$ is _______.

14. The letters of word ZENITH are written in all possible ways. If all these words are written out as in a dictionary, then the rank of the word ZENITH is ______.

15. The coefficient of x^{2009} in the expansion of $(1 - x)^{2008}(1 + x + x^2)^{2007}$ is ______.

SOLUTIONS

1. (c) : Here, $f(x)=\sin^{-1}\left(\frac{|x|+5}{x^2+1}\right)$

$\Rightarrow \left|\frac{|x|+5}{x^2+1}\right| \le 1 \Rightarrow |x| + 5 \le x^2 + 1 \Rightarrow x^2 - 4 - |x| \ge 0$

$\Rightarrow \left(|x|-\frac{1}{2}\right)^2-\frac{17}{4}\ge 0 \Rightarrow \left(|x|-\frac{1}{2}\right)^2-\left(\frac{\sqrt{17}}{2}\right)^2\ge 0$

$\Rightarrow \left(|x|-\frac{1}{2}+\frac{\sqrt{17}}{2}\right)\left(|x|-\frac{1}{2}-\frac{\sqrt{17}}{2}\right)\ge 0$

$\Rightarrow \left(|x|+\frac{\sqrt{17}-1}{2}\right)\left(|x|-\left(\frac{\sqrt{17}+1}{2}\right)\right)\ge 0$

$$\underset{-\left(\frac{\sqrt{17}-1}{2}\right)}{+\quad|}\quad - \quad\underset{\frac{\sqrt{17}+1}{2}}{|\quad +}$$

But $|x|$ can't be negative, therefore $|x|\ge\frac{\sqrt{17}+1}{2}$

$\Rightarrow x\in\left(-\infty,-\left(\frac{\sqrt{17}+1}{2}\right)\right]\cup\left[\frac{\sqrt{17}+1}{2},\infty\right)$

Thus $a=\frac{\sqrt{17}+1}{2}$

[$\because$ Domain of $f(x)$ is $(-\infty, -a] \cup [a, \infty)$]

2. (c) : $\because$ Slope of $3x + 2y = 1$ is $-\frac{3}{2}$

$\therefore\quad f'\left(\frac{1}{2}\right)=-\frac{3}{2}$ ...(i)

Now, $p=\lim\limits_{x\to 0}\frac{x(x-1)}{f\left(\frac{e^{2x}}{2}\right)-f\left(\frac{e^{-2x}}{2}\right)}$ $\left[\frac{0}{0}\text{ form}\right]$

$=\lim\limits_{x\to 0}\frac{2x-1}{f'\left(\frac{e^{2x}}{2}\right)\cdot\frac{1}{2}\cdot e^{2x}\cdot 2-f'\left(\frac{e^{-2x}}{2}\right)\cdot\frac{1}{2}\cdot e^{-2x}(-2)}$

$=\lim\limits_{x\to 0}\frac{2x-1}{f'\left(\frac{e^{2x}}{2}\right)e^{2x}+f'\left(\frac{e^{-2x}}{2}\right)e^{-2x}}$

$=\frac{-1}{f'\left(\frac{1}{2}\right)+f'\left(\frac{1}{2}\right)}=\frac{-1}{2\left(-\frac{3}{2}\right)}$ [From (i)]

$=\frac{1}{3}$

$\therefore\quad \sum_{r=1}^{\infty}p^r=\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+\ldots\text{ to }\infty=\frac{1/3}{1-1/3}=\frac{1}{2}$

3. (c) : $z=\frac{5}{(1-i)(2-i)(3-i)}\times\frac{(1+i)(2+i)(3+i)}{(1+i)(2+i)(3+i)}$

$=\frac{5\times(2+i^2+3i)(3+i)}{2\times5\times10}=\frac{5(10i)}{2\times5\times10}=\frac{i}{2}$

4. (c) : We have $a_1, a_2, a_3, \ldots a_{50}$ are in G.P.

$\frac{a_1-a_3+a_5-\ldots+a_{49}}{a_2-a_4+a_6-\ldots+a_{50}}=\frac{a-ar^2+ar^4-\ldots.+ar^{48}}{ar-ar^3+ar^5-\ldots.+ar^{49}}=\frac{1}{r}$

5. (b): Let $x^3 = u$, then $3x^2dx = du$

Also, $\int f(x)dx=\psi(x)$

Now, $\int x^5 f(x^3)dx=\frac{1}{3}\int u\, f(u)du$

$=\frac{1}{3}\left[u\int f(u)du-\int(\int f(u)du)du\right]$

$=\frac{1}{3}x^3\psi(x^3)-\int x^2\psi(x^3)dx+C$

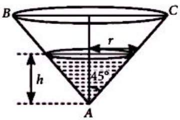
6. (d): Foci are given by $(\pm\, ae, 0)$

As $a^2e^2 = a^2 - b^2 = 7$, we have equation of circle as

$(x - 0)^2 + (y - 3)^2 = (\sqrt{7}-0)^2+(0-3)^2$

$\therefore\quad x^2 + y^2 - 6y - 7 = 0$

7. (b): We have,



$\frac{dV}{dt}=2$ or $\frac{d}{dt}\left(\frac{1}{3}\pi r^3\right)=2$

[$\because\; r = h$ as $\theta = 45°$]

$\Rightarrow \pi r^2 \dfrac{dr}{dt} = 2 \Rightarrow \dfrac{dr}{dt} = \dfrac{2}{\pi r^2}$ ...(i)

Now, perimeter = $2\pi r = p$ (say)

$\therefore \dfrac{d}{dt}(2\pi r) = 2\pi \dfrac{dr}{dt} = 2\pi\left(\dfrac{2}{\pi r^2}\right) = \dfrac{4}{r^2}$ ...(ii) [Using (i)]

When $h = 2$ m then $r = 2$ m

Hence, $\dfrac{dp}{dt} = \dfrac{4}{4} = 1$ m/s

8. (d): It is given that $f(x)$ is continuous at $x = \dfrac{\pi}{2}$

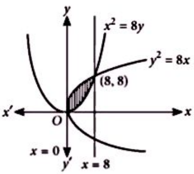
$$\therefore k = \lim_{x\to\pi/2} f(x) \Rightarrow k = \lim_{x\to\pi/2} \frac{\sin(\cos x) - \cos x}{(\pi - 2x)^3}$$

$$\Rightarrow k = \lim_{x\to\pi/2} \frac{\sin(\cos x) - \cos x}{\cos^3 x} \times \frac{\sin^3(\pi/2 - x)}{8(\pi/2 - x)^3}$$

$$\Rightarrow k = -\frac{1}{6} \times \frac{1}{8} = -\frac{1}{48} \quad \left[\because \lim_{x\to 0} \frac{\sin x - x}{x^3} = -\frac{1}{6}\right]$$

9. (d): Given equations of the curves are $y^2 = 8x$ ...(i)
and $x^2 = 8y$ ...(ii)

Equation (i) represents the parabola whose vertex is (0, 0) and axis is x-axis.
Equation (ii) represents the parabola whose vertex is (0, 0) and axis is y-axis.



Solving equations (i) and (ii), we get $\left(\dfrac{x^2}{8}\right)^2 = 8x$

$\Rightarrow x^4 = 8^3 x \Rightarrow x^4 - 8^3 x = 0 \Rightarrow x(x^3 - 8^3) = 0$

$\Rightarrow x = 0$ or $x^3 = 8^3 \Rightarrow x = 0$ or $x = 8$

$$\therefore \text{ Required area } = \int_0^8 \sqrt{8}\sqrt{x}\,dx - \int_0^8 \frac{x^2}{8}\,dx$$

$$= \sqrt{8}\left[\frac{2x^{3/2}}{3}\right]_0^8 - \frac{1}{8}\left[\frac{x^3}{3}\right]_0^8 = \frac{2\sqrt{8}}{3}[8^{3/2} - 0] - \frac{1}{24}[8^3 - 0]$$

$$= \frac{2}{3}(64) - \frac{1}{24}(512) = \frac{128}{3} - \frac{64}{3} = \frac{64}{3} \text{ sq. units}$$

10. (d): We have, $(5\sin x - 12\cos x)(5\cos x + 12\sin x)$

Let $5 = r\cos\alpha$ and $12 = r\sin\alpha$

$\therefore r^2 = 5^2 + 12^2 = 13^2 \Rightarrow r = 13$

$\therefore$ The given expression is

$13(\sin x\cos\alpha - \cos x\sin\alpha)\cdot 13(\cos x\cos\alpha + \sin x\sin\alpha)$

$= 13\sin(x - \alpha)\cdot 13\cos(x - \alpha) = \dfrac{169}{2}\sin 2(x - \alpha)$

$\therefore$ Maximum value is 169/2.

11. (6): Total number of ways in which papers of 4 students can be checked by seven teachers = 7^4

Now, choosing two teachers out of 7 is ${}^7C_2 = 21$

4 papers can be checked by exactly 2 teachers = $2^4 - 2 = 14$

The favourable number of ways = (21) (14)

Required probability = $\dfrac{(21)(14)}{7^4} = \dfrac{6}{49} = A \;\therefore\; 49A = 6$

12. (1): $\dfrac{|\text{adj } B|}{|C|} = \dfrac{|\text{adj}(\text{adj} A)|}{|5A|} = \dfrac{|A|^{(3-1)^2}}{5^3|A|} = \dfrac{|A|^3}{125}$

Now, as $|A| = 5 \quad \therefore \dfrac{|\text{adj } B|}{|C|} = 1$

13. (3): The d.r.'s of the line are given by

$l - 2m + 2n = 0,\; l + 3n = 0 \Rightarrow \dfrac{l}{6} = \dfrac{m}{1} = \dfrac{n}{-2}$

Taking $y = 0$, we get $x + 2z = 4$,

$x + 3z = 11 \Rightarrow x = -10, z = 7$

The line is $\dfrac{x+10}{6} = \dfrac{y}{1} = \dfrac{z-7}{-2}$; $\vec{b} = 6\hat{i} + \hat{j} - 2\hat{k}$

Distance of the point (3, 0, 5) from the line is

$$\frac{|(13\hat{i} - 2\hat{k}) \times (6\hat{i} + \hat{j} - 2\hat{k})|}{\sqrt{41}} = \frac{|2\hat{i} + 14\hat{j} + 13\hat{k}|}{\sqrt{41}} = \frac{\sqrt{369}}{\sqrt{41}} = 3$$

14. (616) : Total number of words = 6! = 720

Writing the letters of word ZENITH alphabetically that is EHINTZ.

Word starting with	Number of words
E	5!
H	5!
I	5!
N	5!
T	5!
ZEH	3!
ZEI	3!
ZENH	2!
ZENIH	1
Total number of words before ZENITH	**615**

Hence, there are 615 words before ZENITH, the rank of ZENITH is 616.

15. (0): $(1 - x)^{2008}(1 + x + x^2)^{2007}$

$= (1 - x)[(1 - x)(1 + x + x^2)]^{2007}$

$= (1 - x)(1 - x^3)^{2007} = (1 - x^3)^{2007} - x(1 - x^3)^{2007}$

All the terms in the expansion of $(1 - x^3)^{2007}$ are of the form x^{3r} and all the terms in the expansion of $x(1 - x^3)^{2007}$ are of the form x^{3r+1} where as x^{2009} is of the form x^{3r+2}. Thus, the desired coefficient is 0.

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- The students must have secured a minimum of 50% marks (45% marks for reserved category) in 10+2.
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- The students should have completed their International Baccalaureate Diploma, after 10 years of schooling, with not less than 50% marks in aggregate and with Mathematics as a compulsory subject of examination.

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Entrance Exams

NATA : National Aptitude Test in Architecture (NATA) is a national-level undergraduate exam which is conducted by the Council of Architecture (COA) for admission to 5-year B. Arch.

AAT : The Architecture Aptitude Test (AAT) is conducted every year for the candidates who wish to take admission to the B.Arch programme offered by the IIT Kharagpur, IIT Varanasi and IIT Roorkee.

JEE Main : JEE Main B. Arch Exam is a national-level entrance examination for architecture. Architecture is one of the two papers that are conducted under JEE Main, the other being engineering. The paper for Architecture is called JEE Main Paper 2. Students who qualify for this prestigious paper get a chance to study the Bachelor of Architecture course in top colleges like IITs, NITs, IIITs, and other government colleges all over India.

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YOU ASK WE ANSWER

Do you have a question that you just can't get answered? Use the vast expertise of our MTG team to get to the bottom of the question. From the serious to the silly, the controversial to the trivial, the team will tackle the questions, easy and tough.

The best questions and their solutions will be printed in this column each month.

1. If p^{th}, q^{th} and r^{th} terms of an H.P., be respectively x, y and z, then prove that $(p - q)xy + (q - r)yz + (r - p)zx = 0$. ***(Preeti, U.P.)***

Ans. Since, x, y, z are respectively the p^{th}, q^{th} and r^{th} terms of an H.P., hence, $\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ will respectively be the p^{th}, q^{th} and r^{th} terms of an A.P.

If a and d be the first term and common difference of the corresponding A.P., then we have

$$\frac{1}{x} = a + (p-1)d \quad \text{...(i)}$$

$$\frac{1}{y} = a + (q-1)d \quad \text{...(ii)}$$

and $$\frac{1}{z} = a + (r-1)d \quad \text{...(iii)}$$

Subtracting equation (ii) from equation (i), we have

$$\frac{1}{x} - \frac{1}{y} = (p-q)d$$

i.e., $$xy(p-q) = \frac{y-x}{d} \quad \text{...(iv)}$$

Similarly, subtracting equation (iii) from equation (ii), we have, $$xz(q-r) = \frac{z-y}{d} \quad \text{...(v)}$$

and subtracting equation (i) from equation (iii), we have

$$zx(r-p) = \frac{x-z}{d} \quad \text{...(vi)}$$

Adding (iv), (v) and (vi), we get the desired result.

2. Evaluate the indefinite integral :

$$\int \frac{1}{\sin^6 x + \cos^6 x} dx$$ ***(Ritu Jain, Gujarat)***

Ans. Let $$I = \int \frac{dx}{\sin^6 x + \cos^6 x} = \int \frac{\sec^6 x}{1 + \tan^6 x} dx$$

$$= \int \frac{(1 + \tan^2 x)^2}{1 + \tan^6 x} \sec^2 x \, dx$$

Putting $\tan x = t$ and $\sec^2 x \, dx = dt$, we have

$$I = \int \frac{(1+t^2)^2}{1+t^6} dt = \int \frac{1+t^2}{t^4 - t^2 + 1} dt$$

$$= \int \frac{1 + \frac{1}{t^2}}{t^2 + \frac{1}{t^2} - 1} dt = \int \frac{d\left(t - \frac{1}{t}\right)}{\left(t - \frac{1}{t}\right)^2 + 1}$$

$$= \tan^{-1}\left(t - \frac{1}{t}\right) + C = \tan^{-1}(\tan x - \cot x) + C.$$

3. If $f(x) = x^3 + (a - 3)x^2 + \frac{4}{3}(a - 4)x + 5$ has a maxima at some $x \in R^-$ and minima at some $x \in R^+$, then find the possible values of a.

(Neetu Singh, Delhi)

Ans. We have, $f(x) = x^3 + (a - 3)x^2 + \frac{4}{3}(a - 4)x + 5$

and $f'(x) = 3x^2 + 2(a - 3)x + \frac{4}{3}(a - 4)$

According to the given condition, $f'(x)$ must vanish at two distinct points say α, β such that $\alpha < 0$ and $\beta > 0$. Then, we have

$f'(x) = 3(x - \alpha)(x - \beta)$

and $f''(x) = 3(2x - \alpha - \beta)$

According to the given condition, α is a point of maxima

$\therefore \quad f''(\alpha) < 0 \Rightarrow 3(\alpha - \beta) < 0$ which is true $\forall\, a$

Also, β is a point of minima

$\therefore \quad f''(\beta) > 0 \Rightarrow 3(\beta - \alpha) > 0$ which is true $\forall\, a$

Now, for $\alpha < 0$ and $\beta > 0$, we have

$\alpha\beta < 0 \quad$ *i.e.*, $\frac{4}{9}(a - 4) < 0 \quad \Rightarrow a < 4$ or $a \in (-\infty, 4)$.

Recipe for Success

"Success is not Final;
Failure is not Fatal:
It is the courage to continue
that counts."

–Winston Churchill

ARE YOU READY FOR JEE 2024?

Top Weightage Topics for JEE

In this article we will discuss about some of the most important topics of mathematics in respect to JEE Main and Advanced. It is based on the analysis of previous years' questions of JEE Main and Advanced.

Sets

A well defined collection of objects is called a set.

Operations on Sets

Union: It consists of all the elements of A and B with the common element taken only once and is denoted by $A \cup B = \{x : x \in A$ or $x \in B\}$

Intersection: It consists of all those elements which belong to both A and B and is denoted by $A \cap B = \{x : x \in A$ and $x \in B\}$

Complement: It is the set of all elements of universal set U which are not the elements of A *i.e.*, $A' = U - A$

Complement law
- $A \cup A' = U$
- $A \cap A' = \phi$

De Morgan's Law
- $(A \cup B)' = A' \cap B'$
- $(A \cap B)' = A' \cup B'$

Difference: It is the set of elements which belong to A but not to B and is denoted by $A - B = \{x : x \in A, x \notin B\}$

Application of Sets

If A, B and C are finite sets and U be the finite universal set, then
- $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
- $n(A \cup B) = n(A) + n(B)$, if A, B are disjoint.
- $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$
- $n(A' \cup B') = n(A \cap B)' = n(U) - n(A \cap B)$
- $n(A' \cap B') = n(A \cup B)' = n(U) - n(A \cup B)$

Relations

R is a relation from A to B (where $A, B \neq \phi$) if $R \subseteq A \times B = \{(a, b) : a \in A, b \in B\}$.

Types of Relations

Empty (Void) Relation
$R = \phi \Rightarrow R$ is void.

Reflexive Relation
R is reflexive in A if $(a, a) \in R \; \forall \; a \in A$

Transitive Relation
R is transitive in A if $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R \; \forall \; a, b, c \in A$

Identity Relation
$R = \{(a, a) \; \forall \; a \in A\}$ is an identity relation in A.

Universal Relation
$R = A \times B \Rightarrow R$ is universal.

Symmetric Relation
R is symmetric in A if $(a, b) \in R \Rightarrow (b, a) \in R \; \forall \; a, b \in A$.

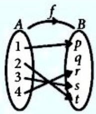
Equivalence Relation
If R is reflexive, symmetric and transitive, then R is an equivalence relation.

Inverse Relation
R^{-1} is the inverse relation of R if $(a, b) \in R$, then $(b, a) \in R^{-1}$.
Domain (R) = Range (R^{-1}) and Range (R) = Domain (R^{-1})

Functions

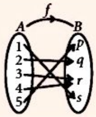
A relation $f : A \to B$, where every element of set A has only one image in set B.

Types of Functions



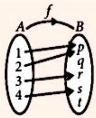
One-one (Injective) Function

A function $f : A \to B$ is one-one, if no two elements of A have same image in B.



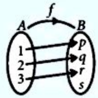
Onto (Surjective) Function

A function $f : A \to B$ is onto, if all the elements of B have atleast one pre-image in A.



Many-one Function

A function $f : A \to B$ is many-one, if two or more than two elements of A have the same images in B.



Into Function

A function $f : A \to B$ is into, if there exists atleast single element in B having no pre-image in A.

Bijective Function

A function which is both one-one & onto.

- $n(A) = n(B)$
- Range = Codomain

Number of Functions

Let $f : A \to B$, $|A| = m$, $|B| = n$.

The number of functions from A to B is n^m.

The number of one-one functions from A to B = $\begin{cases} {}^nP_m, & \text{if } n \geq m \\ 0, & \text{if } n < m \end{cases}$

The number of onto functions from A to B is = $\sum_{r=1}^{n} (-1)^{n-r}\, {}^nC_r\, r^m$, if $n \leq m$

The number of bijections from A to B is $n!$ if $m = n$.

Invertible Functions

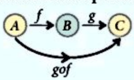
Let $f : A \to B$ be a bijection. Then the inverse function f^{-1} exists from B to A.

i.e., $f^{-1} : B \to A$

$\therefore\ f^{-1}(y) = x \Leftrightarrow y = f(x)$

Composition of Functions

Let $f : A \to B$ and $g : B \to C$ be two functions, then $gof : A \to C$ is known as composition of functions.



Properties

- $fog \neq gof$
- $fo(goh) = (fog)oh$ (Associative property)
- Composition of two bijections is also a bijection.

Matrices

A matrix is an ordered rectangular array of numbers or functions. The numbers or functions are called the elements or the entries of the matrix.

Order of a Matrix

A matrix having m rows and n columns is called a matrix of order $m \times n$.

Transpose of a Matrix

Transpose of a matrix is obtained by interchanging rows and columns of the matrix.

If $A = [a_{ij}]_{m \times n}$, then A' or $A^T = [a_{ji}]_{n \times m}$

Properties

- $(A')' = A$
- $(A \pm B)' = A' \pm B'$
- $(kA)' = kA'$
- $(AB)' = B'A'$
- $(ABC)' = C'B'A'$

Inverse of a Matrix

If A and B are two square matrices such that $AB = BA = I$, then B is the inverse matrix of A and A is the inverse of B. It is denoted by A^{-1}.

Properties

- Inverse of a square matrix is unique, if it exists.
- $(A^{-1})^{-1} = A$
- $(kA)^{-1} = A^{-1}/k$
- $(AB)^{-1} = B^{-1}A^{-1}$
- $(A^T)^{-1} = (A^{-1})^T$

Types of Matrices

- Column Matrix : $A = [a_{ij}]_{m \times 1}$
- Row Matrix : $A = [a_{ij}]_{1 \times n}$
- Square Matrix : $A = [a_{ij}]_{m \times m}$
- Diagonal Matrix : $A = [a_{ij}]_{m \times m}$, where $a_{ij} = 0 \ \forall\ i \neq j$
- Scalar Matrix : $A = [a_{ij}]_{n \times n}$, where $a_{ij} = \begin{cases} 0, \text{ if } i \neq j \\ k, \text{ if } i = j \end{cases}$ for some constant k.
- Zero Matrix : $A = [a_{ij}]_{m \times n}$, where $a_{ij} = 0 \ \forall\ i \ \&\ j$
- Identity Matrix : $A = [a_{ij}]_{n \times n}$, where $a_{ij} = \begin{cases} 1, \text{ if } i = j \\ 0, \text{ if } i \neq j \end{cases}$
- Upper Triangular Matrix : A square matrix in which all the elements below the diagonal are zero.
- Lower Triangular Matrix : A square matrix in which all the elements above the diagonal are zero.
- Involutory Matrix : A square matrix A is involuntory if $A^2 = I$
- Orthogonal Matrix : A square matrix A is orthogonal if $AA^T = A^TA = I$
- Idempotent Matrix : A square matrix A is idempotent if $A^2 = A$
- Symmetric Matrix : A square matrix A is symmetric if $A^T = A$
- Skew Symmetric Matrix : A square matrix A is skew-symmetric if $A^T = -A$

Determinants

Corresponding to every square matrix A, there exists a number called the determinant of A and denoted by $|A|$.

Let $A=\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$. Then,

$|A| = a_{11}(a_{22}a_{33} - a_{32}a_{23}) - a_{12}(a_{21}a_{33} - a_{23}a_{31}) + a_{13}(a_{21}a_{32} - a_{22}a_{31})$

Properties

- If $|A| = \begin{vmatrix} a+k_1 & b+k_2 & c+k_3 \\ d & e & f \\ g & h & k \end{vmatrix}$,

  then $|A| = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & k \end{vmatrix} + \begin{vmatrix} k_1 & k_2 & k_3 \\ d & e & f \\ g & h & k \end{vmatrix}$
- $|A'| = |A|$
- If any two rows (or columns) of a determinant are identical, then the value of determinant is zero.
- If each element of a row (or column) of a determinant is multiplied by a constant k, then its value gets multiplied by k.

Singular and Non-Singular Matrices

A square matrix A of order n is said to be

- Singular if $|A| = 0$
- Non-singular if $|A| \neq 0$

Adjoint of a Matrix

Let $B = [A_{ij}]_{n\times n}$ be the matrix of cofactors of matrix $A = [a_{ij}]_{n\times n}$. Then the transpose of B is called the adjoint of matrix A.

Properties

If A is non-singular matrix of order n, then

- $A(\text{adj } A) = (\text{adj } A)\, A = |A|\, I_n$
- $\text{adj } (AB) = (\text{adj } B)\cdot(\text{adj } A)$
- $\text{adj } (\text{adj } A) = |A|^{n-2} A$
- $\text{adj } (kA) = k^{n-1} \text{ adj } (A)$
- $|A \text{ adj } A| = |A|^n$
- $|\text{adj } A| = |A|^{n-1}$
- $|\text{adj } (\text{adj } A)| = |A|^{(n-1)^2}$
- $\text{adj}A = |A|A^{-1}$

Solution of System of Linear Equations

Let $AX = B$ be the given system of equations.

- If $|A| \neq 0$, the system is consistent and has a unique solution.
- If $|A| = 0$ and $(\text{adj}A)B \neq O$, then the system is inconsistent and hence it has no solution.
- If $|A| = 0$ and $(\text{adj } A)B = O$, then the system may be either consistent or inconsistent according as the system has either infinitely many solutions or no solution.

ALGEBRA

Complex Numbers and Quadratic Equations

Complex Number

A number of the form $z = a + ib$ (where a = Real part, b = Imaginary part and $a, b \in R$) is known as complex number.

Algebra of Complex Numbers

Let $z_1 = a_1 + ib_1$ and $z_2 = a_2 + ib_2$.

Addition : $z_1 + z_2 = (a_1 + a_2) + i(b_1 + b_2)$

Subtraction : $z_1 - z_2 = (a_1 - a_2) + i(b_1 - b_2)$

Multiplication : $z_1z_2 = (a_1a_2 - b_1b_2) + i(a_1b_2 + a_2b_1)$

Division : $\frac{z_1}{z_2} = \frac{a_1a_2 + b_1b_2}{a_2^2 + b_2^2} + i\frac{a_2b_1 - a_1b_2}{a_2^2 + b_2^2}$, where $z_2 \neq 0$

Conjugate of Complex Numbers

Conjugate of $z = a + ib$ is $\overline{z} = a - ib$

Properties

- $\overline{(\overline{z})} = z$
- $z = \overline{z} \Leftrightarrow z$ is purely real
- $z + \overline{z} = 0 \Leftrightarrow z$ is purely imaginary
- $\overline{z_1 \pm z_2} = \overline{z}_1 \pm \overline{z}_2$; $\overline{z_1z_2} = \overline{z}_1\,\overline{z}_2$
- $\overline{(z_1/z_2)} = \overline{z}_1 / \overline{z}_2 (\overline{z}_2 \neq 0)$
- $\overline{(z^n)} = (\overline{z})^n$
- $\alpha = f(z) \Rightarrow \overline{\alpha} = f(\overline{z}), \alpha \in C$

Polar Form

$z = a + ib = r(\cos\theta + i\sin\theta) = r \text{ cis } \theta$,

where r = Modulus of $z = \sqrt{a^2 + b^2}$

and θ = Argument of $z = \tan^{-1}\left(\frac{b}{a}\right)$

Cube Roots of Unity

Let $x = (1)^{1/3} \Rightarrow x^3 - 1 = 0 \Rightarrow x = 1, \frac{-1 + i\sqrt{3}}{2}, \frac{-1 - i\sqrt{3}}{2}$

or $x = 1, \omega, \omega^2$

- $\omega^3 = 1$
- $1 + \omega + \omega^2 = 0$
- $1 + \omega^n + \omega^{2n} = \begin{cases} 0, \text{ if } n \text{ is not multiple of } 3 \\ 3, \text{ if } n \text{ is multiple of } 3 \end{cases}$

De-Moivre's Theorem

$z = r(\text{cis } \theta), z^n = r^n(\text{cis } n\theta)$

Also, n^{th} roots of unity is given by $z^n = 1$ and $z = \text{cis}\left(\frac{2k\pi}{n}\right), k = 0, 1, 2, \ldots, n - 1$.

Euler's Form

$z = re^{i\theta}, \overline{z} = re^{-i\theta}$, where $-\pi < \theta \leq \pi$, θ is the principal argument.

Quadratic Equation

An equation of the form $ax^2 + bx + c$ is called a quadratic equation where $a, b, c \in R$ and $a \neq 0$. If $b^2 - 4ac < 0$, then the solution is given as

$$x = \frac{-b \pm \sqrt{4ac - b^2}\, i}{2a}$$

Nature of the Roots of Quadratic Equation

a, b, c are real and $\Delta = b^2 - 4ac$ is the discriminant of $ax^2 + bx + c = 0$

- If $\Delta > 0$, then the roots are real and unequal.
- If $\Delta = 0$, then the roots are real and equal.
- Let $\Delta < 0$, then the roots are complex conjugates.
- Let a, b, c be integers and Δ be a square of an integer. Then the roots of $ax^2 + bx + c = 0$ are rational numbers.

Common Roots of Quadratic Equations

Condition of one common root

Let $a_1x^2 + b_1x + c_1 = 0$ and $a_2x^2 + b_2x + c_2 = 0$ have a common root α.

$$\therefore \alpha = \frac{b_1c_2 - b_2c_1}{a_2c_1 - a_1c_2} = \frac{a_2c_1 - a_1c_2}{a_1b_2 - a_2b_1}, \alpha \neq 0$$

Condition for both the common roots

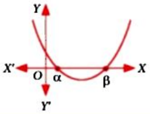
Let $a_1x^2 + b_1x + c_1 = 0$ and $a_2x^2 + b_2x + c_2 = 0$, then the required condition is

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Quadratic Expression and its Graph

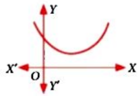
Case-I : For $a > 0$ and $D > 0$

For $\alpha < x < \beta$, $y < 0$



Case-III : For $a > 0$ and $D < 0$

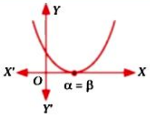
In this case, imaginary roots appears & $y > 0$ for $x \in R$.



Case-II : For $a > 0$ and $D = 0$

$y = a(x - \alpha)^2$

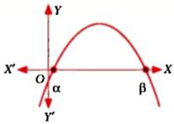
and $y \geq 0$, for $x \in R$.



Case-IV : For $a < 0$ and $D > 0$

$y > 0$, if $\alpha < x < \beta$

$y < 0$, if $x < \alpha$ or $x > \beta$



Logarithm

Logarithm is the power to which a number must be raised in order to get some other number.

Properties of Logarithm

- $\log_b 1 = 0$
- $\log_b b = 1$
- $\log_b b^x = x$
- $b^{\log_b x} = x$
- $\log_b a = \log_c a \log_b c$
- $\log_b a = \dfrac{1}{\log_a b}$
- $\log_b a = \dfrac{\log_c a}{\log_c b}$
- $\log_b (xy) = \log_b x + \log_b y$
- $\log_b (x/y) = \log_b x - \log_b y$
- $\log_b a^\alpha = \alpha \log_b a$
- $\log_{b^\beta} a = \dfrac{1}{\beta}\log_b a$
- $\log_{b^\beta} a^\alpha = \dfrac{\alpha}{\beta}\log_b a$
- $a^{\log_c b} = b^{\log_c a}$ $(a, b, c > 0$ and $c \neq 1)$
- $a^{\sqrt{\log_a b}} = b^{\sqrt{\log_b a}}$
- $\log_2 \log_2 \sqrt{\sqrt{\sqrt{\sqrt{\ldots\sqrt{2}}}}} = -n$, where n is the number of square roots.

Permutations and Combinations

Factorial Numbers

Product of first n natural numbers is denoted by $n!$ *i.e.*, $n! = n(n-1)(n-2)... 3\cdot2\cdot1$

Combinations

Selecting r objects out of n different things

$$= {}^nC_r = \frac{n!}{r!(n-r)!}, 0 \le r \le n$$

Properties

- ${}^nP_r = {}^nC_r\, r!, 0 \le r \le n$
- For $0 \le r \le n, {}^nC_r = {}^nC_{n-r}$
- For $1 \le r \le n, {}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$
- ${}^nC_a = {}^nC_b \Rightarrow a = b$ or $n = a + b$

Permutations

Arranging r objects out of n different things,

- When repetition is not allowed = ${}^nP_r = \frac{n!}{(n-r)!}$, where $0 \le r \le n$
- When repetition is allowed = n^r

Distributions into Groups

Distribution of n distinct things into r groups $G_1, G_2, ..., G_r$ containing $P_1, P_2, ..., P_r$ elements respectively,

- Groups are distinct : $\frac{n!}{P_1!\,P_2!\,...\,P_r!}r!$
- Groups are identical : $\frac{n!}{P_1!\,P_2!\,...\,P_r!}$

Binomial Theorem

Binomial Expansion

For positive integral index 'n', $n \in N$

$$(x+a)^n = {}^nC_0x^n + {}^nC_1x^{n-1}a + ... + {}^nC_na^n$$

$$= \sum_{r=0}^{n} {}^nC_r x^{n-r} a^r$$

Properties of Binomial Coefficients

- $C_0 + C_1 + C_2 ... + C_n = 2^n$
- $C_0 + C_2 + C_4 + ... = C_1 + C_3 + C_5 + ... = 2^{n-1}$
- $\sum_{r=0}^{n} r \cdot C_r = n\,2^{n-1}$
- $C_0 + \frac{C_1}{2} + \frac{C_2}{3} + \ldots + \frac{C_n}{n+1} = \frac{2^{n+1}-1}{n+1}$
- $C_0 - \frac{C_1}{2} + \frac{C_2}{3} - \frac{C_3}{4} + \ldots + \frac{(-1)^nC_n}{n+1} = \frac{1}{n+1}$

General Terms

- $T_{r+1} = {}^nC_rx^{n-r}a^r$
- T_{r+1} from the end = T_{n-r+1} from the beginning

Middle Terms

No. of terms = $n + 1$

If n is even, then $T_{\frac{n}{2}+1}$ is the middle term.

If n is odd, then $T_{\frac{n+1}{2}}, T_{\frac{n+3}{2}}$ are two middle terms.

Multinomial Theorem

$$(x_1 + x_2 + ... + x_r)^n = \sum \frac{n!}{p_1!\,p_2!\ldots p_r!} x_1^{p_1} x_2^{p_2} \ldots x_r^{p_r},$$

where $p_1 + p_2 + p_3 ... + p_r = n$

Sequences and Series

Arithmetic Progression (A.P.)

A sequence whose terms increases or decreases by a fixed number.

n^{th} term : $T_n = a + (n - 1)d$, where d (common difference) $= T_n - T_{n-1}$, a = first term

n^{th} term from end : $T'_n = l - (n - 1)d$, where l = last term

Sum of n terms : $S_n = \frac{n}{2}[2a+(n-1)d] = \frac{n}{2}[a+l]$

Geometric Progression (G.P.)

A sequence of non-zero numbers for which the ratio of a term to its just preceding term is always constant.

n^{th} term : $T_n = ar^{n-1}$, where r(common ratio) $= \frac{T_n}{T_{n-1}}$ and a = first term

n^{th} term from end : $T'_n = \frac{l}{r^{n-1}}$, l = last term

Sum of n terms :

$$S_n = \begin{cases} \frac{a(r^n-1)}{r-1}, & r>1 \\ \frac{a(1-r^n)}{1-r}, & r<1 \\ an, & r=1 \end{cases}$$

Sum of infinite terms :

$$S_\infty = \frac{a}{1-r}, |r|<1$$

Arithmetic Mean (A.M.)

- For two numbers a and b, A.M. is $\frac{a+b}{2}$.
- $A_r = a + r\left(\frac{b-a}{n+1}\right), \forall\ r = 1, 2, \ldots, n$,

where $A_1, ..., A_n$ are n arithmetic means inserted between two numbers a and b.

Geometric Mean (G.M.)

- For two numbers a and b, G.M. is $\sqrt{ab}$.
- $G_k = a\left(\frac{b}{a}\right)^{\frac{k}{n+1}}$, where $k = 1,2,3,\ldots,n$,

where $G_1, ..., G_n$ are n geometric means inserted between two numbers a and b.

Relation between Arithmetic Mean and Geometric Mean

Let a and b are two real positive numbers and A, G are arithmetic and geometric mean respectively between them.

- If $a = b$, then $A = G$
- If $a \neq b$, then $A > G$

Sum of n Terms of Special Series

- Sum of n natural numbers $= \sum n = \frac{n(n+1)}{2}$
- Sum of squares of n natural numbers $= \sum n^2 = \frac{n(n+1)(2n+1)}{6}$
- Sum of cubes of n natural numbers $= \sum n^3 = \frac{n^2(n+1)^2}{4} = \left(\sum n\right)^2$
- $\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \ldots + \frac{1}{n(n+1)} = \frac{n}{n+1}$
- $1\cdot 2 + 2\cdot 3 + \ldots + n(n+1) = \frac{n(n+1)(n+2)}{3}$

Exam on 28th April 2024

WB JEE

PRACTICE PAPER 2024

Category - I (Q.1 to 50)

(Carry 1 mark each. Only one option is correct. Negative marks : ¼)

1. Solve the differential equation : $\frac{dy}{dx} = (e^x + 1)y$

(a) $\log x = xe^x + c$ (b) $\log x = e^x + x + c$
(c) $\log|y| = xe^x + c$ (d) $\log|y| = e^x + x + c$

2. $y = e^{-x} + Ax + B$ is solution of the differential equation

(a) $\frac{d^2y}{dx^2} = e^{-x}$ (b) $\frac{d^2y}{dx^2} = e^x$
(c) $\frac{d^2y}{dx^2} = -e^{-x}$ (d) $\frac{d^2y}{dx^2} = -e^x$

3. Let the tangents drawn from the origin to the circle, $x^2 + y^2 - 8x - 4y + 16 = 0$ touch it at the points A and B. Then $(AB)^2$ is equal to

(a) $\frac{32}{5}$ (b) $\frac{56}{5}$ (c) $\frac{52}{5}$ (d) $\frac{64}{5}$

4. Let $f(x) = \begin{cases} \cos x, & \text{if } x \geq 0 \\ -\cos x, & \text{if } x < 0 \end{cases}$

Which one of the following statements is not true?
(a) $f(x)$ is continuous at $x = 1$
(b) $f(x)$ is continuous at $x = -1$
(c) $f(x)$ is continuous at $x = 2$
(d) $f(x)$ is continuous at $x = 0$

5. Between 1 and 2, how many local maxima the function $f(x) = 6x^5 - 7x^4 + 3x^3 + 2x^2 + 11x - 17$ has ?
(a) 0 (b) 1 (c) 2 (d) 3

6. If $\vec{p}$ and $\vec{q}$ are non-collinear unit vectors and $|\vec{p} + \vec{q}| = \sqrt{3}$, then $(2\vec{p} - 3\vec{q}) \cdot (3\vec{p} + \vec{q})$ is equal to

(a) 0 (b) $\frac{1}{3}$ (c) $-\frac{1}{3}$ (d) $-\frac{1}{2}$

7. The chord of the curve $y = x^2 + 2ax + b$, joining the points where $x = \alpha$ and $x = \beta$, is parallel to the tangent to the curve at abscissa $x =$

(a) $\frac{a+b}{2}$ (b) $\frac{2a+b}{3}$ (c) $\frac{2\alpha+\beta}{3}$ (d) $\frac{\alpha+\beta}{2}$

8. If $\int_0^{\pi/3} \frac{\cos x}{3 + 4\sin x} dx = k \log\left(\frac{3+2\sqrt{3}}{3}\right)$, then k is

(a) 1/2 (b) 1/3 (c) 1/4 (d) 1/8

9. If the mean of the numbers a, b, 8, 5, 10 is 6 and their variance is 6.8, then ab is equal to
(a) 6 (b) 7 (c) 12 (d) 14

10. If a, b, c are distinct and the roots of $(b - c)x^2 + (c - a)x + (a - b) = 0$ are equal, then a, b, c are in
(a) Arithmetic progression
(b) Geometric progression
(c) Harmonic progression
(d) Arithmetico-Geometric progression

11. The value of $\sum_{n=1}^{13} (i^n + i^{n+1})$, where $i = \sqrt{-1}$, equals
(a) 0 (b) i (c) $-i$ (d) $i - 1$

12. If z_1 and z_2 are two different complex numbers with $|z_2| = 1$, then $\left|\frac{1 - \bar{z}_1 z_2}{z_1 - z_2}\right|$ is equal to

(a) 0 (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) 1

13. How many four digit numbers are there with distinct digits?
(a) 5040 (b) 4536 (c) 30240 (d) 5274

14. If $1 - 2i$ is a root of $z^2 + \alpha z + \beta = 0$, where α and β are real, then $\alpha - \beta$ is
(a) 3 (b) 4 (c) −7 (d) 2

15. The number of ways in which the letters of the word ARTICLE can be rearrange so that the even places are always occupied by consonants is

(a) 576 (b) ${}^4C_3 \times (4!)$
(c) 2(4!) (d) None of these

16. If $A=\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}$, then $(aI + bA)^n$ is (where I is the identity matrix of order 2)

(a) $a^2I + a^{n-1}\, b \cdot A$ (b) $a^nI + na^nbA$
(c) $a^nI + n\, a^{n-1}b\, A$ (d) $a^nI + b^n\, A$

17. If $P(n)$: "$2^{2n} - 1$ is divisible by k for all $n \in N$" is true, then the value of 'k' is

(a) 6 (b) 3 (c) 7 (d) 2

18. Let A and B be two sets containing four and two elements respectively. Then the number of subsets of the set $A \times B$, each having at least three elements is

(a) 275 (b) 510 (c) 219 (d) 256

19. Let $X=\begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}$, then $|X^{100}|=$

(a) 1024 (b) 100 (c) 1 (d) –1

20. If $A=\begin{bmatrix}\alpha & 2\\ 2 & \alpha\end{bmatrix}$ and $|A^3| = 27$, then $\alpha =$

(a) ± 2 (b) $\pm\sqrt{5}$ (c) ± 1 (d) $\pm\sqrt{7}$

21. For an invertible matrix A if $A\,(\text{adj}\, A)=\begin{bmatrix}10 & 0\\ 0 & 10\end{bmatrix}$, then $|A|$ is

(a) 100 (b) – 100 (c) 10 (d) – 10

22. A six-faced unbiased die is thrown twice and the sum of the numbers appearing on the upper face is observed to be 7. The probability that the number 3 has appeared at least once is

(a) 1/2 (b) 1/3 (c) 1/4 (d) 1/5

23. Two sets A and B are as

$A = \{(x, y) \in R \times R : |x - 5| < 1 \text{ and } |y - 5| < 1\}$

$B = \{(x, y) \in R \times R : 4(x - 6)^2 + 9(y - 5)^2 \leq 36\}$

Then

(a) $A \cap B = \phi$ (an empty set)
(b) neither $A \subset B$ or nor $B \subset A$
(c) $B \subset A$ (d) $A \subset B$

24. Let the relation ρ be defind on R as $a\rho b$ iff $1 + ab > 0$. Then

(a) ρ is reflexive only
(b) ρ is equivalence relation
(c) ρ is reflexive and transitive but not symmetric
(d) ρ is reflexive and symmetric but not transitive.

25. If the cartesian coordinates of a point are $\left(\frac{-5\sqrt{3}}{2}, \frac{5}{2}\right)$, then its polar coordinates are

(a) $\left(5, \frac{5\pi}{6}\right)$ (b) $\left(5, \frac{2\pi}{3}\right)$
(c) $\left(5, \frac{11\pi}{18}\right)$ (d) $\left(5, \frac{13\pi}{18}\right)$

26. A coin is tossed three times. If X denotes the absolute difference between the number of heads and the number of tails, then $P(X = 1)$ is

(a) 1/2 (b) 2/3 (c) 1/6 (d) 3/4

27. If $\cos\alpha+\cos\beta=\frac{3}{2}$ and $\sin\alpha+\sin\beta=\frac{1}{2}$ and θ is the arithmetic mean of α and β, then $\sin 2\theta + \cos 2\theta$ is equal to

(a) 3/5 (b) 4/5 (c) 7/5 (d) 8/5

28. Let a and b be non-zero reals such that $a \neq b$. Then the equation of the line passing through the origin and the point of intersection of $\frac{x}{a}+\frac{y}{b}=1$ and $\frac{x}{b}+\frac{y}{a}=1$ is

(a) $ax + by = 0$ (b) $bx + ay = 0$
(c) $y - x = 0$ (d) $x + y = 0$

29. A straight line has its extremities on two fixed straight lines and cuts off from them a triangle of constant area C^2. Then the locus of the middle point of the line is

(a) $2xy = C^2$ (b) $xy + C^2 = 0$
(c) $4x^2y^2 = C$ (d) None of these

30. The line $5x + y - 1 = 0$ coincides with one of the lines given by $5x^2 + xy - kx - 2y + 2 = 0$, then the value of k is

(a) –11 (b) 31 (c) 11 (d) –31

31. The parametric equation of the circle $x^2 + y^2 - 6x - 2y + 9 = 0$ are

(a) $x = 3 + \cos\theta, y = 1 + \sin\theta$
(b) $x = 1 + \cos\theta, y = 3 + \sin\theta$
(c) $x = \cos\theta, y = \sin\theta$
(d) $x = 3 + \sin\theta, y = 1 + \cos\theta$

32. The equation $x^2 + y^2 + 4x + 6y + 13 = 0$ represents

(a) a pair of coincident lines
(b) a pair of concurrent straight lines
(c) a parabola (d) a point circle

33. If PQ is a double ordinate of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ such that ΔOPQ is equilateral, O being the centre. Then the eccentricity e satisfies

(a) $1<e<\frac{2}{\sqrt{3}}$ (b) $e=\frac{2}{\sqrt{2}}$

(c) $e=\frac{\sqrt{3}}{2}$ (d) $e>\frac{2}{\sqrt{3}}$

34. Let S_1, S_2 be the foci of the ellipse $\frac{x^2}{16}+\frac{y^2}{8}=1$. If $A(x, y)$ is any point on the ellipse, then the maximum area of the triangle AS_1S_2 (in square units) is

(a) $2\sqrt{2}$ (b) $2\sqrt{3}$ (c) 8 (d) 4

35. A parabola $y^2 = 32x$ is drawn. From its focus, a line of slope 1 is drawn. The equation of the line is

(a) $y = x + 8$ (b) $y = x - 4$

(c) $y = x$ (d) $y = x - 8$

36. If the angle between the planes

$\vec{r}\cdot(m\hat{i}-\hat{j}+2\hat{k})+3=0$ and $\vec{r}\cdot(2\hat{i}-m\hat{j}-\hat{k})-5=0$ is $\frac{\pi}{3}$, then $m =$

(a) 2 (b) ± 3 (c) 3 (d) -2

37. If a plane meets the coordinate axes at A, B and C such that the centroid of the triangle is (1, 2, 4), then the equation of the plane is

(a) $x + 2y + 4z = 12$ (b) $4x + 2y + z = 12$

(c) $x + 2y + 4z = 3$ (d) $4x + 2y + z = 3$

38. If $f(x)=\begin{cases}\frac{x-4}{|x-4|}+a & \text{for } x<4\\ a+b & \text{for } x=4\\ \frac{x-4}{|x-4|}+b & \text{for } x>4\end{cases}$

is continuous at $x = 4$, then

(a) $a = 1, b = 1$ (b) $a = 1, b = -1$

(c) $a = 0, b = 0$ (d) $a = -1, b = 1$

39. If $f(x)=\begin{cases}\tan x, & \text{if } 0\le x\le\frac{\pi}{4}\\ ax+b, & \text{if } \frac{\pi}{4}<x<\frac{\pi}{2}\end{cases}$. If $f(x)$ is differentiable at $x=\frac{\pi}{4}$, then the values of a and b are respectively

(a) $2, \frac{2-\pi}{2}$ (b) $2, \frac{4-\pi}{4}$

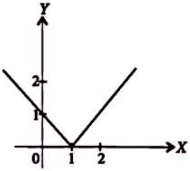
(c) $1, \frac{-\pi}{4}$ (d) $2, \frac{-\pi}{4}$

40. If $y = ae^{mx} + be^{-mx}$, then $\frac{d^2y}{dx^2}-m^2y$ is equal to

(a) $m^2(ae^{mx} + be^{-mx})$ (b) 1

(c) 0 (d) -1

41. The function represented by the following graph is



(a) Continuous but not differentiable at $x = 1$

(b) Differentiable but not continuous at $x = 1$

(c) Continuous and differentiable at $x = 1$

(d) Neither continuous nor differentiable at $x = 1$

42. $\int\frac{1}{\sin x\cdot\cos^2 x}\,dx$ is equal to

(a) $\sec x + \log|\sec x + \tan x| + c$

(b) $\sec x\cdot\tan x + c$

(c) $\sec x + \log|\sec x - \tan x| + c$

(d) $\sec x + \log|\operatorname{cosec} x - \cot x| + c$

43. $\int\frac{2x-1}{(x-1)(x+2)(x-3)}\,dx = A\log|x-1| + B\log|x+2| + C\log|x-3| + K$, then A, B, C are respectively

(a) $\frac{-1}{6},\frac{1}{3},\frac{-1}{2}$ (b) $\frac{1}{6},\frac{-1}{3},\frac{1}{3}$

(c) $\frac{1}{6},\frac{1}{3},\frac{1}{5}$ (d) $\frac{-1}{6},\frac{-1}{3},\frac{1}{2}$

44. The value of

$I=\int_0^{\pi/4}(\tan^{n+1}x)\,dx+\frac{1}{2}\int_0^{\pi/2}\tan^{n-1}\left(\frac{x}{2}\right)dx$ is equal to

(a) $\frac{1}{n}$ (b) $\frac{n+2}{2n+1}$ (c) $\frac{2n-1}{n}$ (d) $\frac{2n-3}{3n-2}$

45. The value of

$\lim_{n\to\infty}\frac{1}{n}\left[\sec^2\frac{\pi}{4n}+\sec^2\frac{2\pi}{4n}+......+\sec^2\frac{n\pi}{4n}\right]$ is

(a) $\log_e 2$ (b) $\pi/2$ (c) $4/\pi$ (d) e

46. The value of $\int_0^{4042}\frac{\sqrt{x}\,dx}{\sqrt{x}+\sqrt{4042-x}}$ is equal to

(a) 4042 (b) 2021 (c) 8084 (d) 1010

47. $y = a\cos(\log x) + b\sin(\log x)$ is a solution of the differential equation

(a) $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=0$ (b) $x\frac{d^2y}{dx^2}+\frac{dy}{dx}+y=0$

(c) $\frac{d^2y}{dx^2}+\frac{dy}{dx}+y=0$ (d) $x\frac{d^2y}{dx^2}+x^2\frac{dy}{dx}+y=0$

48. The value of $\lim_{n\to\infty}\left\{\frac{1+2+3+......+n}{n+2}-\frac{n}{2}\right\}$ is

(a) 1/2 (b) 1 (c) –1 (d) –1/2

49. $\lim_{x\to 0}\frac{\alpha x-(e^{4x}-1)}{\alpha x(e^{4x}-1)}=\beta$ if limit exist then $2(\alpha+\beta)$ is

(a) –1 (b) –7 (c) 1 (d) 7

50. $\lim_{x\to 0}\left(\frac{\int_0^{x^2}\sin\sqrt{t}\,dt}{x^2}\right)$ is equal to

(a) 2/3 (b) 2/9 (c) 1/3 (d) 0

Category - II (Q.51 to 65)

(Carry 2 marks each. Only one option is correct. Negative marks : ½)

51. Let α, β, be the roots of $x^2 - x + p = 0$ and γ, δ be the roots of $x^2 - 4x + q = 0$. If α, β, γ and δ are in G.P., then the integer values of p and q respectively are

(a) –2, 3 (b) –6, 3
(c) –6, –32 (d) –2, –32

52. If $[\vec{a}\times\vec{b}\;\;\vec{b}\times\vec{c}\;\;\vec{c}\times a]=\lambda[\vec{a}\;\vec{b}\;\vec{c}]^2$, then λ is equal to

(a) 3 (b) 0 (c) 1 (d) 2

53. If the ellipse $\frac{x^2}{4}+y^2=1$ meets the ellipse $x^2+\frac{y^2}{a^2}=1$ in four distinct points and $a = b^2 - 5b + 7$, then b does not lie in

(a) [4, 5] (b) $(-\infty, 2)\cup(3, \infty)$
(c) $(-\infty, 0)$ (d) [2, 3]

54. If $A=\begin{bmatrix}k & l\\ m & n\end{bmatrix}$ and $kn \neq lm$, then the value of $A^2 - (k + n)A + (kn - lm)I$ equals (where I is the identity matrix of order 2 × 2)

(a) The zero matrix of order 2 × 2
(b) A
(c) $-A$
(d) $2A$

55. The solution of the differential equation

$$\frac{dy}{dx}+y\cos x=\frac{\sin 2x}{2}$$ is

(a) $ye^{\sin x} = C + e^{2\sin x}(\sin x - 1)$
(b) $ye^{\sin x} = C + e^{\sin x}(\sin x - 1)$
(c) $ye^{\sin x} = C + e^{-\sin x}(\sin x + 1)$
(d) $ye^{\sin x} = C + e^{-2\sin x}(\sin x - 1)$; C being an arbitrary constant

56. Here, $[x]$ denotes the greatest integer less than or equal to x. Given that $f(x) = [x] + x$. The value obtained when this function is integrated with respect to x with lower limit as $\frac{3}{2}$ and upper limit as $\frac{9}{2}$, is

(a) 12 (b) 10.5 (c) 8 (d) 16.5

57. Which of the following statements is always true?

(a) If $f(x)$ is increasing, then $f^{-1}(x)$ is decreasing
(b) If $f(x)$ is increasing, then $1/f(x)$ is also increasing
(c) If f and g are positive function and f is increasing and g is decreasing, then f/g is a decreasing function
(d) If f and g are positive and f is decreasing and g is increasing, then f/g is a decreasing function

58. The derivative of $f(\tan x)$ w.r.t. $g(\sec x)$ at $x=\frac{\pi}{4}$, where $f'(1) = 2$ and $g'(\sqrt{2})=4$ is

(a) $\frac{1}{\sqrt{2}}$ (b) $\sqrt{2}$ (c) 1 (d) $\frac{1}{2}$

59. Consider the following relations in the real number

$R_1 = \{(x, y)\,|x^2 + y^2 \le 25\}$, $R_2=\left\{(x,y)\,|\,y\ge\frac{4x^2}{9}\right\}$,

then the range of $R_1 \cap R_2$ is

(a) [0, 5] (b) [–3, 3]
(c) [–5, 5] (d) [–3, 5]

60. If $\sum_{r=1}^{n}(2r-1)=x$, then $\lim_{n\to\infty}\left[\frac{1^3}{x^2}+\frac{2^3}{x^2}+\frac{3^3}{x^2}+.....+\frac{n^3}{x^2}\right]=$

(a) $\frac{1}{2}$ (b) 1 (c) $\frac{1}{4}$ (d) 4

61. ΔABC has vertices at $A \equiv (2, 3, 5)$, $B \equiv (-1, 3, 2)$ and $C \equiv (\lambda, 5, \mu)$. If the median through A is equally inclined to the axes, then the values of λ and μ respectively are

(a) 10, 7 (b) 9, 10 (c) 7, 9 (d) 7, 10

62. If $A=\begin{pmatrix}0 & -\tan\frac{\theta}{2}\\ \tan\frac{\theta}{2} & 0\end{pmatrix}$ and $(I_2 + A)(I_2 - A)^{-1}$ $=\begin{pmatrix}a & -b\\ b & a\end{pmatrix}$, where I_2 denote the 2 × 2 identity matrix, then $13(a^2 + b^2)$ is equal to

(a) 1 (b) 0 (c) 13 (d) 2

63. A plane is given as $3x + 4y - 12z + 13 = 0$. The points$(1, 1, k)$ and $(-3, 0, 1)$ are equidistant from the plane. What will be the quadratic equation that will have roots as values of k?

(a) $4x^2 - 15x + 9 = 0$ (b) $x^2 - 10x + 25 = 0$
(c) $3x^2 - 8x + 4 = 0$ (d) $3x^2 - 10x + 7 = 0$

64. Let $X \sim B(n, p)$, if $E(X) = 5$, Var $(X) = 2.5$ then $P(X < 1) =$

(a) $\left(\frac{1}{2}\right)^{11}$ (b) $\left(\frac{1}{2}\right)^{10}$ (c) $\left(\frac{1}{2}\right)^{6}$ (d) $\left(\frac{1}{2}\right)^{9}$

65. The area bounded by the curve

$$y=\begin{cases} x^{1/\ln x}, & x \neq 1 \\ e, & x=1 \end{cases} \text{ and } y=|x-e| \text{ is}$$

(a) $e^2/2$ (b) e^2 (c) $2e^2$ (d) 1

Category - III (Q.66 to 75)

(Carry 2 marks each. One or more options are correct. No negative marks)

66. Consider all possible permutations of the letters of the word ENDEANOEL. The number of permutations in which letters A, E, O occur only in odd position is

(a) $5 \times 5!$ (b) $3 \times 5!$ (c) $2 \times 5!$ (d) $7 \times 5!$

67. If A and B are two equivalence relations defined on set C, then, which of the following statements is (are) definitely correct?

(a) $A \cap B$ is an equivalence relation
(b) $A \cap B$ is not an equivalence relation
(c) $A \cup B$ is an equivalence relation
(d) $A^{-1} \cap B^{-1}$ is an equivalence relation

68. Let X and Y be two arbitrary, 3×3 non-zero, skew-symmetric matrices and Z be an arbitrary 3×3, non-zero, symmetric matrix. Then which of the following matrices is (are) skew-symmetric?

(a) $Y^3Z^4 - Z^4Y^3$ (b) $X^{44} + Y^{44}$
(c) $X^4Z^3 - Z^3X^4$ (d) $X^{23} + Y^{23}$

69. Consider the system of equations:
$x + y + z = 0$; $\alpha x + \beta y + \gamma z = 0$; $\alpha^2 x + \beta^2 y + \gamma^2 z = 0$
Then the system of equations has

(a) a unique solution for all values of α, β, γ
(b) infinite number of solutions if any two of α, β, γ are equal
(c) a unique solution if α, β, γ are distinct
(d) more than one, but finite number of solutions depending on values of α, β, γ

70. The coordinates of the foot of the perpendicular drawn from the origin to the plane $2x + 6y - 3z = 63$ are

(a) $\left(\frac{2}{7}, \frac{6}{7}, \frac{-3}{7}\right)$ (b) $(4, 2, -4)$
(c) $\left(\frac{9}{7}, \frac{6}{7}, \frac{-3}{7}\right)$ (d) $\left(\frac{18}{7}, \frac{54}{7}, \frac{-27}{7}\right)$

71. If L_1 and L_2 are two lines belonging to the family of lines $(3 + 2\lambda)x + (4 + 3\lambda)y = 7 + 5\lambda$ such that they are at maximum and minimum distances from the point $(2, 3)$, then the equation of line through the point $(1, 2)$ and making equal angles with L_1 and L_2 is

(a) $x - 3y + 5 = 0$ (b) $3x + y - 5 = 0$
(c) $x + 2y - 7 = 0$ (d) $2x - y = 0$

72. Let P and Q be distinct points on the parabola $y^2 = 2x$ such that a circle with PQ as diameter passes through the vertex O of the parabola. If P lies in the first quadrant and the area of the triangle OPQ is $3\sqrt{2}$ sq. units, then which of the following is (are) the coordinates of P ?

(a) $(4, 2\sqrt{2})$ (b) $(9, 3\sqrt{2})$
(c) $\left(\frac{1}{4}, \frac{1}{\sqrt{2}}\right)$ (d) $(1, \sqrt{2})$

73. If $x^2 + y^2 = 25$, then $\log_5 [\text{Max}(3x + 4y)]$ is

(a) 2 (b) 3 (c) 4 (d) 5

74. Let $\lim\limits_{x\to a} f(x)\cdot g(x)=9$ and $\lim\limits_{x\to a} \frac{f(x)}{g(x)} = 4.$

If $\lim\limits_{x\to a} f(x)$ and $\lim\limits_{x\to a} g(x)$ both exist, then the value of $\lim\limits_{x\to a} (f(x)+g(x))$ is equal to

(a) $\frac{15}{2}$ (b) $\frac{9}{2}$ (c) $\frac{-9}{2}$ (d) $\frac{-15}{2}$

75. The volume of the parallelepiped whose co-terminous edges are represented by the vectors $2\bar{b}\times\bar{c}, 3\bar{c}\times\bar{a}$ and $4\bar{a}\times\bar{b}$ where

$$\bar{a} = (1+\sin\theta)\hat{i}+\cos\theta\,\hat{j}+\sin 2\theta\hat{k},$$

$$\bar{b} = \sin\left(\theta+\frac{2\pi}{3}\right)\hat{i}+\cos\left(\theta+\frac{2\pi}{3}\right)\hat{j}+\sin\left(2\theta+\frac{4\pi}{3}\right)\hat{k},$$

$$\bar{c} = \sin\left(\theta-\frac{2\pi}{3}\right)\hat{i}+\cos\left(\theta-\frac{2\pi}{3}\right)\hat{j}+\sin\left(2\theta-\frac{4\pi}{3}\right)\hat{k}$$

is 18 cubic units, then the values of θ, in the interval $\left(0, \frac{\pi}{2}\right)$, is/are

(a) $\frac{\pi}{9}$ (b) $\frac{2\pi}{9}$ (c) $\frac{\pi}{3}$ (d) $\frac{4\pi}{9}$

SOLUTIONS

1. **(d) :** We have, $\frac{dy}{dx} = (e^x + 1)y$

$\Rightarrow \int \frac{dy}{y} = \int (e^x + 1)dx$ [Integrating on both sides]

$\Rightarrow \log|y| = e^x + x + c$

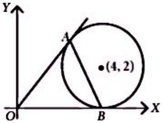
2. **(a) :** We have, $y = e^{-x} + Ax + B$

$\Rightarrow \frac{dy}{dx} = -e^{-x} + A \Rightarrow \frac{d^2y}{dx^2} = e^{-x}$

3. **(d) :** Length of tangent from the point $O(0, 0)$ to the given circle is $L = \sqrt{16} = 4$

Radius of the circle,

$R = \sqrt{16+4-16} = 2$



Length of chord of contact

i.e., $AB = \frac{2 \times L \times R}{\sqrt{L^2 + R^2}} = \frac{2 \times 4 \times 2}{\sqrt{16+4}} = \frac{16}{\sqrt{20}} = \frac{8}{\sqrt{5}}$

$\therefore (AB)^2 = \frac{64}{5}$

4. **(d) :** We have, R.H.L. $= \lim_{x \to 0^+} f(x) = \lim_{h \to 0} \cos h = 1$

L.H.L. $= \lim_{x \to 0^-} f(x) = \lim_{h \to 0} (-\cos h) = -1$

Since L.H.L. $\neq$ R.H.L.

$\therefore$ $f(x)$ is not continuous at $x = 0$

5. **(a) :** We have, $f(x) = 6x^5 - 7x^4 + 3x^3 + 2x^2 + 11x - 17$

$f'(x) = 30x^4 - 28x^3 + 9x^2 + 4x + 11$

Here, $f'(x) > 0 \; \forall \; x \in (1, 2)$

$\therefore$ $f(x)$ is an increasing function for $x \in (1, 2)$

Thus, $f(x)$ has no local maxima $\forall \; x \in (1, 2)$.

6. **(d) :** We have, $|\vec{p} + \vec{q}| = \sqrt{3} \Rightarrow |\vec{p}|^2 + |\vec{q}|^2 + 2\vec{p}\cdot\vec{q} = 3$

Since $\vec{p}$ and $\vec{q}$ are unit vectors

So, $1 + 1 + 2\vec{p}\cdot\vec{q} = 3 \Rightarrow 2\vec{p}\cdot\vec{q} = 1 \Rightarrow \vec{p}\cdot\vec{q} = \frac{1}{2}$

Now, $(2\vec{p} - 3\vec{q})\cdot(3\vec{p} + \vec{q}) = 6|\vec{p}|^2 + 2\vec{p}\cdot\vec{q} - 9\vec{q}\cdot\vec{p} - 3|\vec{q}|^2$

$= 6 - \frac{7}{2} - 3 = 3 - \frac{7}{2} = \frac{-1}{2}$

7. **(d) :** Using mean value theorem, $f'(c) = \frac{f(b) - f(a)}{b - a}$

$\Rightarrow 2c + 2a = \frac{f(\beta) - f(\alpha)}{\beta - \alpha}$

$= \frac{(\beta^2 + 2a\beta + b) - (\alpha^2 + 2a\alpha + b)}{\beta - \alpha} = \beta + \alpha + 2a$

$\therefore \; c = \frac{\alpha + \beta}{2}$

8. **(c) :** Let $I = \int_0^{\pi/3} \frac{\cos x}{3 + 4\sin x} dx$

Put $\sin x = t \Rightarrow \cos x \, dx = dt$

when $x = 0, t = 0$ and $x = \frac{\pi}{3} \Rightarrow t = \frac{\sqrt{3}}{2}$

$\therefore \; I = \int_0^{\sqrt{3}/2} \frac{dt}{3 + 4t} = \frac{1}{4}\log|3 + 4t| \Big|_0^{\sqrt{3}/2}$

$= \frac{1}{4}\left[\log\left|3 + \frac{4\sqrt{3}}{2}\right| - \log|3|\right]$

$= \frac{1}{4}\left[\log|3 + 2\sqrt{3}| - \log|3|\right] = \frac{1}{4}\log\left|\frac{3 + 2\sqrt{3}}{3}\right|$

On comparing with $k\log\left|\frac{3 + 2\sqrt{3}}{3}\right|$, we get $k = \frac{1}{4}$

9. **(c) :** According to question,

Mean $= 6 \Rightarrow a + b + 8 + 5 + 10 = 30$

$\Rightarrow \; a + b = 7$ ...(i)

Also, variance $= 6.8$

$\Rightarrow \; (a - 6)^2 + (b - 6)^2 + 2^2 + (-1)^2 + 4^2 = (6.8) \times 5$

$\Rightarrow \; a^2 + b^2 = 25$ ...(ii)

On solving (i) and (ii), we get $ab = 12$

10. **(a) :** Clearly $x = 1$ is the solution.

$\therefore$ Product of the roots $= \frac{a - b}{b - c}$ $\therefore \; (1)(1) = \frac{a - b}{b - c}$

$\Rightarrow b - c = a - b \Rightarrow 2b = a + c \Rightarrow a, b, c$ are in A.P.

11. **(d) :** Since, $i^2 = -1, i^3 = -i$ and $i^4 = 1$

$\therefore \; \sum_{n=1}^{13}(i^n + i^{n+1}) = (i + i^2) + (i^2 + i^3) + (i^3 + i^4) + \ldots + (i^{13} + i^{14})$

$= i + 2(i^2 + i^3 + i^4 + \ldots + i^{13}) + i^{14}$

$= i + 2[3(i + i^2 + i^3 + i^4)] + i^{14}$

$= i + 6(i - 1 - i + 1) + i^{2 \times 7} = i + 0 - 1 = i - 1$

12. **(d) :** We have, $\left|\frac{1 - \bar{z}_1 z_2}{z_1 - z_2}\right| = \sqrt{\frac{(1 - \bar{z}_1 z_2)\overline{(1 - \bar{z}_1 z_2)}}{(z_1 - z_2)\overline{(z_1 - z_2)}}}$

$= \sqrt{\frac{(1 - \bar{z}_1 z_2)(1 - z_1 \bar{z}_2)}{(z_1 - z_2)(\bar{z}_1 - \bar{z}_2)}}$

$= \sqrt{\frac{1 - z_1\bar{z}_2 - \bar{z}_1 z_2 + (z_1\bar{z}_1)(z_2\bar{z}_2)}{z_1\bar{z}_1 - z_1\bar{z}_2 - z_2\bar{z}_1 + z_2\bar{z}_2}}$

$= \sqrt{\frac{1 - z_1\bar{z}_2 - \bar{z}_1 z_2 + |z_1|^2}{|z_1|^2 - z_1\bar{z}_2 - \bar{z}_1 z_2 + 1}} = 1$ $(\because \; |z_2| = 1)$

13. (b) : Thousand's digit can be filled in 9 ways from 1 to 9
Hundred's digit can be filled in 9 ways
Ten's digit can be filled in 8 ways
One's digit can be filled in 7 ways
So, number of 4 digit numbers = $9 \times 9 \times 8 \times 7 = 4536$

14. (c) : We have, $1 - 2i$ is a root of
$z^2 + \alpha z + \beta = 0; \alpha, \beta \in R$
$\Rightarrow$ $1 + 2i$ will be the another root.
Where, $\alpha = -\text{sum of roots} = -((1-2i) + (1+2i)) = -2$
and $\beta = \text{product of roots} = (1-2i)(1+2i) = 5$
$\therefore \quad \alpha - \beta = -2 - 5 = -7$

15. (a) : ARTICLE is a seven letter word, in which there are 3 vowels and 4 consonants.
There are 3 even places with the 7 letter word ARTICLE
So, we have to arrange 4 consonants in these 3 places in ${}^4P_3$ ways and the remaining 4 letters can be arranged among themselves in ${}^4P_4$ ways.
$\therefore$ Total number of arrangements $= {}^4P_3 \times {}^4P_4 = 576$

16. (c) : We have, $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

$$\Rightarrow \quad A^2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Now, $(aI + bA)^n = {}^nC_0(aI)^n + {}^nC_1(aI)^{n-1}\, bA + {}^nC_2(aI)^{n-2}(bA)^2 + ...$
$= a^nI + n\,a^{n-1}\,I\,bA + 0 \qquad [\because A^2 = 0]$
$= a^nI + na^{n-1}\,bA$

17. (b) : Let $P(n) = 2^{2n} - 1$ is divisible by k for all $n \in N$
$\therefore \quad P(1) = 2^2 - 1$ is divisible k
$\Rightarrow$ 3 is divisible by k. So, the value of $k = 3$

18. (c) : We have, $n(A) = 4$ and $n(B) = 2$
Thus the number of elements in $A \times B$ is 8.
Number of subsets having at least 3 elements
$= {}^8C_3 + {}^8C_4 + {}^8C_5 + {}^8C_6 + {}^8C_7 + {}^8C_8$
$= ({}^8C_0 + {}^8C_1 + {}^8C_2 + ... + {}^8C_8) - ({}^8C_0 + {}^8C_1 + {}^8C_2)$
$= 2^8 - (1 + 8 + 28) = 256 - 37 = 219$

19. (c) : We have, $X = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

$$\Rightarrow \quad |X| = \begin{vmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{vmatrix} = \cos^2\alpha + \sin^2\alpha = 1$$

Now, $|X^{100}| = (|X|)^{100} = (1)^{100} = 1$

20. (d) : Given, $|A^3| = 27 \Rightarrow |A|^3 = 27$

$$\Rightarrow \quad |A| = 3 \Rightarrow \begin{vmatrix} \alpha & 2 \\ 2 & \alpha \end{vmatrix} = 3$$

$\Rightarrow \quad \alpha^2 - 4 = 3 \Rightarrow \alpha^2 = 7 \Rightarrow \alpha = \pm\sqrt{7}$

21. (c) : We have, $A(\text{adj}\,A) = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix} = 10\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 10\,I$

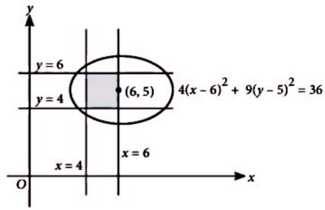
We know that $A(\text{adj}\,A) = |A|\,I \Rightarrow |A| = 10$

22. (b) : Let A = event that 3 has appeared atleast once.
B = event that sum of the numbers is 7.

$$\therefore \quad P(A \cap B) = \frac{2}{36},\ P(B) = \frac{6}{36}$$

$$\text{So,} \quad P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{2}{36}}{\frac{6}{36}} = \frac{1}{3}$$

23. (d) :



We have, $A = \{(x, y) \in R \times R \mid 4 < x < 6 \text{ and } 4 < y < 6\}$

and $B = \left\{(x, y) \in R \times R \mid \frac{(x-6)^2}{9} + \frac{(y-5)^2}{4} \le 1\right\}$

Clearly, $A \subset B$

24. (d) : Relation ρ be defined on R as $a\rho b$ iff $1 + ab > 0$.
(i) Reflexivity : Let $a \in R$
Since $1 + a^2 > 0$
$\therefore \quad a\rho a \quad \therefore\ \rho$ is reflexive.
(ii) Symmetry : Let $a, b \in R$
Now $1 + ab = 1 + ba > 0$
$\therefore \quad a\rho b \Rightarrow b\rho a \ \therefore\ \rho$ is symmetric
(iii) Transitivity : Let $a, b, c \in R$
Now, $1 + ab > 0$ and $1 + bc > 0 \not\Rightarrow 1 + ac > 0$
$\therefore \quad a\rho b$ and $b\rho c \not\Rightarrow a\rho c \quad \therefore\ \rho$ is not transitive.

25. (a) : If (x, y) are cartesian coordinates then polar coordinates can be written as (r, θ), where

$$r = \sqrt{x^2 + y^2},\ \ \theta = \tan^{-1}\left(\frac{y}{x}\right)$$

Given, $x = \frac{-5\sqrt{3}}{2}$, $y = \frac{5}{2}$. So, $r = \sqrt{\left(\frac{-5\sqrt{3}}{2}\right)^2 + \left(\frac{5}{2}\right)^2} = 5$

$$\theta = \tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) = \frac{5\pi}{6}$$

Hence, polar coordinates are $\left(5, \frac{5\pi}{6}\right)$.

26. (d) : Total number of outcomes = 8 (HHH, HTH, THH, HHT, HTT,THT, TTH, TTT)
Favorable outcomes = {HTH, THH, HHT, HTT, THT, TTH} which are 6 in number.

$\therefore$ Required probability = $\frac{6}{8}=\frac{3}{4}$

27. (c) : Since, $\cos\alpha+\cos\beta=\frac{3}{2}$

$\Rightarrow 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)=\frac{3}{2}$ ...(i)

Also, $\sin\alpha+\sin\beta=\frac{1}{2}$

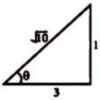
$\Rightarrow 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)=\frac{1}{2}$ ...(ii)

Dividing (ii) by (i), $\tan\left(\frac{\alpha+\beta}{2}\right)=\frac{1}{3}$

Since, θ is the arithmetic mean of α and β

$\therefore \quad \theta=\frac{\alpha+\beta}{2}$

$\tan\theta=\frac{1}{3} \Rightarrow \sin\theta=\frac{1}{\sqrt{10}}$ and $\cos\theta=\frac{3}{\sqrt{10}}$



Now, $\sin 2\theta+\cos 2\theta$

$=2\sin\theta\cos\theta+2\cos^2\theta-1$

$=2\times\frac{1}{\sqrt{10}}\times\frac{3}{\sqrt{10}}+2\left(\frac{9}{10}\right)-1=\frac{6}{10}+\frac{18}{10}-1=\frac{7}{5}$

28. (c) : Given equation of lines are

$\frac{x}{a}+\frac{y}{b}=1$ ...(i), and $\frac{x}{b}+\frac{y}{a}=1$ ...(ii)

$\Rightarrow \quad bx+ay=ab$ ...(iii) and $ax+by=ab$ ...(iv)

Solving (iii) and (iv), we get

$(a^2-b^2)y=a^2b-ab^2=ab(a-b)\Rightarrow y=\frac{ab}{a+b}$

Substituting the value of y in (iii), we get

$bx+a\cdot\frac{ab}{a+b}=ab \Rightarrow bx=ab-\frac{a^2b}{a+b}\Rightarrow x=\frac{ab}{a+b}$

$\therefore$ Point of intersection is $\left(\frac{ab}{a+b},\frac{ab}{a+b}\right)$.

Since, equation of the line passing through origin is $y=mx$

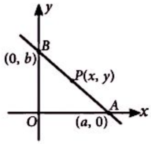
$\therefore$ When it pass through $\left(\frac{ab}{a+b},\frac{ab}{a+b}\right)$ then, we get $m=1$

Hence, required equation of line is $y-x=0$

29. (a) : Let the equation of line be $\frac{x}{a}+\frac{y}{b}=1$ ...(i)

(i) cuts the x-axis at $A(a, 0)$ and y-axis at $B(0, b)$.

Let mid-point of line be $P(x, y)$.



$\therefore \quad \frac{a+0}{2}=x$ and $\frac{0+b}{2}=y$

$\Rightarrow \quad a=2x$ and $b=2y$

Area of $\Delta OAB=\frac{1}{2}(a\times b)=C^2$ (given)

$\Rightarrow \frac{1}{2}(2x)(2y)=C^2 \Rightarrow 2xy=C^2$, which is the required locus.

30. (c) : We have, $5x^2+xy-kx-2y+2=0$

As y^2 is absent in given equation, let the equation of second line be $ax+c=0$.

$\Rightarrow \quad (5x+y-1)(ax+c)=0$

$\Rightarrow \quad 5ax^2+5cx+axy+cy-ax-c=0$

Given equation is $5x^2+xy-kx-2y+2=0$

$\therefore \quad a=1, c=-2$ and $-k=5c-a$

$\Rightarrow \quad -k=5(-2)-1\Rightarrow k=11$

31. (a) : If (x_1, y_1) is center of circle and r is radius, then parametric equation of circle can be written as

$x=x_1+r\cos\theta$

$y=y_1+r\sin\theta$

Given equation of circle is

$x^2+y^2-6x-2y+9=0 \Rightarrow (x-3)^2+(y-1)^2=1$

$\therefore$ Center of circle is (3, 1) and radius of circle is 1.

Equation in parametric form is

$x=3+\cos\theta$

$y=1+\sin\theta$

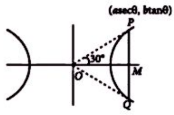
32. (d) : We have, $x^2+y^2+4x+6y+13=0$

$\Rightarrow \quad (x+2)^2+(y+3)^2=0$

which represents a point circle at (–2, –3).

33. (d) : Let $OM=a\sec\theta$, $PM=b\tan\theta$

$\tan 30^\circ=\frac{PM}{OM}=\frac{b\sin\theta}{a}$



$\Rightarrow \sin\theta=\frac{a}{b\sqrt{3}}$

$\Rightarrow \sin^2\theta=\frac{a^2}{3b^2}$

$\because \quad 0<\sin^2\theta<1 \qquad \therefore \quad 0<\frac{a^2}{3b^2}<1$

$\Rightarrow \frac{3b^2}{a^2}>1\Rightarrow 3(e^2-1)>1 \left[\because \frac{b^2}{a^2}=e^2-1\right]$

$\Rightarrow e^2>\frac{4}{3}\Rightarrow e>\frac{2}{\sqrt{3}}$

34. (c) : Equation of ellipse is $\frac{x^2}{16}+\frac{y^2}{8}=1$

where, $a = 4,\ b=2\sqrt{2}$

Eccentricity, $e=\sqrt{1-\frac{b^2}{a^2}}=\sqrt{1-\frac{8}{16}}=\frac{1}{\sqrt{2}}$

Area is maximum, when vertex is $(0, b)$.

$\therefore$ Maximum area

$=\frac{1}{2}\times 2ae\times b=\frac{1}{2}\times 2\times 4\times\frac{1}{\sqrt{2}}\times 2\sqrt{2}=8$ sq. units

35. (d)

36. (c) : Direction ratios of $\vec{n}_1$ are $m, -1, 2$.

Direction ratios of $\vec{n}_2$ are $2, -m, -1$.

Now, $\cos\theta=\left|\frac{\vec{n}_1\cdot\vec{n}_2}{|\vec{n}_1||\vec{n}_2|}\right|$

$\Rightarrow \frac{1}{2}=\left|\frac{2m+m-2}{\sqrt{m^2+5}\sqrt{m^2+5}}\right| \qquad \left(\because \theta=\frac{\pi}{3}\right)$

$\Rightarrow \frac{1}{2}=\left|\frac{3m-2}{m^2+5}\right|\Rightarrow m^2+5=\pm(6m-4)$

$\Rightarrow m^2+5=6m-4$ or $m^2+5=-6m+4$

$\Rightarrow m^2-6m+9=0$ or $m^2+6m+1=0$

$\Rightarrow (m-3)^2=0\Rightarrow m=3$ or $m=-3\pm 2\sqrt{2}$

37. (b) : Let the equation of the plane is $\frac{x}{\alpha}+\frac{y}{\beta}+\frac{z}{\gamma}=1$

Then, $A(\alpha, 0, 0)$, $B(0, \beta, 0)$ and $C(0, 0, \gamma)$ are the points on the coordinate axes.

Since, the centroid of the triangle is $(1, 2, 4)$.

$\therefore \frac{\alpha}{3}=1\Rightarrow\alpha=3, \frac{\beta}{3}=2\Rightarrow\beta=6$ and $\frac{\gamma}{3}=4\Rightarrow\gamma=12$

$\therefore$ The equation of the plane is $\frac{x}{3}+\frac{y}{6}+\frac{z}{12}=1$

$\Rightarrow 4x+2y+z=12$

38. (b) : Since, $f(x)$ is continuous at $x = 4$

So, $\lim_{x\to 4^-} f(x)=\lim_{x\to 4^+} f(x)=f(x)$ at $x=4$

$\Rightarrow -1+a=a+b=1+b$

Now, $a-1=a+b \Rightarrow b=-1$

and $a+b=1+b \Rightarrow a=1$

Hence, $a=1, b=-1$.

39. (a) : We have, $f(x)=\begin{cases}\tan x, & \text{if } 0\le x\le\frac{\pi}{4}\\ ax+b, & \text{if } \frac{\pi}{4}<x<\frac{\pi}{2}\end{cases}$

$\Rightarrow f'(x)=\begin{cases}\sec^2 x, & \text{if } 0\le x\le\frac{\pi}{4}\\ a, & \text{if } \frac{\pi}{4}<x<\frac{\pi}{2}\end{cases}$

$\because f(x)$ is differentiable at $x=\frac{\pi}{4}$

$\therefore \sec^2\left(\frac{\pi}{4}\right)=a\Rightarrow(\sqrt{2})^2=a\Rightarrow a=2$

Also, $f(x)$ should be continuous at $x=\frac{\pi}{4}$

$\therefore \lim_{x\to\pi/4^-}\tan x=\lim_{x\to\pi/4^+}2x+b$

$\Rightarrow 1=\frac{\pi}{2}+b\Rightarrow b=\frac{2-\pi}{2}$

40. (c) : Given, $y=ae^{mx}+be^{-mx}$ ...(i)

$\therefore \frac{dy}{dx}=ame^{mx}-bme^{-mx}$ and $\frac{d^2y}{dx^2}=am^2e^{mx}+bm^2e^{-mx}$

$=m^2(ae^{mx}+be^{-mx})=m^2y$ [Using (i)]

$\therefore \frac{d^2y}{dx^2}-m^2y=0$

41. (a) : From the given graph, the function is continuous. The given graph is that of $y=|x-1|$

Now, $f(x)=y=\begin{cases}x-1, & x\ge 1\\ 1-x, & x<1\end{cases}$

$Rf'(1)=\lim_{h\to 0}\frac{f(1+h)-f(1)}{h}=\lim_{h\to 0}\frac{(1+h-1)-0}{h}=1$

$Lf'(1)=\lim_{h\to 0}\frac{f(1-h)-f(1)}{-h}=\lim_{h\to 0}\frac{(1-1+h)-0}{-h}=-1$

Since $Rf'(1)\ne Lf'(1)$

So, the given function is not differentiable at $x = 1$.

42. (d)

43. (d) : Let $I=\int\frac{2x-1}{(x-1)(x+2)(x-3)}dx$

Let $\frac{2x-1}{(x-1)(x+2)(x-3)}=\frac{P}{x-1}+\frac{Q}{x+2}+\frac{R}{x-3}$

$\Rightarrow 2x-1=P(x+2)(x-3)+Q(x-1)(x-3)+R(x-1)(x+2)$

Putting $x=1$, $1=P(3)(-2)\Rightarrow P=\frac{-1}{6}$

Putting $x=-2$, $-5=Q(-3)(-5)\Rightarrow Q=\frac{-1}{3}$

Putting $x=3$, $5=R(2)(5)\Rightarrow R=\frac{1}{2}$

$$\therefore \quad I=\frac{-1}{6}\int\frac{dx}{x-1}-\frac{1}{3}\int\frac{dx}{x+2}+\frac{1}{2}\int\frac{dx}{x-3}$$

$$=\frac{-1}{6}\log|x-1|-\frac{1}{3}\log|x+2|+\frac{1}{2}\log|x-3|+K$$

$$\therefore \quad A=\frac{-1}{6},\ B=\frac{-1}{3},\ C=\frac{1}{2}$$

44. (a) : $I=\int_0^{\pi/4}\tan^{n+1}x\cdot dx+\frac{1}{2}\int_0^{\pi/2}\tan^{n-1}(x/2)\cdot dx$

$$=\int_0^{\pi/4}\tan^{n+1}t\cdot dt+\int_0^{\pi/4}\tan^{n-1}t\cdot dt \qquad \left[\text{Put } \frac{x}{2}=t\right]$$

$$=\int_0^{\pi/4}\tan^{n-1}t(\tan^2 t+1)dt$$

Put $\tan t=z$ $\therefore$ $I=\int_0^1 z^{n-1}\cdot dz=\left[\frac{z^n}{n}\right]_0^1=\frac{1}{n}$

45. (c) : Required value $=\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}\sec^2\left(\frac{\pi}{4}\cdot\frac{r}{n}\right)$

$$=\int_0^1\sec^2\frac{\pi x}{4}dx=\left[\frac{\tan(\pi x/4)}{\pi/4}\right]_0^1=\frac{4}{\pi}$$

46. (b) : Let $I=\int_0^{4042}\frac{\sqrt{x}\,dx}{\sqrt{x}+\sqrt{4042-x}}$ ...(i)

$$\Rightarrow I=\int_0^{4042}\frac{\sqrt{4042-x}}{\sqrt{4042-x}+\sqrt{x}}dx \qquad \text{...(ii)}$$

$$\left[\because \int_a^b f(x)dx=\int_a^b f(a+b-x)dx\right]$$

Adding (i) and (ii), we get

$$2I=\int_0^{4042}1\,dx=[x]_0^{4042} \Rightarrow I=\frac{4042}{2}=2021$$

47. (a) : $y = a\cos(\log x) + b\sin(\log x)$ ...(i)

$$\frac{dy}{dx}=-a\sin(\log x)\times\frac{1}{x}+b\cos(\log x)\times\frac{1}{x} \qquad \text{...(ii)}$$

$$\frac{d^2y}{dx^2}=-\frac{a}{x}\cos(\log x)\times\frac{1}{x}-a\sin(\log x)\times\left(\frac{-1}{x^2}\right)$$
$$-\frac{b}{x}\sin(\log x)\times\frac{1}{x}+b\cos(\log x)\left(\frac{-1}{x^2}\right)$$

$$\Rightarrow \frac{d^2y}{dx^2}=\frac{1}{x^2}\ [-a\cos(\log x)+a\sin(\log x)-b\sin(\log x)-b\cos(\log x)]$$

$$\Rightarrow x^2\frac{d^2y}{dx^2}=-[a\cos(\log x)+b\sin(\log x)]+[a\sin(\log x)-b\cos(\log x)]$$

$$\Rightarrow x^2\frac{d^2y}{dx^2}=-y-x\frac{dy}{dx} \qquad \text{[Using (i) \& (ii)]}$$

$$\Rightarrow x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=0$$

48. (d) : We have, $\lim_{n\to\infty}\left\{\frac{1+2+3+...+n}{n+2}-\frac{n}{2}\right\}$

$$=\lim_{n\to\infty}\left\{\frac{n(n+1)}{2(n+2)}-\frac{n}{2}\right\}=\lim_{n\to\infty}\frac{-n}{2(n+2)}$$

$$=-\lim_{n\to\infty}\frac{1}{2\left(1+\frac{2}{n}\right)}=-\frac{1}{2}$$

49. (d) : Let $L=\lim_{x\to 0}\frac{\alpha x-(e^{4x}-1)}{\alpha x(e^{4x}-1)}$ $\left(\frac{0}{0}\text{ form}\right)$

$$=\lim_{x\to 0}\frac{\alpha-4e^{4x}}{\alpha(e^{4x}-1)+\alpha x(4e^{4x})} \qquad \text{(By L' Hospital's rule)}$$

The limit to be exist finitely, $\alpha = 4$

$\therefore$ We have, $L=\lim_{x\to 0}\frac{4-4e^{4x}}{4(e^{4x}-1)+4x(4e^{4x})}$

$$=\lim_{x\to 0}\frac{1-e^{4x}}{e^{4x}-1+4xe^{4x}} \quad \left(\frac{0}{0}\text{ form}\right)$$

$$=\lim_{x\to 0}\frac{-4e^{4x}}{4e^{4x}+16xe^{4x}+4e^{4x}} \qquad \text{(By L'Hospital's rule)}$$

$$=\frac{-4}{4+4}=\frac{-1}{2}=\beta$$

$$\Rightarrow 2(\alpha+\beta)=2\left(4-\frac{1}{2}\right)=2\left(\frac{8-1}{2}\right)=7$$

50. (d) : We have, $\lim_{x\to 0}\left(\frac{\int_0^{x^2}\sin\sqrt{t}\,dt}{x^2}\right)$

Applying L' Hospital's rule, we get

$$\lim_{x\to 0}\left(\frac{\sin\sqrt{x^2}\cdot 2x}{2x}\right)=\lim_{x\to 0}\sin x=0$$

51. (d) : α, β, γ and δ are in G.P.

Let $\alpha = a, \beta = ar, \gamma = ar^2, \delta = ar^3$

$\alpha + \beta = 1 \Rightarrow a + ar = 1 \Rightarrow a(1 + r) = 1$ ...(i)

Also, $\gamma + \delta = 4 \Rightarrow ar^2 + ar^3 = 4 \Rightarrow ar^2(1 + r) = 4$ ...(ii)

From (i) and (ii), $\frac{1}{r^2}=\frac{1}{4} \Rightarrow r=\pm 2$

When $r = 2 \Rightarrow a = \alpha = 1/3$

$\beta = ar = \frac{1}{3}\times 2 = \frac{2}{3} \quad \therefore\ \alpha\beta = p$

$\frac{1}{3}\times\frac{2}{3} = p\ ;\ p = \frac{2}{9}\ ;\ \gamma = ar^2 = \frac{1}{3}\times 4 = \frac{4}{3}$

$\delta = ar^3 = \frac{1}{3}(2^3) = \frac{8}{3}\ ;\ q = \gamma\delta = \frac{32}{3}$

When $r = -2$

$a = \alpha = -1; \beta = ar = -1 \times -2 = 2\ \therefore p = -2$

$\gamma = ar^2 = (-2)^2 \times (-1) = -4, \delta = ar^3 = (-1)(-2)^3 = 8$

$\therefore\ \gamma\delta = -32 = q$

52. (c) : $[\vec{a}\times\vec{b}\ \ \vec{b}\times\vec{c}\ \ \vec{c}\times\vec{a}] = (\vec{a}\times\vec{b})\cdot\{(\vec{b}\times\vec{c})\times(\vec{c}\times\vec{a})\}$

$= (\vec{a}\times\vec{b})\cdot\{(\vec{b}\times\vec{c}\cdot\vec{a})\vec{c} - (\vec{b}\times\vec{c}\cdot\vec{c})\vec{a}\}$

$= (\vec{a}\times\vec{b})\cdot[\vec{a}\ \vec{b}\ \vec{c}]\vec{c} = [\vec{a}\ \vec{b}\ \vec{c}]\cdot[\vec{a}\ \vec{b}\ \vec{c}] = [\vec{a}\ \vec{b}\ \vec{c}]^2$

$\therefore$ On comparison, $\lambda = 1$

53. (d) : Given, ellipse $\frac{x^2}{4}+\frac{y^2}{1}=1$ ...(i) meets the ellipse $\frac{x^2}{1}+\frac{y^2}{a^2}=1$ ...(ii) in four distinct points.

Now ellipse (ii) meets ellipse (i) at four distinct points iff $a > 1$

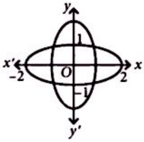
$\therefore\ b^2 - 5b + 7 > 1$

$[\because\ a = b^2 - 5b + 7$ (Given)]

$\Rightarrow\ b^2 - 5b + 6 > 0$

$\Rightarrow\ (b-2)(b-3) > 0$

$\Rightarrow\ b \in (-\infty, 2) \cup (3, \infty)$



54. (a) : We have, $A = \begin{bmatrix} k & l \\ m & n \end{bmatrix}$

$A^2 = \begin{bmatrix} k & l \\ m & n \end{bmatrix}\begin{bmatrix} k & l \\ m & n \end{bmatrix} = \begin{bmatrix} k^2+lm & kl+ln \\ mk+mn & ml+n^2 \end{bmatrix}$

$A^2 - (k+n)A + (kn - lm)I$

$= \begin{bmatrix} k^2+lm & kl+ln \\ mk+mn & ml+n^2 \end{bmatrix} - \begin{bmatrix} k^2+nk & kl+ln \\ km+mn & kn+n^2 \end{bmatrix} + \begin{bmatrix} kn-lm & 0 \\ 0 & kn-lm \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

55. (b) : The given equation is $\frac{dy}{dx} + y\cos x = \frac{\sin 2x}{2}$

Now, $\text{I.F.} = e^{\int \cos x\, dx} = e^{\sin x}$

$\therefore$ The required solution is $y\cdot e^{\sin x} = \frac{1}{2}\int e^{\sin x}\cdot \sin 2x\, dx$

$\Rightarrow\ y\cdot e^{\sin x} = \frac{1}{2}\int e^{\sin x}\cdot 2\sin x\cos x\, dx$

$\Rightarrow\ y e^{\sin x} = \int t\cdot e^t dt$ [Putting $\sin x = t \Rightarrow \cos x\, dx = dt$]

$\Rightarrow\ y e^{\sin x} = e^t(t-1) + C$

$\Rightarrow\ y e^{\sin x} = e^{\sin x}(\sin x - 1) + C$

56. (d) : We have, $f(x) = [x] + x$

Now, $\int_{3/2}^{9/2} f(x)dx = \int_{3/2}^{9/2} [x]dx + \int_{3/2}^{9/2} x\,dx$

$= \int_{3/2}^{2} 1dx + \int_{2}^{3} 2dx + \int_{3}^{4} 3dx + \int_{4}^{9/2} 4dx + \int_{3/2}^{9/2} x\,dx$

$= [x]_{3/2}^{2} + 2[x]_2^3 + 3[x]_3^4 + 4[x]_4^{9/2} + \left[\frac{x^2}{2}\right]_{3/2}^{9/2}$

$= \left(2-\frac{3}{2}\right) + 2(3-2) + 3(4-3) + 4\left(\frac{9}{2}-4\right) + \frac{1}{2}\left[\left(\frac{9}{2}\right)^2 - \left(\frac{3}{2}\right)^2\right]$

$= \frac{1}{2} + 2 + 3 + \frac{4}{2} + \frac{72}{8} = \frac{1}{2} + 16 = 16.5$

57. (d) : (a) Not true, if we take $f(x) = \sin x$, where $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, then both $f(x)$ and $f^{-1}(x)$ are increasing.

(b) Not true, if we take $f(x) = e^x$, then $f(x)$ is increasing but $\frac{1}{f(x)} = e^{-x}$ is decreasing.

(c) Not true, if we take $f(x) = e^x$ and $g(x) = e^{-x}$, then both f and g are positive, also f is increasing and g is decreasing but $\frac{f}{g} = e^{2x}$ is increasing function.

Hence, option (d) is correct.

58. (a) : $\frac{d(f(\tan x))}{d(x)} = f'(\tan x)\sec^2 x$

and $\frac{d(g(\sec x))}{dx} = g'(\sec x)\sec x\cdot\tan x$

Now, $\frac{d(f(\tan x))}{d(g(\sec x))} = \frac{f'(\tan x)\sec^2 x}{g'(\sec x)\sec x\cdot\tan x} = \frac{f'(\tan x)\sec x}{g'(\sec x)\tan x}$

$\therefore\ \left[\frac{d(f(\tan x))}{d(g(\sec x))}\right]_{x=\frac{\pi}{4}} = \frac{f'(1)\cdot\sqrt{2}}{g'(\sqrt{2})\cdot 1} = \frac{2\sqrt{2}}{4\cdot 1}$

$[\because\ f'(1) = 2 \text{ and } g'(\sqrt{2}) = 4]$

$= \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$

59. (a) : We have, $R_1 = \{(x, y) | x^2 + y^2 \le 25\}$

$\Rightarrow \quad -5 \le x \le 5, -5 \le y \le 5$

Range of $R_1 = [-5, 5]$

$$R_2 = \left\{(x, y) \mid y \ge \frac{4x^2}{9}\right\} \Rightarrow \text{Range of } R_2 = [0, \infty)$$

$\therefore \quad$ Range of $R_1 \cap R_2 = [0, 5]$

60. (c) : Given, $x = \sum_{r=1}^{n} (2r-1)$

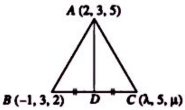
$\Rightarrow \quad x = 1 + 3 + 5 + ... + (2n-1) \Rightarrow x = n^2 \Rightarrow x^2 = n^4$

Now, $\lim_{n\to\infty}\left[\frac{1^3}{x^2}+\frac{2^3}{x^2}+\frac{3^3}{x^2}+.....+\frac{n^3}{x^2}\right]$

$$= \lim_{n\to\infty}\frac{1^3+2^3+3^3+....+n^3}{x^2}$$

$$= \lim_{n\to\infty}\frac{n^2(n+1)^2}{4n^4} = \lim_{n\to\infty}\frac{n^4\left(1+\frac{1}{n}\right)^2}{4n^4} = \frac{1}{4}$$

61. (d) : Let AD is the median which is equally inclined to the axes. So, D is the mid-point of BC.



$$\therefore D \equiv \left(\frac{\lambda-1}{2}, \frac{5+3}{2}, \frac{\mu+2}{2}\right)$$

$$\equiv \left(\frac{\lambda-1}{2}, 4, \frac{\mu+2}{2}\right)$$

Direction ratios of AD are $\left\langle\frac{\lambda-1}{2}-2, 4-3, \frac{\mu+2}{2}-5\right\rangle$

i.e., $\left\langle\frac{\lambda-5}{2}, 1, \frac{\mu-8}{2}\right\rangle$

Since, the line AD is equally inclined to coordinate axes, its direction ratios are $\pm 1; \pm 1; \pm 1$

$\therefore \quad \frac{\lambda-5}{2} = \pm 1$ and $\frac{\mu-8}{2} = \pm 1$

$\Rightarrow \quad \lambda = 7$ or 3 and $\mu = 10$ or 6.

62. (c)

63. (d) : Let plane $3x + 4y - 12z + 13 = 0$ is equidistant from the points $A(1, 1, k)$ and $B(-3, 0, 1)$.

$$\therefore \quad \frac{|3(-3)+4(0)-12(1)+13|}{\sqrt{(3)^2+(4)^2+(-12)^2}} = \frac{|3(1)+4(1)-12(k)+13|}{\sqrt{(3)^2+(4)^2+(-12)^2}}$$

$\Rightarrow \quad |-9 - 12 + 13| = |3 + 4 - 12k + 13|$

$\Rightarrow \quad 8 = \pm(20 - 12k) \Rightarrow k = 1$ or $\frac{7}{3}$

So, the required equation is $3x^2 - 10x + 7 = 0$

64. (b) : $\because E(X) = np = 5$ ...(i)

and $\text{Var}(X) = npq = 2.5$ ...(ii)

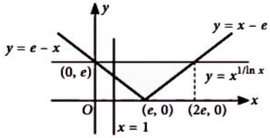
From (i) and (ii), we get $\frac{npq}{np} = \frac{2.5}{5} \Rightarrow q = \frac{1}{2}$

Since, $p + q = 1 \Rightarrow q = 1 - \frac{1}{2} = \frac{1}{2}$

$\therefore \quad n = \frac{5}{1} \times 2 = 10$ (From (i))

$$\Rightarrow P(X < 1) = P(X = 0) = {}^{10}C_0\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^{10} = \left(\frac{1}{2}\right)^{10}$$

65. (b) : We have, $y = \begin{cases} x^{1/\ln x}, & x \ne 1 \\ e, & x = 1 \end{cases}$ and $y = |x - e|$



$$\therefore \quad \text{Required area} = (2e \times e) - \frac{1}{2}(e \times e) - \frac{1}{2}(e \times e)$$

$$= 2e^2 - \frac{e^2}{2} - \frac{e^2}{2} = e^2$$

66. (c) : The number of permutations for letters A, E, O $= \frac{5!}{3!}$ and the number of permutations for remaining letters $= \frac{4!}{2!}$

$\therefore$ Required number of permutations $= \frac{5!}{3!} \times \frac{4!}{2!} = 2 \times 5!$

67. (a, d) : If A and B are equivalence relations, then $A \cap B$ and $A^{-1} \cap B^{-1}$ are equivalence relations.

68. (c, d) : Given, $X^T = -X$, $Y^T = -Y$, $Z^T = Z$

(a) $(Y^3Z^4 - Z^4Y^3)^T = (Y^3Z^4)^T - (Z^4Y^3)^T$
$= -Z^4Y^3 + Y^3Z^4 = Y^3Z^4 - Z^4Y^3$

Hence, symmetric.

(b) $(X^{44} + Y^{44})^T = X^{44} + Y^{44}$, so symmetric.

(c) $(X^4Z^3 - Z^3X^4)^T = (X^4Z^3)^T - (Z^3X^4)^T = Z^3X^4 - X^4Z^3$
$= -(X^4Z^3 - Z^3X^4)$

Hence, skew-symmetric.

(d) $(X^{23} + Y^{23})^T = -X^{23} - Y^{23} = -(X^{23} + Y^{23})$

Hence, skew-symmetric.

69. (b, c) : $D = \begin{vmatrix} 1 & 1 & 1 \\ \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ \alpha & \beta-\alpha & \gamma-\alpha \\ \alpha^2 & \beta^2-\alpha^2 & \gamma^2-\alpha^2 \end{vmatrix}$

(Applying $C_2 \to C_2 - C_1$, $C_3 \to C_3 - C_1$)

Expand along R_1

$$= (\beta-\alpha)(\gamma-\alpha)\begin{vmatrix} 1 & 1 \\ \beta+\alpha & \gamma+\alpha \end{vmatrix} = (\beta-\alpha)(\gamma-\alpha)(\gamma-\beta)$$

$= (\alpha - \beta)(\beta - \gamma)(\gamma - \alpha)$

$\because$ $D = 0 \Rightarrow$ Trivial as well as non-trivial solution and so the number of solutions will be infinite.

$\because$ $D \neq 0$ $\Rightarrow$ system has only trivial solution.

$\therefore$ (a) is incorrect. (b) is correct.

(c) is correct as trivial solution (*i.e.* a unique solution) exists only if α, β, γ are distinct implying $D \neq 0$.

(d) is incorrect.

70. (d) : Given, equation of plane is $2x + 6y - 3z = 63$

$\therefore$ Direction ratios of the normal to the plane are $\langle 2, 6, -3 \rangle$.

$\therefore$ Direction cosines are $l = \dfrac{2}{\sqrt{2^2 + 6^2 + (-3)^2}} = \dfrac{2}{7}$,

$m = \dfrac{6}{7}$ and $n = \dfrac{-3}{7}$

The normal form of the plane is $\dfrac{2}{7}x + \dfrac{6}{7}y - \dfrac{3}{7}z = \dfrac{63}{7} = 9$

$\therefore$ Coordinates of the foot of perpendicular are

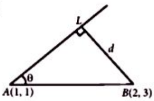
$$\left(\left(\frac{2}{7}\right)\times 9, \left(\frac{6}{7}\right)\times 9, \left(\frac{-3}{7}\right)\times 9\right) = \left(\frac{18}{7}, \frac{54}{7}, \frac{-27}{7}\right)$$

71. (a, b) : The family of lines

$(3 + 2\lambda)x + (4 + 3\lambda)y - 7 - 5\lambda = 0$

$\Rightarrow 3x + 4y - 7 + \lambda(2x + 3y - 5) = 0$

The lines are concurrent at the point given by $3x + 4y - 7 = 0$, $2x + 3y - 5 = 0$ which is $(1, 1)$.



Let d be the distance of $B(2, 3)$ from a line through $A(1, 1)$.

Now, $d = AB \sin\theta$

$d = 0$, minimum when L is along AB.

$\therefore$ $L_1 : \dfrac{y - 1}{x - 1} = 2 \Rightarrow 2x - y - 1 = 0$

$d = AB$, maximum when L is perpendicular to AB.

$\therefore$ $L_2 : x + 2y - 3 = 0$

The line which is equally inclined to L_1 and L_2 is along a bisector of angle between L_1 and L_2,

$$\frac{2x - y - 1}{\sqrt{5}} = \pm\frac{(x + 2y - 3)}{\sqrt{5}}$$

or $x - 3y + 2 = 0$ and $3x + y - 4 = 0$

Lines passing through the point $(1, 2)$ and parallel to the above two lines are $x - 3y + 5 = 0$ and $3x + y - 5 = 0$.

72. (a, d) : Let t_1 and t_2 be parameters of P and Q respectively. Since OP and OQ are perpendicular

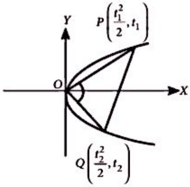
$\therefore$ Product of slopes $= -1 \Rightarrow t_1 t_2 = -4$

Again, area of $\Delta OPQ = 3\sqrt{2}$

$$\Rightarrow \frac{1}{4}\left|t_1^2 t_2 - t_1 t_2^2\right| = 3\sqrt{2} \Rightarrow |t_1 t_2||t_1 - t_2| = 12\sqrt{2}$$

$$\Rightarrow |t_1 - t_2| = 3\sqrt{2}$$

$$\Rightarrow \left|t_1 + \frac{4}{t_1}\right| = 3\sqrt{2}$$



$$\therefore\ t_1 + \frac{4}{t_1} = 3\sqrt{2}\ (\because t_1 > 0)$$

$$\Rightarrow t_1^2 - 3\sqrt{2}t_1 + 4 = 0$$

$$\therefore\ t_1 = 2\sqrt{2}, \sqrt{2}$$

Thus, $P \equiv (4, 2\sqrt{2})$ or $(1, \sqrt{2})$

73. (a) : Let $P = 3x + 4y$ ...(i)

Given, $x^2 + y^2 = 25 \Rightarrow y = \sqrt{25 - x^2}$

$\therefore$ $P = 3x + 4\sqrt{25 - x^2}$

$$\frac{dP}{dx} = 3 + \frac{4(-2x)}{2\sqrt{25 - x^2}} = 3 - \frac{4x}{\sqrt{25 - x^2}}$$

For critical points, $\dfrac{dP}{dx} = 0 \Rightarrow x = 3$ $\because$ $\left(\dfrac{d^2P}{dx^2}\right)_{x=3} < 0$

So, P is maximum when $x = 3$ and $y = 4$

$\therefore$ Maximum value of $P = 3 \times 3 + 4 \times 4 = 25$

$\therefore$ $\log_5[\text{Max}(3x + 4y)] = \log_5(5^2) = 2$

74. (a, d) : Let $\lim\limits_{x\to a} f(x) = p$ and $\lim\limits_{x\to a} g(x) = q$

According to the question, $pq = 9$ and $p/q = 4$

$\therefore$ $p^2 = 36 \Rightarrow p = -6$ or 6

If $p = 6$, then $q = \dfrac{3}{2}$ $\therefore p + q = \dfrac{15}{2}$

and if $p = -6$, then $q = \dfrac{-3}{2}$ $\therefore p + q = \dfrac{-15}{2}$

75. (a, b, d) : Volume $= \left|[2\vec{b}\times\vec{c}\ \ 3\vec{c}\times\vec{a}\ \ 4\vec{a}\times\vec{b}]\right| = 18$

$$\Rightarrow 24[\vec{a}\,\vec{b}\,\vec{c}]^2 = 18 \Rightarrow \left|[\vec{a}\,\vec{b}\,\vec{c}]\right| = \frac{\sqrt{3}}{2}$$

Now,

$$[\vec{a}\,\vec{b}\,\vec{c}] = \begin{vmatrix} (1+\sin\theta) & \cos\theta & \sin 2\theta \\ \sin\left(\theta + \frac{2\pi}{3}\right) & \cos\left(\theta + \frac{2\pi}{3}\right) & \sin\left(2\theta + \frac{4\pi}{3}\right) \\ \sin\left(\theta - \frac{2\pi}{3}\right) & \cos\left(\theta - \frac{2\pi}{3}\right) & \sin\left(2\theta - \frac{4\pi}{3}\right) \end{vmatrix}$$

Applying $R_1 \to R_1 + R_2 + R_3$ and expanding, we get

$$\left|[\vec{a}\,\vec{b}\,\vec{c}]\right| = \sqrt{3}|\cos 3\theta| = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \cos 3\theta = \pm\frac{1}{2} \Rightarrow 3\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3} \Rightarrow \theta = \frac{\pi}{9}, \frac{2\pi}{9}, \frac{4\pi}{9}.$$

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Challenging PROBLEMS For JEE

ON

Coordinate Geometry

SINGLE OPTION CORRECT TYPE

1. If P be a point inside an equilateral ΔABC such that $PA = 3$, $PB = 4$ and $PC = 5$, then the side length of the equilateral ΔABC is
   (a) $\sqrt{25-12\sqrt{3}}$ (b) 13
   (c) $\sqrt{25+12\sqrt{3}}$ (d) 17

2. In triangle ABC, equation of the side BC is $x - y = 0$. Circumcentre and orthocenter of the triangle are (2, 3) and (5, 8) respectively. Equation of the circumcircle of the triangle is
   (a) $x^2 + y^2 - 4x + 6y - 27 = 0$
   (b) $x^2 + y^2 - 4x - 6y - 27 = 0$
   (c) $x^2 + y^2 + 4x + 6y - 27 = 0$
   (d) $x^2 + y^2 + 4x - 6y - 27 = 0$

3. Circles C_1 and C_2 having centres G_1 and G_2 respectively intersect each other at the points A and B, secants L_1 and L_2 are drawn to the circles C_1 and C_2 to intersect them in the points A_1, B_1 and A_2, B_2 respectively. If the secants L_1 and L_2 intersect each other at a point P in the exterior region of circles C_1 and C_2 and $PA_1 \times PB_1 = PA_2 \times PB_2$, then which of the following statement is false?
   (a) Points P, A and B are collinear
   (b) Line joining G_1 and G_2 is perpendicular to line joining P and A
   (c) $PA_1 \times PB_1 = PA \times PB$
   (d) $PA = PA_1$

4. The locus of the centre of a circle which cuts orthogonally the parabola $y^2 = 4x$ at (1, 2) will pass through points
   (a) (3, 4) (b) (4, 3) (c) (5, 3) (d) (2, 4)

5. Let AB be any chord of the circle $x^2 + y^2 - 4x - 4y + 4 = 0$ which subtends an angle of 90° at the point (2, 3), then the locus of the midpoint of AB is circle whose centre is
   (a) (1, 5) (b) $\left(1, \frac{5}{2}\right)$ (c) $\left(1, \frac{3}{2}\right)$ (d) $\left(2, \frac{5}{2}\right)$

6. If a pair of variable straight lines $x^2 + 4y^2 + \alpha xy = 0$ (where α is a real parameter) cut the ellipse $x^2 + 4y^2 = 4$ at two points A and B, then locus of point of intersection of tangents at A and B is
   (a) $x^2 - 4y^2 + 8xy = 0$ (b) $(2x - y)(2x + y) = 0$
   (c) $x^2 - 4y^2 + 4xy = 0$ (d) $(x - 2y)(x + 2y) = 0$

7. If directions of two sides of a triangle are fixed and length of third side is constant and is sliding between these sides, then locus of the orthocenter of the triangle is
   (a) circle (b) ellipse
   (c) straight line (d) hyperbola

8. If an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ has least area but contain the circle $(x - 1)^2 + y^2 = 1$, then
   (a) equation of ellipse is $2x^2 + 6y^2 = 9$
   (b) equation of ellipse is $6x^2 + 2y^2 = 9$
   (c) eccentricity of ellipse is $2e=\sqrt{\frac{2}{3}}$
   (d) eccentricity of ellipse is $e = 1/2$

9. Consider two concentric circle $C_1 : x^2 + y^2 = 1$ and $C_2 : x^2 + y^2 - 4 = 0$. A parabola is drawn through the points where C_1 meet the x-axis and having arbitrary tangent of C_2 as its directrix. If locus of focus of drawn parabola is $\frac{3}{4}x^2+y^2=k$, then value of k is
   (a) 3 (b) 7 (c) 8 (d) 4

10. All chords of the curve $3x^2 - y^2 - 2x + 4y = 0$ that subtend a right angle at the origin, pass through a fixed point (h, k), then $h - k$ is equal to
   (a) 1 (b) 2 (c) 3 (d) 4

By : Tapas Kr. Yogi, Visakhapatnam Mob : 09533632105

11. How many tangents to the circle $x^2 + y^2 = 3$ are there which are normal to the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$?

(a) 3 (b) 2 (c) 1 (d) 0

12. If the ellipse $\frac{x^2}{a^2-3}+\frac{y^2}{a+4}=1$ is inscribed in a square of side length $a\sqrt{2}$, then a is equal to

(a) 4 (b) 2
(c) 1 (d) None of these

13. The points of intersection of the two ellipse $x^2+2y^2-6x-12y+23=0$ and $4x^2+2y^2-20x-12y+35=0$

(a) lie on a circle centered at $\left(\frac{8}{3},3\right)$ and of radius $\frac{1}{3}\sqrt{\frac{47}{2}}$.

(b) lie on a circle centered at $\left(\frac{8}{3},-3\right)$ and of radius $\frac{1}{3}\sqrt{\frac{47}{3}}$.

(c) lie on a circle centered at (8, 9) and of radius $\frac{1}{3}\sqrt{\frac{47}{2}}$.

(d) are not concyclic.

14. If the chord of contact of tangents from 3 points A, B and C to the circle $x^2 + y^2 = a^2$ are concurrent, then A, B and C will

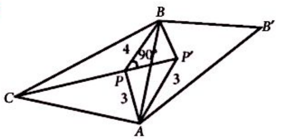
(a) be concyclic (b) be collinear
(c) form the vertices of triangle
(d) None of these

15. Let $ABCD$ be a quadrilateral with area 18, with side AB parallel to CD and $AB = 2CD$. Let AD be perpendicular to AB and CD. If a circle is drawn inside the quadrilateral $ABCD$ touching all the sides, then its radius is

(a) 3 (b) 2 (c) 3/2 (d) 1

SOLUTIONS

1. (c) :



Rotate the triangle in clockwise direction through an angle 60°.

Let the points A, B, C and P will be A, B', B and P' respectively after the rotation.

We have, $PA = P'A = 3$ and $\angle PAP' = 60° \Rightarrow PP' = 3$.

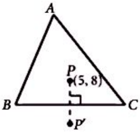
Also, $CP = BP' = 5$.

So $\Delta BPP'$ is right angle triangle with $\angle BPP' = 90°$.

Now apply cosine rule in ΔBPA, where $\angle BPA = 90° + 60° = 150°$, $PA = 3$ and $BP = 4$

we get $AB = \sqrt{25+12\sqrt{3}}$

2. (b) : Let coordinates of P be (5, 8). Reflection of P in BC will lie on circumcircle.



$\therefore$ Equation of circumcircle is

$(x-2)^2 + (y-3)^2 = (8-2)^2 + (5-3)^2$

$\Rightarrow x^2 + y^2 - 4x - 6y - 27 = 0$

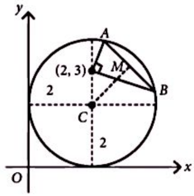
3. (d) : Line joining PAB will be the radical axis of the two circles. So options, (a), (b) and (c) are correct statements.

4. (a) : Tangent to parabola $y^2 = 4x$ at (1, 2) will be the locus.

i.e., $y \cdot 2 = 2(x + 1) \Rightarrow y = x + 1$

5. (d) : Let midpoint of AB is $M(h, k)$.

AB subtends angle 90° at (2, 3).



$\Rightarrow AM = MB$

$= \sqrt{(h-2)^2+(k-3)^2}$

Also, $CM^2 + MB^2 = CB^2$

$\Rightarrow (h-2)^2 + (k-2)^2 + (h-2)^2 + (k-3)^2 = 4$

$\Rightarrow x^2+y^2-4x-5y+\frac{17}{2}=0$

6. (d) : Let the point of intersection of tangents at A and B be $P(h, k)$, then equation of AB is

$$\frac{xh}{4}+\frac{yk}{1}=1 \quad \text{...(i)}$$

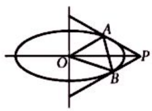
Homogenizing the ellipse with (i), we get

$$\frac{x^2}{4}+\frac{y^2}{1}=\left(\frac{xh}{4}+\frac{yk}{1}\right)^2$$

$$\Rightarrow x^2\left(\frac{h^2-4}{16}\right)+y^2(k^2-1)+\frac{2hk}{4}xy=0 \quad \text{...(ii)}$$

Given, equation of OA and OB is

$x^2 + 4y^2 + \alpha xy = 0$ ...(iii)



Since, (ii) and (iii) are same

$\therefore (h - 2k)(h + 2k) = 0$

Thus, required locus is

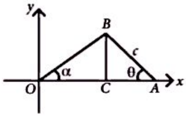
$(x - 2y)(x + 2y) = 0$

7. (a) : Let fixed directions be OA and OB inclined at a constant angle α and $AB = c$.

Let $\angle BAO = \theta$, then

$BC = c\sin\theta$ and $AC = c\cos\theta$.

$\therefore \quad OC = c\sin\theta \cdot \cot\alpha$



Equation of the line passing through point A and perpendicular to OB is

$y = -\cot\alpha(x - c\sin\theta\cot\alpha - c\cos\theta)$ and equation of BC is $x = c\sin\theta \cdot \cot\alpha$

Orthocenter is $(c\sin\theta \cdot \cot\theta, c\cos\theta \cdot \cot\alpha)$

$\therefore$ Required locus is $x^2 + y^2 = c^2\cot^2\alpha$, which is the equation of a circle.

8. (a) : On solving given conditions, we get

$(b^2 - a^2)x^2 + 2a^2x - a^2b^2 = 0$

Now, $D = 0 \Rightarrow a^2 - a^2e^2b^2 = 0 \Rightarrow b = 1/e$

Also, $a^2 = \dfrac{b^2}{1-e^2} \Rightarrow a^2 = \dfrac{1}{e^2(1-e^2)} \Rightarrow a = \dfrac{1}{e\sqrt{1-e^2}}$

Now, $s = \pi ab = \dfrac{\pi}{e^2\sqrt{1-e^2}}$

$$\Rightarrow \frac{ds}{de} = \pi\left(\frac{e(3e^2-2)}{e^4(1-e^2)^{3/2}}\right) \qquad [\because\ 0 < e < 1]$$

$\Rightarrow$ s is least, when $e = \sqrt{\dfrac{2}{3}}$

$\therefore$ Equation of ellipse is $2x^2 + 6y^2 = 9$

9. (a) : $(h-1)^2 + k^2 = (\cos\theta - 2)^2$ ... (i)

$(h+1)^2 + k^2 = (\cos\theta + 2)^2$ ... (ii)

(ii) – (i) $\Rightarrow \cos\theta = \dfrac{h}{2}$

(ii) + (i) $\Rightarrow 2(h^2 + k^2 + 1) = 2(\cos^2\theta + 4) \Rightarrow \dfrac{3}{4}x^2 + y^2 = 3$

10. (c) : Let the equation of the chord is $y = mx + c$.

Combined equation of the line joining the point of intersection with origin is

$$3x^2 - y^2 - 2(x - 2y)\left(\frac{y - mx}{c}\right) = 0$$

$\Rightarrow x^2(3c + 2m) - y^2(c - 4) - 2xy(1 + 2m) = 0$

From the condition of perpendicularity, we get

$3c + 2m - c + 4 = 0 \Rightarrow m + c = -2$

$\therefore$ The line $y = mx + c$, passes through $(1, -2)$.

11. (d) : Equation of normal at $P(3\cos\theta, 2\sin\theta)$ is

$3x\sec\theta - 2y\,\mathrm{cosec}\,\theta = 5$

$$\therefore \frac{5}{\sqrt{9\sec^2\theta + 4\,\mathrm{cosec}^2\theta}} = \sqrt{3}$$

But minimum of $9\sec^2\theta + 4\,\mathrm{cosec}^2\theta = 25$

$\therefore$ No such θ exists.

12. (d) : Sides of the square will be perpendicular tangents to the ellipse. So, vertices of the square will lie on the director circle. So, diameter of director circle is

$$2\sqrt{(a^2 - 3) + (a + 4)} = \sqrt{2a^2 + 2a^2}$$

$$\Rightarrow 2\sqrt{a^2 + a + 1} = 2a \Rightarrow a = -1$$

But for ellipse $a^2 > 3$ and $a > -4$

So, a cannot take the value -1.

13. (a) : If $S_1 = 0$ and $S_2 = 0$ are the equations, then $\lambda S_1 + S_2 = 0$ is a second degree curve passing through the points of intersection of $S_1 = 0$ and $S_2 = 0$.

$$\Rightarrow (\lambda + 4)x^2 + 2(\lambda + 1)y^2 - 2(3\lambda + 10)x - 12(\lambda + 1)y + (23\lambda + 35) = 0 \quad \text{...(i)}$$

For it to be a circle, choose λ such that the coefficients of x^2 and y^2 are equal.

$\therefore \lambda = 2$, which gives the equation of the circle as

$6(x^2 + y^2) - 32x - 36y + 81 = 0$ [Using (i)]

$\Rightarrow x^2 + y^2 - \dfrac{16}{3}x - 6y + \dfrac{27}{2} = 0,$

whose centre is $C\left(\dfrac{8}{3}, 3\right)$ and radius is

$$r = \sqrt{\frac{64}{9} + 9 - \frac{27}{2}} = \frac{1}{3}\sqrt{\frac{47}{2}}$$

14. (b) : We have, $xx_1 + yy_1 = a^2$, $xx_2 + yy_2 = a^2$ and $xx_3 + yy_3 = a^2$

These lines will be concurrent, iff

$$\begin{vmatrix} x_1 & y_1 & -a^2 \\ x_2 & y_2 & -a^2 \\ x_3 & y_3 & -a^2 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0,$$

which is the condition to the collinearity of points A, B and C.

15. (b) : $A(0, 0)$; $B(2a, 0)$; $C(a, 2r)$; $D(0, 2r)$

Area of quadrilateral $ABCD = 1/2(2a + a) \times 2r = 18$

$\Rightarrow ar = 6$ ...(i)

Also, $a^2 + (2r)^2 = [(2a - r) + (a - r)]^2 \Rightarrow 8a^2 = 12ar$

$\Rightarrow a = (3/2)r$ ...(ii)

From (i) and (ii), we get $(3/2)r^2 = 6 \Rightarrow r = 2$

QUANTITATIVE APTITUDE

For Various Competitive Exams

1. The square root of $\frac{(0.96)^3-(0.1)^3}{(0.96)^2+0.096+(0.1)^2}$ equals

(a) $\frac{\sqrt{77}}{10}$ (b) $\frac{\sqrt{86}}{10}$

(c) $\frac{\sqrt{87}}{10}$ (d) None of these

2. $2\frac{1}{2}$ is added to a number. The sum is multiplied by $4\frac{1}{2}$. Then 3 is added to the product and then dividing the sum by $1\frac{1}{5}$, the quotient becomes 25. What is the number?

(a) $5\frac{1}{2}$ (b) $7\frac{1}{2}$ (c) $6\frac{1}{2}$ (d) $3\frac{1}{2}$

3. Due to 25% increase in the price of rice per kg, a person is able to purchase 20 kg less for ₹ 400. What is the increased price of rice?

(a) ₹ 6 (b) ₹ 5 (c) ₹ 4 (d) ₹ 7

4. A man gain 10% by selling an article for a certain price. If he sells it at double of that price, then profit made is

(a) 120% (b) 12%

(c) 1.2% (d) None of these

5. What is the mean proportional of $(15+\sqrt{200})$ and $(27-\sqrt{648})$?

(a) $3\sqrt{5}$ (b) $5\sqrt{3}$ (c) $7\sqrt{5}$ (d) $5\sqrt{7}$

6. 10 boys and 15 girls together can complete the work in 6 days. It takes 100 days for one boy alone to complete the same work. How many days will be required for one girl alone to complete the same work?

(a) 90 days (b) 145 days

(c) 150 days (d) 225 days

7. *A*, *B* and *C* together finish the work in 8 days and received ₹ 640. *A* and *C* together complete the same work in 5 days and received ₹ 250, *B* and *C* do the same in 6 days and received ₹ 420, what is the daily wages of *C*?

(a) ₹ 40 (b) ₹ 50 (c) ₹ 70 (d) ₹ 80

8. Pipe *A* can fill an empty reservoir in 6 hours and pipe *B* in 9 hours. If both the pipes are opened and after 3 hours the pipe *B* is closed, how much time pipe *A* will take to fill the remaining part of reservoir?

(a) 3 hours (b) 2 hours

(c) 1 hour (d) None of these

9. A train of length 150 m takes 10 s to cross another train 100 m long coming from opposite direction. If speed of first train is 30 km/h. What is the speed of second train?

(a) 60 km/h (b) 55 km/h (c) 50 km/h (d) 45 km/h

10. There are two vessels A and B. Vessel A is containing 40 L of pure milk and vessel B is containing 22 litres of pure water. From vessel A, 8 L of milk is taken out and poured into the vessel B. Then 6 L of mixture (water and milk) from vessel B is taken out and poured into the vessel A. What is the respective ratio of quantity of pure milk in vessel A and quantity of pure water in vessel B?

(a) 21 : 13 (b) 14 : 5 (c) 21 : 11 (d) 24 : 13

11. Satish invested an amount of ₹ 8000 at 10% p.a. and another amount at 20% p.a. At the end of one year, he gets 12% rate of interest per annum for total amount. Find the total amount invested.

(a) ₹ 15000 (b) ₹ 12000 (c) ₹ 10000 (d) ₹ 9000

12. Ranjeet makes a deposit of ₹ 100,000 in Punjab National Bank for a period of 2 years. If the rate of interest is 12% p.a. compounded half yearly, what will be the maturity value of the money deposited by him?

(a) ₹ 122247.89 (b) ₹ 122436.89

(c) ₹ 126247.69 (d) ₹ 122436.79

13. If $5^{3-6x}=2^{6x-3}$, then find the value of *x*.

(a) 2 (b) –1/2 (c) 1/2 (d) 3

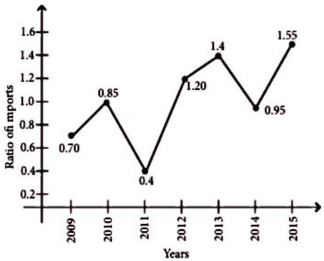
14. If the perimeters of a rectangle and a square are equal and the ratio of two adjacent sides of the rectangle is 1 : 2, then the ratio of area of the rectangle to that of the square is

(a) 1 : 2 (b) 3 : 2 (c) 8 : 9 (d) 1 : 4

15. A frustum of a right circular cone has radii of base and top are 12 cm and 7 cm respectively. If the height of frustum is 12 cm, then total surface area and volume respectively are
(a) 240π cm^2 and 1180 cm^3
(b) 480π cm^2 and 1080 cm^3
(c) 880π cm^2 and 2216 cm^3
(d) 440π cm^2 and 1108 πcm^3

Direction (Q. 16-20) : The following line graph gives the ratio of the amounts of imports by a company to the amount of exports from that company over the period from 2009 to 2015. Study the graph carefully and answer the questions given below.

Ratio of values of imports to exports by a company over the years.



16. The number of years in which exports was more than the imports is
(a) 4 (b) 3 (c) 2 (d) 1

17. What was the percent increase in import from 2012 to 2013?
(a) 80 (b) Data insufficient
(c) 45 (d) None of these

18. If imports of the company in 2010 was ₹ 289 crores, the exports from the company in the same year was
(a) ₹ 240 crores (b) ₹ 300 crores
(c) ₹ 340 crores (d) ₹ 380 crores

19. The imports were minimum proportionate to the exports of the company in the year
(a) 2014 (b) 2013 (c) 2012 (d) 2011

20. If the imports in 2009 was 161 crores and the total exports in 2009 and 2010 together was 245 crores, then the imports in 2010 was
(a) 12.75 crores (b) 15 crores
(c) 20.25 crores (d) 25 crores

SOLUTIONS

1. **(b) :** Let $a = 0.96$, $b = 0.1$

$\therefore$ Given expression $= \dfrac{a^3 - b^3}{a^2 + ab + b^2}$

$$= \frac{(a-b)(a^2+ab+b^2)}{a^2+ab+b^2} = a - b$$

$$\therefore \sqrt{\frac{(0.96)^3-(0.1)^3}{(0.96)^2+(0.96\times0.1)+(0.1)^2}} = \sqrt{0.86} = \frac{\sqrt{86}}{10}$$

2. **(d) :** Let the number be x.

According to the question, we have

$$\frac{\left(4\frac{1}{2}\right)\left(x+2\frac{1}{2}\right)+3}{1\frac{1}{5}} = 25 \Rightarrow \frac{\frac{9}{2}\left(x+\frac{5}{2}\right)+3}{\frac{6}{5}} = 25$$

$$\Rightarrow \frac{9}{2}\left(x+\frac{5}{2}\right) = 30 - 3 \Rightarrow x + \frac{5}{2} = \frac{27\times 2}{9}$$

$$\Rightarrow x = 6 - \frac{5}{2} = \frac{7}{2} = 3\frac{1}{2}$$

3. **(b) :** Let the original price of rice per kg be ₹ x.

$\therefore$ Amount of rice bought with ₹ 400 $= \dfrac{400}{x}$ kg

Now, increase in price is 25%.

$\therefore$ Price after increase $= \dfrac{₹\,125x}{100} = ₹\,\dfrac{5}{4}x$

$\therefore$ Amount of rice bought with ₹ 400

$$= \frac{400}{\frac{5}{4}x}\text{ kg} = \frac{1600}{5x}\text{ kg}$$

According to the question, we have

$$\frac{400}{x} - \frac{1600}{5x} = 20$$

$$\Rightarrow \frac{2000-1600}{20\times 5} = x \Rightarrow x = 4$$

$\therefore$ Increased price $= ₹\,\dfrac{5}{4}\times x = ₹\,\dfrac{5}{4}\times 4 = ₹\,5$

4. **(a) :** Let C.P. be ₹ x.

$\therefore$ S.P. = 110% of C.P. $= \dfrac{110\times ₹x}{100}$

Making the S.P. double, we have

$$\text{S.P.} = \frac{2\times110\times ₹x}{100} = ₹\,\frac{220}{100}x$$

$$\therefore \text{Profit} = ₹\left(\frac{220}{100}x - x\right) = ₹\,\frac{120}{100}x$$

$\therefore$ % Profit $= \frac{\text{Profit}}{\text{C.P.}} \times 100 = \frac{120x}{100} \times \frac{1}{x} \times 100 = 120\%$

5. **(a) :** We have, $15 + \sqrt{200} = 15 + 10\sqrt{2} = (\sqrt{10} + \sqrt{5})^2$

and $27 - \sqrt{648} = 27 - 18\sqrt{2} = 9(3 - 2\sqrt{2}) = 9(\sqrt{2} - 1)^2$

$\therefore$ Mean proportional between $(15 + \sqrt{200})$ and $(27 - \sqrt{648})$ is

$\sqrt{(15+\sqrt{200})(27-\sqrt{648})} = \sqrt{9(\sqrt{10}+\sqrt{5})^2(\sqrt{2}-1)^2}$

$\Rightarrow 3(\sqrt{10}+\sqrt{5})(\sqrt{2}-1) = 3(\sqrt{20}+\sqrt{10}-\sqrt{10}-\sqrt{5})$

$\Rightarrow 3(2\sqrt{5}-\sqrt{5}) = 3\sqrt{5}$

6. **(d) :** From the question, we have

1 boy's one day's work $= \frac{1}{100}$

1 boy's six day's work $= 6 \times \frac{1}{100} = \frac{3}{50}$

$\therefore$ 10 boy's six day's work $= \frac{3}{50} \times 10 = \frac{3}{5}$

$\therefore$ Remaining work $= 1 - \frac{3}{5} = \frac{2}{5}$

Now, the remaining $\frac{2}{5}$ of the work is done by 15 girls in 6 days

$\therefore$ Complete work is done by 15 girls in $\frac{6 \times 5}{2}$ = 15 days

$\therefore$ One girl alone will finish the work in 15 × 15 = 225 days

7. **(a) :** $(A + B + C)$'s one day wages

$= ₹\left(\frac{640}{8}\right) = ₹\,80$ ...(i)

Also, $(B + C)$'s one day wages

$= ₹\left(\frac{420}{6}\right) = ₹\,70$ ..(ii)

and $(A + C)$'s one day wages

$= ₹\left(\frac{250}{5}\right) = ₹\,50$ ...(iii)

From (i) and (ii), we get

A's 1 day wages = ₹(80 – 70) = ₹ 10

From (i) and (iii), we get

B's 1 day wages = ₹(80 – 50) = ₹ 30

$\therefore$ C's 1 day wages = ₹(80 – 30 – 10) = ₹ 40

8. **(c) :** Pipe A and B can fill the reservoir in 6 and 9 hours respectively.

$\therefore$ Part of reservoir filled by A and B together in 1 hour

$= \frac{1}{6} + \frac{1}{9} = \frac{5}{18}$

$\therefore$ Part of reservoir is filled by both pipes A and B in 3 hours $= 3 \times \frac{5}{18} = \frac{5}{6}$

$\therefore$ Empty part $= 1 - \frac{5}{6} = \frac{1}{6}$ which is to be filled by pipe A.

Now, A can fill the reservoir in 6 hours.

$\therefore$ Remaining $\frac{1}{6}$ part of the reservoir can be filled by A in $\left(\frac{1}{6} \times 6\right)$ hours *i.e.* 1 hour

9. **(a) :** Speed of I train = 30 km/h

$= \frac{30 \times 5}{18}$ m/s $= \frac{25}{3}$ m/s.

Total length of both the trains = 250 m

Let speed of II train be x m/s.

Total time $= \frac{\text{Total distance}}{\text{Speed of I train + Speed of II train}}$

$\Rightarrow 10 = \frac{250}{\frac{25}{3} + x} \Rightarrow \frac{250 \times 3}{3x + 25} = 10$

$\Rightarrow 3x + 25 = 75 \Rightarrow x = \frac{50}{3}$

$\therefore$ Speed of II train $= \frac{50}{3} \times \frac{18}{5}$ km/h $= 60$ km/h

10. **(c) :** Quantity of pure milk in vessel A = 40 L

On taking out 8 L of pure milk from vessel A and taking out 6 L of mixture (milk and water) from vessel B and mixing in vessel A, then quantity of pure milk in the vessel A

$\Rightarrow (40 - 8) + \frac{8}{22+8} \times 6 = 32 + \frac{8}{30} \times 6 = 32 + \frac{8}{5} = \frac{168}{5}$ L

Similarly, on taking out 6 L mixture from vessel B then quantity of pure water in vessel B left

$\Rightarrow 22 - \frac{22}{(22+8)} \times 6 = 22 - \frac{22}{5} = \frac{88}{5}$ L

$\therefore$ Required ratio of pure milk in vessel A and pure water in vessel B

$\Rightarrow \frac{168}{5} : \frac{88}{5} = 168 : 88 = 21 : 11.$

11. **(c) :** Case I : Principal (sum invested) = ₹ 8000, r = 10% p.a.

$\therefore$ $SI_1 = \frac{8000 \times 10 \times 1}{100} = ₹\,800$

Case II : Let sum invested be ₹ P, $r = 20\%$ p.a.

$\therefore\ SI_2 = \frac{P \times 20 \times 1}{100}$

$\therefore$ Total interest = Interest from Case I + Interest from Case II

$= SI_1 + SI_2 = ₹\left(800 + \frac{20P}{100}\right)$ ...(i)

Again, total sum invested = $(8000 + P)$

$\therefore$ SI at 12% p.a. $= \frac{(8000+P)\times 12}{100}$ ...(ii)

From (i) and (ii), we have

$800 + \frac{20P}{100} = \frac{(8000+P)\times 12}{100}$

$\Rightarrow (20 - 12)P = 96000 - 80000 \Rightarrow P = ₹\,2000$

$\therefore$ Total amount invested = ₹(8000 + 2000) = ₹ 10000

12. (c) : $P = ₹\,100{,}000$,

$n = 2 \times 2$ half years = 4 half year and

$r = \frac{12}{2}\% = 6\%$ half yearly

$\therefore\ A = P\left(1+\frac{r}{100}\right)^n = 100000 \times \left(1 + \frac{6}{100}\right)^4$

$= 100000 \times \left(\frac{53}{50}\right)^4 = ₹\,126247.69$

$\therefore$ Required maturity amount = ₹ 126247.69

13. (c) : $5^{3-6x} = 2^{6x-3} = 2^{-(3-6x)}$

$\Rightarrow (3-6x)\log 5 = -(3-6x)\log 2 \Rightarrow (3-6x)(\log 5 + \log 2) = 0$

$\Rightarrow (3-6x)\log(5\times 2) = 0 \Rightarrow (3-6x)\log 10 = 0$

$\Rightarrow 3 - 6x = 0 \Rightarrow x = 1/2$

14. (c) : Let length of rectangle be $2l$ units and breadth be l units & let the side of square be a units.

$\therefore$ According to problem, we have

$4 \times$ side of square $= 2(l + 2l) \Rightarrow 4a = 6l \Rightarrow l = \frac{2}{3}a$

Now, area of rectangle $= 2l \times l = 2l^2$

$= 2\left(\frac{2}{3}a\right)^2 = \frac{8}{9}a^2$ sq.units

and area of square of side 'a' $= a \times a = a^2$ sq. units

$\therefore$ Required ratio $= \frac{8}{9}a^2 : a^2 = 8:9$

15. (d) : Radius of base of frustum = 12 cm = (R)

Radius of top of frustum = 7 cm = (r)

Height = 12 cm = (h)

$\therefore$ Slant height of frustum $l = \sqrt{h^2 + (R-r)^2}$

$\Rightarrow l = \sqrt{12^2 + (12-7)^2} = \sqrt{144+25} = 13$ cm

$\therefore$ Total surface area of frustum $= \pi(R^2 + r^2 + l(R + r))$

$= \pi(144 + 49 + 13(19)) = \pi(144 + 49 + 247) = 440\pi\ \text{cm}^2$

and volume of frustum $= \frac{\pi h}{3}(R^2 + r^2 + Rr)$

$= \frac{\pi}{3} \times 12(144 + 49 + 84) = 4\pi(277)\ \text{cm}^3 = 1108\pi\ \text{cm}^3$

16. (a) : If the exports are higher than the imports, it means ratio of the value of import to exports must be less than 1 and such ratio is less than 1 in 2009, 2010, 2011 and 2014.

$\therefore$ Number of years = 4

17. (b) : From the line graph only the ratio of imports to exports for different years can be calculated. To find the percentage increase in import or exports we need some more details such as the value of import or exports during these years. Hence, data is insufficient.

18. (c) : From the line graph the ratio of imports to exports in 2010 = 0.85

Let the exports in the same year = ₹ x crores

Therefore we have, $\frac{289}{x} = 0.85 \Rightarrow x = 340$

$\therefore$ Exports in 2010 = 340 crores

19. (d) : The imports are minimum proportionate to the exports indicate that the ratio of the value of imports to exports has the minimum value.

From the line bar graph we notice that the ratio has the minimum value 0.40 in year 2011, therefore the imports are minimum proportionate to the exports in 2011.

20. (a) : The ratio of imports to exports for the year 2009 and 2010 are 0.70 and 0.85 respectively.

Let the exports in the year 2009 = x crores

$\therefore$ The exports in the year 2010 = $(245 - x)$ crores

Now, $0.7 = \frac{161}{x} \Rightarrow x = \frac{1610}{7} = 230$

Thus, the exports in the year 2010 = (245 – 230) = 15 crores

Let the imports in the year 2010 = y crores

Then, $0.85 = \frac{y}{15} \Rightarrow y = 0.85 \times 15 = 12.75$

$\therefore$ Imports in the year 2010 = 12.75 crores

Brain Teaser

LOGICAL REASONING

For Various Competitive Exams

Direction Q.(1 and 2) : There are two series given below, of which one series is complete and follows a certain pattern. Based on the pattern of complete series, you have to determine the different answering terms of the second series.

1. 47, 96, 147, 200, 255, 312

27, A, B, C, D, E

Which number will come at the place of C?

(a) 235 (b) 180

(c) 292 (d) None of these

2. 6, 8, 11, 15, 20, 26, 33

9, A, B, C, D, E, F

Which number will come at the place of E?

(a) 29 (b) 36 (c) 23 (d) 18

Direction Q.(3 and 4) : Identify what will come in the place of '?'.

3. Visitor : Invitation : : Witness : ?

(a) Subpoena (b) Permission

(c) Assent (d) Document

4. Mash : Horse : : Mast : ?

(a) Cow (b) Monkey

(c) Chimpanzee (d) Pig

Direction Q.(5 and 6) : Find the odd element in the following sequences.

5. 65, 82, 101, 120, 145, 170

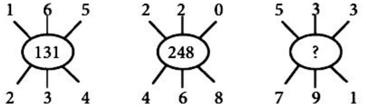
(a) 82 (b) 120 (c) 145 (d) 170

6. 9, 12, 29, 94, 386

(a) 94 (b) 386 (c) 29 (d) 12

Direction Q.(7 and 8) : In each of the following questions, find the number which replaces the sign of '?'.

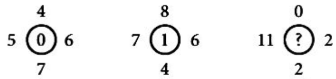
7.



A B C

(a) 320 (b) 274 (c) 262 (d) 132

8.



A B C

(a) 0 (b) 2

(c) 11 (d) 12

Directions Q.(9 and 10) : Study the following information to answer the given questions :

'TRAVEL' is related to 'UDKUSB' and 'CORNER' is related to 'MDQDPS'.

9. 'SURVEY' is related to _____ .

(a) UDXSTV (b) UXDTSV

(c) TVSUDX (d) UDXTVS

10. 'GROUPS' is related to _____ .

(a) TORHSP (b) TOHRSP

(c) TORPHS (d) ROTHSP

Direction Q.(11 and 12) : Read the information carefully and answer the following questions:

In a family of 6 members there are 3 men A, B and C and 3 women X, Y and Z. The six are Accountant, Lawyer, Chemist, Pilot, Dancer and Entertainer by profession but not in the same order.

I. There are two married couples and 2 unmarried persons.

II. C is not X's husband.

III. The dancer is married to the lawyer.

IV. X's father is a pilot.

V. B is not A's son, nor an accountant or pilot.

VI. The lawyer is Z's daughter-in-law.

VII. A is married to the chemist.

11. Who is the accountant?

(a) A (b) B

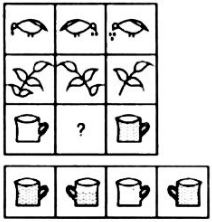
(c) X (d) Z

12. Which of the following is a married couple?

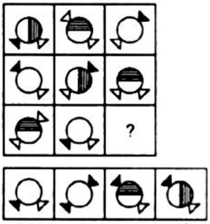
(a) A and Y (b) A and Z

(c) B and X (d) C and X

Direction Q.(13 and 14) : Select a figure from the options which will complete the given figure matrix.

13. 

(a) (b) (c) (d)

14. 

(a) (b) (c) (d)

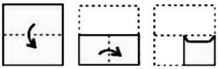
Direction Q.(15 and 16) : In each of the following questions, there is a set of three figures X, Y and Z showing a sequence of folding of a piece of paper. Fig. (Z) shows the manner in which the folded paper has been cut. Select a figure from the options that would most closely resemble the unfolded from figure of fig. (Z).

15. 

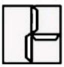
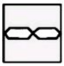
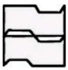
X Y Z

(a)  (b) 

(c)  (d) 

16. 

X Y Z

(a)  (b)  (c)  (d) 

*Direction Q.(17 to 20) : In the following questions, the symbols @, %, $, # and * are used with the following meaning as illustrated below:*

'P $ Q' means 'P is not smaller than Q'.
'P * Q' means 'P is neither smaller than nor equal to Q'.
'P @ Q' means 'P is not greater than Q'.
'P # Q' means 'P is neither greater than nor smaller than Q'.
'P % Q' means 'P is neither greater than nor equal to Q'.
Now in each of the following questions assuming the given statements to be true, find which of the two conclusions (I) and (II) given below them is/ are definitely true?

Give answer

(a) If only conclusion (I) is true.
(b) If only conclusion (II) is true.
(c) If either conclusion (I) or (II) is true.
(d) If both conclusions (I) and (II) are true.

17. **Statements :** M * T, T $ K, K # D
Conclusions:
I. D % M II. M * K

18. **Statements :** R @ J, M # J, D * M
Conclusions:
I. D * J II. R # M

19. **Statements :** F $ M, N @ M, N % W
Conclusions:
I. F # N II. N % F

20. **Statements:** B # J, J @ D, F $ D
Conclusions:
I. B # F II. F * B

SOLUTIONS

1. **(b) :**

47 96 147 200 255 312
+49 +51 +53 +55 +57

Now, the second series follows the pattern of first series, the first term of the second series is 27, therefore the second series will be :

27 76(A) 127(B) 180(C) 235(D) 292(E)
+49 +51 +53 +55 +57

∴ The value of C = 180.

2. **(a) :** The first series is 6, 8, 11, 15, 20, 26, 33 which is obtained by adding 2, 3, 4, 5, 6, 7 respectively to its previous terms therefore, the second series will have the terms 9, 11, 14, 18, 23, 29, 36 respectively.
∴ The value of $E = 29$.

3. **(a) :** A visitor is given an invitation to attend an occasion. Similarly, the witness is delivered a subpoena providing for attendance at the court.

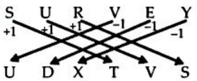
4. **(d) :** First is a food for the second.

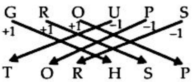
5. **(b) :** Series pattern is : $t_1 = 65 = 8^2 + 1$,
$t_2 = 82 = 9^2 + 1$, $t_3 = 10^2 + 1 = 101$,
$\boxed{t_4 = 11^2 + 1 = 122}$, $t_5 = 12^2 + 1 = 145$,
$t_6 = 13^2 + 1 = 170$
$\therefore$ Odd element = 120

6. **(b) :** In the given series, the rule followed is :
$t_n = t_{n-1} \times (n - 1) + (2n - 1)$
$t_2 = 9 \times (2 - 1) + (2 \times 2 - 1) = 9 + 3 = 12$,
$t_3 = 12 \times (3 - 1) + (2 \times 3 - 1) = 29$,
$t_4 = 29 \times (4 - 1) + (2 \times 4 - 1) = 94$,
$t_5 = 94 \times (5 - 1) + (2 \times 5 - 1) = 385 \neq 386$
$\therefore$ Odd element is 386.

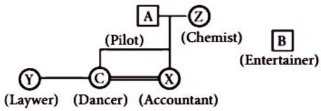
7. **(c) :** The digits of the number inside the circle are the differences between the corresponding numbers above and below the circle. Thus,
In fig. (A), $1 = (2 - 1)$, $3 = (6 - 3)$, $1 = (5 - 4)$.
In fig. (B), $2 = (4 - 2)$, $4 = (6 - 2)$, $8 = (8 - 0)$.
So, in fig. (C), the digits of the missing number are :
$(7 - 5)$, $(9 - 3)$, $(3 - 1)$ *i.e.*, 2, 6, 2.
$\therefore$ Missing number = 262.

8. **(c) :** The number inside the circle is equal to the difference between the sum of the numbers at the extremities of the horizontal diameter and the sum of numbers at the extremities of the vertical diameter.
In fig. (A), $(5 + 6) - (7 + 4) = 0$
In fig. (B), $(7 + 6) - (8 + 4) = 1$
$\therefore$ In fig. (C), missing number $= (11 + 2) - (0 + 2) = 11$.

9. **(d) :**


10. **(a) :**


Solutions for questions 11 and 12 :
From the given information we can draw the following family tree.



11. **(c) :** X is the Accountant.
12. **(b) :** AZ is a married couple.
13. **(b) :** Directions are changing and the quantities are either increasing or decreasing from left to right in each row.

14. **(d) :** In each row, one figure is unshaded, the other has its upper part shaded and the third one has its R.H.S. part shaded. There are three specified positions of the two triangles each of which is used only once in a row. Also, two of the figures in each row have one triangle shaded.

15. **(d) :**


16. **(b) :**


17. **(d) :** Statements:
M * T ⇒ M > T
T \$ K ⇒ T ≥ K
K # D ⇒ K = D
Therefore, M > T ≥ K = D
Conclusions:
I. D % M ⇒ D < M : True
II. M * K ⇒ M > K : True
Hence, both conclusions (I) and (II) are true.

18. **(a) :** Statements:
R @ J ⇒ R ≤ J
M # J ⇒ M = J
D * M ⇒ D > M
Therefore, R ≤ J = M < D
Conclusions:
I. D * J ⇒ D > J : True
II. R # M ⇒ R = M : Not true
Hence, only conclusion (I) is true.

19. **(c) :** Statements:
F \$ M ⇒ F ≥ M
N @ M ⇒ N ≤ M
N % W ⇒ N < W
Therefore, F ≥ M ≥ N < W
Conclusions:
I. F # N ⇒ F = N : May or may not be true
II. N % F ⇒ N < F : May or may not be true
Hence, either conclusion (I) or (II) is true.

20. **(c) :** Statements:
B # J ⇒ B = J
J @ D ⇒ J ≤ D
F \$ D ⇒ F ≥ D
Therefore, B = J ≤ D ≤ F
Conclusions:
I. B # F ⇒ B = F : May or not be true
II. F * B ⇒ F > B : May or may not be true
Hence, either conclusion (I) or (II) is true.

Quiz CLUB

Unlock Your Knowledge!

1. If set A has 3 elements and the set $B = \{3, 4, 5\}$, then find the number of elements in $(A \times B)$.
2. If $x \sin\frac{\pi}{4}\cos^2\frac{\pi}{3} = \frac{\tan^2(\pi/3)\operatorname{cosec}(\pi/6)}{\sec(\pi/4)\cot^2(\pi/6)}$, then find the value of x.
3. Find the modulus of $\frac{1}{1+i}$.
4. Solve the inequality $\left|x+\frac{1}{x}\right| > 2$.
5. Find the number of 4 letter words, with or without meaning, which can be formed out of the letters of the word ROSE, where the repetition of the letters is not allowed.
6. Find the total number of terms in the expansion of $(x + a)^{51} - (x - a)^{51}$.
7. In a G.P., the 3rd term is 24 and the 6th term is 192. Find the 10th term.
8. If the points $(p + 1, 1)$, $(2p + 1, 3)$ and $(2p + 2, 2p)$ are collinear, then find the value of p.
9. If $f(x) = \frac{\sqrt{4+x}-2}{x}$, $x \neq 0$ be continuous at $x = 0$, then find $f(0)$.
10. Find the maximum value of $\sin x \cdot \cos x$.
11. Find the integrating factor of the differential equation $\frac{dy}{dx} + y = \frac{1+y}{x}$.
12. Find the derivative of $2x^4 + x$.
13. Find the standard deviation of the data 6, 5, 9, 13, 12, 8, 10.
14. Let $S = \{1, 2, 3, 4, 5, 6\}$ and $E = \{1, 3, 5\}$, then find $\bar{E}$.
15. If $|\vec{a}| = 3$, $|\vec{b}| = 4$, then find the value of λ for which $\vec{a} + \lambda\vec{b}$ is perpendicular to $\vec{a} - \lambda\vec{b}$.
16. A single letter is selected at random from the word "FAVOURABLE". Find the probability that it is a vowel.
17. If $A = \{1, 2, 3\}$, $B = \{1, 4, 6, 9\}$ and R is a relation from A to B defined by 'x is greater than y'. Find the range of R.
18. Find the value of $\cos^{-1}\left(\frac{-1}{2}\right) + 2\sin^{-1}\left(\frac{-1}{2}\right)$.
19. If $A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$, $B = \begin{bmatrix} x & 1 \\ y & -1 \end{bmatrix}$ and $(A+B)^2 = A^2 + B^2$, then find the value of $x + y$.
20. Evaluate : $\int (2\tan x - 3\cot x)^2\, dx$

Readers can send their responses at editor@mtg.in or post us with complete address by 10th of every month. Winners' names and answers will be published in next issue.

COMIC CAPSULE

GK CORNER

Enhance Your General Knowledge with Current Updates!

INDIA AND THE WORLD

1. India hosted the BIMSTEC (Bay of Bengal Initiative for Multi-Sectoral Technical and Economic Cooperation) Aquatic Championship 2024 in New Delhi from 5-10 February 2024, as an initiative to foster regional cooperation spirit in the Bay of Bengal Region.

2. The new guidelines of the Reserve Bank of India, which allow multiple financial services between India and Nepal. Nepali citizens can now send ₹ 2 lakh per transaction to Nepal and walk-in customers can remit ₹ 50,000 per transaction.

3. The US Federal Communications Commission (FCC) bans companies from using AI-generated voices in robocalls effective immediately. AI can now create convincing fake audio and video, making it easy to scam people. The FCC aims to stop fraudsters using AI voice cloning in illegal robocalls to extort money or spread misinformation.

4. The 7^{th} Indian Ocean Conference on the theme "Towards a Stable and Sustainable Indian Ocean" was held in Perth, Australia on 9-10 February 2024. President, Ranil Wickremesinghe was a special invitee to deliver the keynote speech at the inauguration.

5. A joint session of parliament at the Palace of Versailles ended on Monday with the lawmakers endorsing the Bill to anchor the woman's right to abortion in the French constitution. This will make France the first country to explicitly protect the right to terminate a pregnancy in its basic legal articles.

6. Former Prime Minister, Alexander Stubb was sworn as Finland's new president and he said the Nordic country enters 'a new era' as a NATO member.

7. To promote economic collaborations and encouraging tourism and business ties, Dubai has introduced a five-year multiple-entry visa for Indian tourists. The multiple entry visa allows Indian passport holders to freely enter Dubai and stay up to 90 days at a stretch and it is extendable once for a similar period, with a total stay not exceeding 180 days in a year.

8. Maryam Nawaz took oath as the first-ever woman Chief Minister of Pakistan's most populous and politically crucial Punjab province.

9. The Biden administration on Friday announced more than 500 sanctions on Russia, its "enablers," and its "war machine" as the world marks two years since Russia attacked Ukraine. The American sanctions cover a range of economic and strategic areas like Russian financial networks, military supply chains, technology sectors, future energy production and other areas. They amount to largest single tranche of sanctions since the invasion.

10. India's Union Minister of Petroleum and Natural Gas & Housing and Urban Affairs, Hardeep Singh Puri, has inaugurated 201 compressed natural gas stations across 17 states as well as launched the country's first small-scale LNG unit set up by GAIL at Vijaipur in Madhya Pradesh, reflecting India's accelerated move to a gas-based economy.

11 South Eastern Coalfield Limited's (SECL) Gevra mine, located in Chhattisgarh, is on the verge of becoming the largest coal mine in Asia. With environmental clearance granted for a substantial expansion, the mine's production capacity is set to increase from 52.5 million tons per annum to an impressive 70 million tons.

12 Kerala, known for its strides in education, has taken another innovative step by introducing its first generative AI teacher, Iris. Developed in collaboration with Makerlabs Edutech Private Limited, Iris marks a significant advancement in the integration of artificial intelligence in education.

Test Yourself!

1. Where was the 7th Indian Ocean Conference held?
   (a) Dhaka, Bangladesh (b) Abu Dhabi, UAE
   (c) Perth, Australia (d) Male, Maldives

2. Kerala was recently in news for a new invention. What is it?
   (a) India's first AI teacher
   (b) India's first E-learning school
   (c) India's first AI on-edge Drone
   (d) Both (b) and (c)

3. According to new guidelines of RBI, what is the maximum amount Nepali citizens can send per transaction to Nepal?
   (a) ₹ 1 Lakh (b) ₹ 2 Lakh
   (c) ₹ 3 Lakh (d) ₹ 4 Lakh

4. Which city has recently introduced a five-year multiple-entry visa for Indian tourists?
   (a) Al Ain (b) Abu Dhabi
   (c) Sharjah (d) Dubai

5. In which state India's first small-scale LNG unit was set up by GAIL recently?
   (a) Uttar Pradesh (b) Madhya Pradesh
   (c) Gujarat (d) Chhattisgarh

6. Recently, which country became the first country to include the right to abortion in its constitution?
   (a) Germany (b) France
   (c) Poland (d) Malaysia

7. South Eastern Coalfield Limited's Gevra mine is situated in which state?
   (a) Maharshtra (b) Gujarat
   (c) Chhattisgarh (d) Odisha

8. What is the expanded form of FCC?
   (a) Foundations of Communications Council
   (b) Federal Commission of Commute
   (c) Foundations of Commission Communications
   (d) Federal Communications Commission

9. Alexander Stubb became the ______ President of Finland, a Nordic country.
   (a) 12^{th} (b) 13^{th}
   (c) 14^{th} (d) 15^{th}

10. Which of the following is not the member of BIMSTEC?
    (a) Nepal (b) Sri Lanka
    (c) Pakistan (d) Myanmar

Answer Key

1. (c) 2. (a) 3. (b) 4. (d) 5. (b)
6. (b) 7. (c) 8. (d) 9. (b) 10. (c)

SAMURAI SUDOKU

ANSWER · MARCH 2024

4	6	1	8	2	7	3	9	5				3	2	8	7	6	5	9	4	1
5	3	7	4	9	6	1	8	2				6	4	7	1	9	2	5	3	8
2	9	8	1	5	3	7	4	6				1	5	9	3	4	8	7	2	6
7	1	2	6	8	5	9	3	4				9	3	1	5	7	4	8	6	2
8	5	9	7	3	4	2	6	1				2	8	5	9	3	6	4	1	7
6	4	3	9	1	2	8	5	7				4	7	6	8	2	1	3	5	9
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3	7	6	2	4	8	5	1	9	8	2	4	7	6	3	4	1	9	2	8	5
						2	9	6	5	8	7	4	3	1						
						1	3	5	4	6	2	9	7	8						
						8	4	7	3	9	1	6	2	5						
1	2	3	9	6	8	7	5	4	1	3	6	2	8	9	4	6	3	7	1	5
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8	5	9	1	4	6	2	3	7				4	1	5	3	2	9	6	7	8
6	3	7	5	9	2	1	4	8				6	9	2	7	8	1	3	5	4
7	8	2	6	5	9	4	1	3				8	7	1	2	5	4	9	3	6
3	6	1	8	2	4	5	7	9				9	2	3	6	1	8	5	4	7
4	9	5	3	7	1	8	2	6				5	6	4	9	3	7	2	8	1

Class 12

CBSE

SOLVED PAPER 2024

Held on 9^{th} March

Hurray!! We are happy to inform our readers that in CBSE 2024 Mathematics question papers more than 70% questions were either exactly same or of similar type from **MTG Books**.

The references of few questions of paper having code : 65/2/1 are given here :

Paper Q. No.	P. No.	Q. No.	MTG Book
2	5	26	CBSE Champion
5	109	3	CBSE Champion
14	209	2	CBSE Champion
15	219	34	CBSE Champion
18	241	10	CBSE Champion
22 (a)	97	7	CBSE Champion
28 (b)	192	10	CBSE Champion
32 (b)	6	9	CBSE Champion

Paper Q. No.	P. No.	Q. No.	MTG Book
4	156	7	100 percent
6	337	15	100 percent
21 (b)	33	2	100 percent
23	267	8	100 percent
25	240	7	100 percent
26 (a)	203	11	100 percent
32 (a)	26	16	100 percent

and many more ……

General Instructions :

Time Allowed : 3 hours
Maximum Marks : 80

Read the following instructions very carefully and strictly follow them :

(i) This question paper contains **38** questions. All questions are **compulsory**.
(ii) This question paper is divided into **five** Sections – **A, B, C, D** and **E**.
(iii) In **Section A,** Questions no. **1** to **18** are multiple choice questions (MCQs) and Questions no. **19** and **20** are Assertion-Reason based questions of **1** mark each.
(iv) In **Section B,** Questions no. **21** to **25** are very short answer (VSA) type questions, carrying **2** marks each.
(v) In **Section C,** Questions no. **26** to **31** are short answer (SA) type questions, carrying **3** marks each.
(vi) In **Section D,** Questions no. **32** to **35** are long answer (LA) type questions, carrying **5** marks each.
(vii) In **Section E,** Questions no. **36** to **38** are case study based questions, carrying **4** marks each.
(viii) There is no overall choice. However, an internal choice has been provided in **2** questions in **Section B**, **3** questions in **Section C**, **2** questions in **Section D** and **2** questions in **Section E**.
(ix) Use of calculators is **not** allowed.

SECTION A

This section comprises multiple choice questions (MCQs)

1. If the sum of all elements of a 3×3 scalar matrix is 9, then the product of all its elements is :
(a) 0 (b) 9 (c) 27 (d) 729

2. Let $f : R_+ \to [-5, \infty)$ be defined as $f(x) = 9x^2 + 6x - 5$, where R_+ is the set of all non-negative real numbers. Then, f is :
(a) one-one
(b) onto
(c) bijective
(d) neither one-one nor onto

3. If $\begin{vmatrix} -a & b & c \\ a & -b & c \\ a & b & -c \end{vmatrix} = kabc$, then the value of k is :
(a) 0 (b) 1 (c) 2 (d) 4

4. The number of points of discontinuity of
$$f(x) = \begin{cases} |x|+3 &, \text{ if } x \le -3 \\ -2x &, \text{ if } -3 < x < 3 \\ 6x+2 &, \text{ if } x \ge 3 \end{cases}$$ is :
(a) 0 (b) 1 (c) 2 (d) infinite

5. The function $f(x) = x^3 - 3x^2 + 12x - 18$ is :
(a) strictly decreasing on R
(b) strictly increasing on R
(c) neither strictly increasing nor strictly decreasing on R
(d) strictly decreasing on $(-\infty, 0)$

6. $\int_0^{\pi/2} \frac{\sin x - \cos x}{1 + \sin x \cos x} dx$ is equal to :
(a) π (b) Zero (0)
(c) $\int_0^{\pi/2} \frac{2 \sin x}{1 + \sin x \cos x} dx$ (d) $\frac{\pi^2}{4}$

7. The differential equation $\frac{dy}{dx} = F(x, y)$ will not be a homogeneous differential equation, if $F(x, y)$ is :
(a) $\cos x - \sin\left(\frac{y}{x}\right)$ (b) $\frac{y}{x}$
(c) $\frac{x^2 + y^2}{xy}$ (d) $\cos^2\left(\frac{x}{y}\right)$

8. For any two vectors $\vec{a}$ and $\vec{b}$, which of the following statements is always true?
(a) $\vec{a} \cdot \vec{b} \ge |\vec{a}|\,|\vec{b}|$ (b) $\vec{a} \cdot \vec{b} = |\vec{a}|\,|\vec{b}|$
(c) $\vec{a} \cdot \vec{b} \le |\vec{a}|\,|\vec{b}|$ (d) $\vec{a} \cdot \vec{b} < |\vec{a}|\,|\vec{b}|$

9. The coordinates of the foot of the perpendicular drawn from the point (0, 1, 2) on the x-axis are given by :
(a) (1, 0, 0) (b) (2, 0, 0)
(c) $(\sqrt{5}, 0, 0)$ (d) (0, 0, 0)

10. The common region determined by all the constraints of a linear programming problem is called :
(a) an unbounded region
(b) an optimal region
(c) a bounded region
(d) a feasible region

11. Let E be an event of a sample space S of an experiment, then $P(S|E) =$
(a) $P(S \cap E)$ (b) $P(E)$
(c) 1 (d) 0

12. If $A = [a_{ij}]$ be a 3×3 matrix, where $a_{ij} = i - 3j$, then which of the following is **false**?
(a) $a_{11} < 0$ (b) $a_{12} + a_{21} = -6$
(c) $a_{13} > a_{31}$ (d) $a_{31} = 0$

13. The derivative of $\tan^{-1}(x^2)$ w.r.t. x is :
(a) $\frac{x}{1+x^4}$ (b) $\frac{2x}{1+x^4}$
(c) $-\frac{2x}{1+x^4}$ (d) $\frac{1}{1+x^4}$

14. The degree of the differential equation $(y'')^2 + (y')^3 = x\sin(y')$ is :
(a) 1 (b) 2
(c) 3 (d) not defined

15. The unit vector perpendicular to both vectors $\hat{i} + \hat{k}$ and $\hat{i} - \hat{k}$ is :
(a) $2\hat{j}$ (b) $\hat{j}$ (c) $\frac{\hat{i} - \hat{k}}{\sqrt{2}}$ (d) $\frac{\hat{i} + \hat{k}}{\sqrt{2}}$

16. Direction ratios of a vector parallel to line $\frac{x-1}{2} = -y = \frac{2z+1}{6}$ are :
(a) 2, –1, 6 (b) 2, 1, 6 (c) 2, 1, 3 (d) 2, –1, 3

17. If $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $[F(x)]^2 = F(kx)$, then the value of k is :
(a) 1 (b) 2 (c) 0 (d) –2

18. If a line makes an angle of 30° with the positive direction of x-axis, 120° with the positive direction of y-axis, then the angle which it makes with the positive direction of z-axis is :

(a) 90° (b) 120° (c) 60° (d) 0°

Questions number 19 and 20 are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) options as given below.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is **not** the correct explanation of the Assertion (A).

(c) Assertion (A) is true, but Reason (R) is false.

(d) Assertion (A) is false, but Reason (R) is true.

19. **Assertion (A) :** For any symmetric matrix A, $B'AB$ is a skew-symmetric matrix.

Reason (R) : A square matrix P is skew-symmetric if $P' = -P$.

20. **Assertion (A) :** For two non-zero vectors $\vec{a}$ and $\vec{b}$, $\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{a}$.

Reason (R) : For two non-zero vectors $\vec{a}$ and $\vec{b}$, $\vec{a}\times\vec{b}=\vec{b}\times\vec{a}$.

SECTION B

This section comprises very short answer (VSA) type questions of 2 marks each.

21. (a) Find the value of

$$\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)+\cot^{-1}\left(\frac{1}{\sqrt{3}}\right)+\tan^{-1}\left[\sin\left(-\frac{\pi}{2}\right)\right].$$

OR

(b) Find the domain of the function $f(x) = \sin^{-1}(x^2 - 4)$. Also, find its range.

22. (a) If $f(x) = |\tan 2x|$, then find the value of $f'(x)$ at $x=\frac{\pi}{3}$.

OR

(b) If $y = \operatorname{cosec}(\cot^{-1}x)$, then prove that

$$\sqrt{1+x^2}\,\frac{dy}{dx}-x=0.$$

23. If M and m denote the local maximum and local minimum values of the function $f(x)=x+\frac{1}{x}(x\neq 0)$ respectively, find the value of $(M-m)$.

24. Find : $\int\frac{e^{4x}-1}{e^{4x}+1}dx$

25. Show that $f(x) = e^x - e^{-x} + x - \tan^{-1} x$ is strictly increasing in its domain.

SECTION C

This section comprises short answer (SA) type questions of 3 marks each.

26. (a) If $x = e^{\cos 3t}$ and $y = e^{\sin 3t}$, prove that $\frac{dy}{dx}=-\frac{y\log x}{x\log y}$.

OR

(b) Show that : $\frac{d}{dx}(|x|)=\frac{x}{|x|}, x\neq 0$

27. (a) Evaluate : $\int_{-2}^{2}\sqrt{\frac{2-x}{2+x}}\,dx$

OR

(b) Find : $\int\frac{1}{x[(\log x)^2-3\log x-4]}dx$

28. (a) Find the particular solution of the differential equation given by $2xy + y^2 - 2x^2\frac{dy}{dx}=0$; $y = 2$, when $x = 1$.

OR

(b) Find the general solution of the differential equation : $y\,dx = (x + 2y^2)dy$

29. The position vectors of vertices of ΔABC are $A(2\hat{i}-\hat{j}+\hat{k})$, $B(\hat{i}-3\hat{j}-5\hat{k})$ and $C(3\hat{i}-4\hat{j}-4\hat{k})$. Find all the angles of ΔABC.

30. A pair of dice is thrown simultaneously. If X denotes the absolute difference of the numbers appearing on top of the dice, then find the probability distribution of X.

31. Find : $\int x^2\cdot\sin^{-1}(x^{3/2})dx$

SECTION D

This section comprises long answer (LA) type questions of 5 marks each.

32. (a) Show that a function $f : R \to R$ defined by $f(x)=\frac{2x}{1+x^2}$ is neither one-one nor onto.

Further, find set A so that the given function $f: R \to A$ becomes an onto function.

OR

(b) A relation R is defined on $N \times N$ (where N is the set of natural numbers) as :

$(a, b)\ R\ (c, d) \Leftrightarrow a - c = b - d$

Show that R is an equivalence relation.

33. Find the equation of the line which bisects the line segment joining points $A(2, 3, 4)$ and $B(4, 5, 8)$ and is perpendicular to the lines

$$\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7} \text{ and } \frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$$

34. (a) Solve the following system of equation, using matrices

$$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4,\ \frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1,\ \frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$$

where $x, y, z \neq 0$

OR

(b) If $A = \begin{bmatrix} 1 & \cot x \\ -\cot x & 1 \end{bmatrix}$, show that

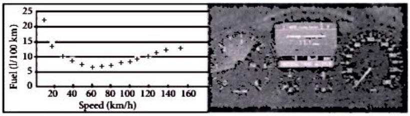
$$A'A^{-1} = \begin{bmatrix} -\cos 2x & -\sin 2x \\ \sin 2x & -\cos 2x \end{bmatrix}.$$

35. If A_1 denotes the area of region bounded by $y^2 = 4x$, $x = 1$ and x-axis in the first quadrant and A_2 denotes the area of region bounded by $y^2 = 4x$, $x = 4$, find $A_1 : A_2$.

SECTION E

Case Study 1

36. Overspeeding increases fuel consumption and decreases fuel economy as a result of tyre rolling friction and air resistance. While vehicles reach optimal fuel economy at different speeds, fuel mileage usually decreases rapidly at speeds above 80 km/h.



The relation between fuel consumption $F(l/100\text{ km})$ and speed $V(\text{km/h})$ under some constraints is given as $F = \frac{V^2}{500} - \frac{V}{4} + 14$.

On the basis of the above information, answer the following questions :

(i) Find F, when $V = 40$ km/h.

(ii) Find $\frac{dF}{dV}$.

(iii) (a) Find the speed V for which fuel consumption F is minimum.

OR

(iii) (b) Find the quantity of fuel required to travel 600 km at the speed V at which $\frac{dF}{dV} = -0.01$.

Case Study 2

37. The month of September is celebrated as the Rashtriya Poshan Maah across the country. Following a healthy and well-balanced diet is crucial in order to supply the body with the proper nutrients it needs. A balanced diet also keeps us mentally fit and promotes improved level of energy.

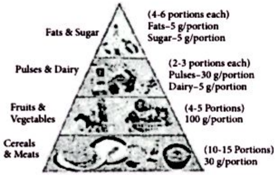


Figure-1

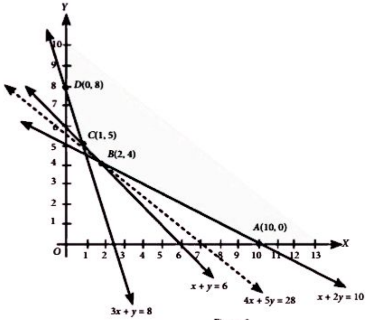


Figure-2

A dietician wishes to minimize the cost of a diet involving two types of foods, food X (x kg) and food Y (y kg) which are available at the rate of ₹ 16/kg and ₹ 20/kg respectively. The feasible region satisfying the constraints is shown in Figure-2.

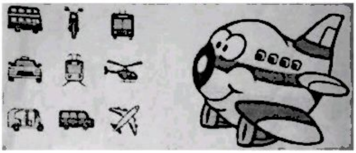
On the basis of the above information, answer the following questions :

(i) Identify and write all the constrains which determine the given feasible region in Figure-2.

(ii) If the objective is to minimize cost $Z = 16x + 20y$, find the values of x and y at which cost is minimum. Also, find minimum cost assuming that minimum cost is possible for the given unbounded region.

Case Study 3

38. Airplanes are by far the safest mode of transportation when the number of transported passengers are measured against personal injuries and fatality totals.



Previous records state that the probability of an airplane crash is 0.00001%. Further, there are 95% chances that there will be survivors after a plane crash. Assume that in case of no crash, all travellers survive.

Let E_1 be the event that there is a plane crash and E_2 be the event that there is no crash. Let A be the event that passengers survive after the journey.

On the basis of the above information, answer the following questions :

(i) Find the probability that the airplane will not crash.

(ii) Find $P(A|E_1) + P(A|E_2)$.

(iii) (a) Find $P(A)$

OR

(iii) (b) Find $P(E_2|A)$.

SOLUTIONS

1. (a) : As we know that in a scalar matrix, every non-diagonal element is zero.

$\therefore$ Product of all elements of the given scalar matrix = 0.

2. (c) : Let $f(x_1) = f(x_2)$

$\Rightarrow 9x_1^2 + 6x_1 - 5 = 9x_2^2 + 6x_2 - 5$

$\Rightarrow 9(x_1^2 - x_2^2) + 6(x_1 - x_2) = 0$

$\Rightarrow (x_1 - x_2)\,[9(x_1 + x_2) + 6] = 0$

$\Rightarrow x_1 - x_2 = 0$ as $9(x_1 + x_2) + 6 > 0$

($\because x_1, x_2$ are non-negative real numbers)

$\Rightarrow x_1 = x_2$

Thus, f is one-one.

Let $y \in [-5, \infty)$ be such that $f(x) = y$

Now, $f(x) = 9x^2 + 6x - 5 = 9x^2 + 6x + 1 - 6 = (3x + 1)^2 - 6$

$\Rightarrow y + 6 = (3x + 1)^2$

$\Rightarrow 3x + 1 = \sqrt{y+6} \Rightarrow x = \dfrac{-1+\sqrt{y+6}}{3}$

$\therefore f\left(\dfrac{-1+\sqrt{y+6}}{3}\right) = y$

$\therefore f(x)$ is onto.

So, the given function is bijective.

3. (d) : We have ,

$$\begin{vmatrix} -a & b & c \\ a & -b & c \\ a & b & -c \end{vmatrix} = -a(bc - bc) - b(-ac - ac) + c(ab + ab)$$

$= 2abc + 2abc = 4abc \quad \Rightarrow \quad k\,abc = 4abc \Rightarrow k = 4$

4. (b) : We have,

$$f(x) = \begin{cases} |x| + 3 &, \text{ if } x \le -3 \\ -2x &, \text{ if } -3 < x < 3 \\ 6x + 2 &, \text{ if } x \ge 3 \end{cases}$$

$$\Rightarrow f(x) = \begin{cases} -x + 3 &, \text{ if } x \le -3 \\ -2x &, \text{ if } -3 < x < 3 \\ 6x + 2 &, \text{ if } x \ge 3 \end{cases}$$

Now, $\lim\limits_{x \to -3^-} f(x) = -(-3) + 3 = 6$

and $\lim\limits_{x \to -3^+} f(x) = -2(-3) = 6$

Also, $f(-3) = -(-3) + 3 = 3 + 3 = 6$

As $\lim\limits_{x \to -3^-} f(x) = \lim\limits_{x \to -3^+} f(x) = f(-3)$

$\therefore f(x)$ is continuous at $x = -3$

Again $\lim\limits_{x \to 3^-} f(x) = -2(3) = -6$

and $\lim\limits_{x \to 3^+} f(x) = 6(3) + 2 = 20 \ne -6$

$\therefore f(x)$ is discontinuous at $x = 3$.

So, only one point of discontinuity.

5. (b) : $f(x) = x^3 - 3x^2 + 12x - 18$

$\Rightarrow f'(x) = 3x^2 - 6x + 12 = 3(x^2 - 2x + 1^2) + 9$

$= 3(x - 1)^2 + 3^2 > 0 \ \forall \ x \in R$

$\Rightarrow f(x)$ is strictly increasing on R

6. (b) : Let $I = \displaystyle\int_0^{\pi/2} \frac{\sin x - \cos x}{1 + \sin x \cos x}\,dx$ ...(i)

$$\Rightarrow \quad I = \int_0^{\pi/2} \frac{\sin(\pi/2 - x) - \cos(\pi/2 - x)}{1 + \sin(\pi/2 - x)\cos(\pi/2 - x)} dx$$

$$\Rightarrow \quad I = \int_0^{\pi/2} \frac{\cos x - \sin x}{1 + \cos x \cdot \sin x} dx \quad \text{...(ii)}$$

Adding (i) and (ii), we get

$$2I = \int_0^{\pi/2} 0\, dx = 0 \Rightarrow I = 0$$

7. (a) : Let $F(x, y) = \cos x - \sin \frac{y}{x}$

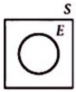
$$\Rightarrow \quad F(\lambda x, \lambda y) = \cos \lambda x - \sin \frac{\lambda y}{\lambda x} = \cos \lambda x - \sin \frac{y}{x}$$

$$\neq \lambda\left(\cos x - \sin \frac{y}{x}\right)$$

$\therefore$ $\cos x - \sin \frac{y}{x}$ is not homogeneous.

8. (c) **9. (d)** **10. (d)**

11. (c) : $P(S|E) = \frac{P(S \cap E)}{P(E)} = \frac{P(E)}{P(E)} = 1$



12. (c) : We have, $a_{ij} = i - 3j$

(a) $a_{11} = 1 - 3 \times 1 = -2 < 0$

(b) $a_{12} + a_{21} = (1 - 3 \times 2) + (2 - 3 \times 1) = (-5) + (-1) = -6$

(c) $a_{13} = 1 - 3 \times 3 = -8$ and $a_{31} = 3 - 3 \times 1 = 0 > -8$

$\Rightarrow a_{31} > a_{13}$

(d) $a_{31} = 0$

13. (b) : Let $y = \tan^{-1}(x^2)$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{d}{dx}(\tan^{-1}(x^2)) = \frac{1}{1+(x^2)^2} \times 2x = \frac{2x}{1+x^4}$$

14. (d) : Since, the given differential equation is not a polynomial in $\frac{dy}{dx}$.

$\therefore$ Its degree is not defined.

15. (b) : Let the required vector be $x\hat{i} + y\hat{j} + z\hat{k}$.

Then, $x^2 + y^2 + z^2 = 1$ ...(i)

Also, $(x\hat{i} + y\hat{j} + z\hat{k}) \cdot (\hat{i} + \hat{k}) = 0$

$\Rightarrow x + z = 0$ ...(ii)

and $(x\hat{i} + y\hat{j} + z\hat{k}) \cdot (\hat{i} - \hat{k}) = 0$

$\Rightarrow x - z = 0$ ...(iii)

Solving (ii) and (iii), we get $x = z = 0$

$\therefore$ From (i), $y^2 = 1 \Rightarrow y = \pm 1$

So, required vector is $\pm\hat{j}$.

16. (d) : The given line can be written as

$$\frac{x-1}{2} = \frac{y}{-1} = \frac{z+1/2}{3}$$

So, direction ratios of line parallel to given line is $< 2, -1, 3 >$.

17. (b) : We have, $[F(x)]^2 = F(kx)$

$$\Rightarrow \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos kx & -\sin kx & 0 \\ \sin kx & \cos kx & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \cos 2x & -\sin 2x & 0 \\ \sin 2x & \cos 2x & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos kx & -\sin kx & 0 \\ \sin kx & \cos kx & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\Rightarrow k = 2$

18. (a) : Let the angle made with positive direction of z-axis be γ.

Then, $\cos^2 30° + \cos^2 120° + \cos^2\gamma = 1$

$$\Rightarrow \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{-1}{2}\right)^2 + \cos^2\gamma = 1$$

$$\Rightarrow \cos^2\gamma = 1 - \frac{3}{4} - \frac{1}{4} = 0 \Rightarrow \gamma = 90°$$

19. (d) : $\because$ A is symmetric matrix.

$\Rightarrow A' = A$ ...(i)

Now, $(B'AB)' = B'A'(B')' = B'AB$ (using (i))

$\Rightarrow B'AB$ is a symmetric matrix.

So, assertion is false but reason is true.

20. (c) : Assertion is true but reason is false.

As, $\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$.

21. (a) We have,

$$\tan^{-1}\left(\frac{-1}{\sqrt{3}}\right) + \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) + \tan^{-1}\left[\sin\left(\frac{-\pi}{2}\right)\right]$$

$$= -\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) + \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) + \tan^{-1}(-1)$$

$$= -\frac{\pi}{6} + \frac{\pi}{3} - \frac{\pi}{4} = -\frac{\pi}{12}$$

OR

(b) For domain, $-1 \le x^2 - 4 \le 1$

$$\Rightarrow 3 \le x^2 \le 5 \Rightarrow \sqrt{3} \le |x| \le \sqrt{5}$$

$$\Rightarrow x \in [-\sqrt{5}, -\sqrt{3}] \cup [\sqrt{3}, \sqrt{5}]$$

Let $y = \sin^{-1}(x^2 - 4) \Rightarrow \sin y + 4 = x^2$

Now, $3 \le x^2 \le 5 \Rightarrow 3 \le \sin y + 4 \le 5 \Rightarrow -1 \le \sin y \le 1$

$$\Rightarrow -\frac{\pi}{2} \le y \le \frac{\pi}{2} \qquad \therefore R_f = \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$$

22. (a) We have, $f(x) = |\tan 2x|$

$$\Rightarrow f'(x) = \frac{\tan 2x}{|\tan 2x|} \cdot \frac{d}{dx}(\tan 2x) = \frac{\tan 2x}{|\tan 2x|} \cdot 2\sec^2 2x$$

$$\therefore f'\left(\frac{\pi}{3}\right) = \frac{\tan 2\pi/3}{|\tan 2\pi/3|} \times 2\sec^2\left(2 \cdot \frac{\pi}{3}\right)$$

$$= \frac{-\sqrt{3}}{\sqrt{3}} \times 2 \cdot (-2)^2 = -8$$

OR

(b) $y = \operatorname{cosec}(\cot^{-1} x) \Rightarrow y = \operatorname{cosec}(\operatorname{cosec}^{-1}\sqrt{1+x^2})$

$$\Rightarrow y = \sqrt{1+x^2} \Rightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{1+x^2}} \times 2x$$

$$\Rightarrow \sqrt{1+x^2}\,\frac{dy}{dx} - x = 0$$

Hence proved.

23. We have, $f(x) = x + \frac{1}{x}$

$$\Rightarrow f'(x) = 1 - \frac{1}{x^2} \text{ and } f''(x) = \frac{2}{x^3}$$

For maxima/minima, put $f'(x) = 0$

$$\Rightarrow 1 - \frac{1}{x^2} = 0 \Rightarrow \frac{1}{x^2} = 1$$

$\Rightarrow x^2 = 1 \Rightarrow x = \pm 1$

At $x = 1$, $f''(x) = \frac{2}{1} = 2 > 0$ (Minimum)

At $x = -1$, $f''(x) = \frac{2}{-1} < 0$ (Maximum)

$$\therefore M = -1 + \frac{1}{(-1)} = -2 \text{ and } m = 1 + \frac{1}{1} = 2$$

So, $M - m = -2 - 2 = -4$

24. Let $I = \int \frac{e^{4x} - 1}{e^{4x} + 1} dx = \int \left(\frac{2e^{4x}}{e^{4x}+1} - 1\right) dx$

$$= 2\int \frac{e^{4x}}{e^{4x}+1} dx - \int 1 dx = 2\int \frac{e^{4x}}{e^{4x}+1} dx - x + C$$

Put $e^{4x} + 1 = t \Rightarrow 4e^{4x}\,dx = dt$

$$\therefore I = \frac{2}{4}\int \frac{dt}{t} - x + C$$

$$\Rightarrow I = \frac{1}{2}\log|t| - x + C$$

$$\Rightarrow I = \frac{1}{2}\log|e^{4x} + 1| - x + C$$

25. We have, $f(x) = e^x - e^{-x} + x - \tan^{-1} x$

$$\Rightarrow f'(x) = e^x + e^{-x} + 1 - \frac{1}{1+x^2}$$

$$= e^x + \frac{1}{e^x} + \frac{1+x^2-1}{1+x^2} = e^x + \frac{1}{e^x} + \frac{x^2}{1+x^2} > 0 \text{ for all}$$

$x \in D_f$ $\qquad$ ($\because e^x > 0$ for all x)

So, $f(x)$ is strictly increasing in its domain.

Hence proved.

26. (a) We have, $x = e^{\cos 3t}$ and $y = e^{\sin 3t}$

$$\Rightarrow \frac{dx}{dt} = e^{\cos 3t} \times (-\sin 3t) \times 3$$

$$\Rightarrow \frac{dx}{dt} = -3\sin 3t \cdot e^{\cos 3t}$$

and $\frac{dy}{dt} = e^{\sin 3t} \times \cos 3t \times 3 \Rightarrow \frac{dy}{dt} = 3\cos 3t \cdot e^{\sin 3t}$

$$\text{Now, } \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-3\cos 3t \cdot e^{\sin 3t}}{3\sin 3t \cdot e^{\cos 3t}} = \frac{-\cos 3t \cdot y}{\sin 3t \cdot x}$$

$$= -\frac{y\log x}{x\log y}$$

($\because \log x = \log e^{\cos 3t} = \cos 3t$ and $\log y = \log e^{\sin 3t} = \sin 3t$)

$$\therefore \frac{dy}{dx} = \frac{-y\log x}{x\log y}$$

Hence proved.

OR

(b) Let $y = |x| \Rightarrow y = (x^2)^{1/2}$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2(x^2)^{1/2}} \times 2x = \frac{x}{\sqrt{x^2}} \Rightarrow \frac{dy}{dx} = \frac{x}{|x|}, \quad x \ne 0$$

Hence proved.

27. **(a)** Let $I=\int_{-2}^{2}\sqrt{\frac{2-x}{2+x}}\,dx$

$$\Rightarrow\quad I=\int_{-2}^{2}\sqrt{\frac{(2-x)(2-x)}{(2+x)(2-x)}}\,dx$$

$$\Rightarrow\quad I=\int_{-2}^{2}\frac{(2-x)}{\sqrt{4-x^2}}\,dx$$

$$\Rightarrow\quad I=\int_{-2}^{2}\frac{2}{\sqrt{4-x^2}}\,dx-\int_{-2}^{2}\frac{x}{\sqrt{4-x^2}}\,dx$$

$$\Rightarrow\quad I=\int_{-2}^{2}\frac{2}{\sqrt{2^2-x^2}}\,dx-0$$

$$\left(\because \frac{x}{\sqrt{4-x^2}} \text{ is an odd function}\right)$$

$$\Rightarrow\quad I=2\left|\sin^{-1}\left(\frac{x}{2}\right)\right|_{-2}^{2}=2[\sin^{-1}(1)-\sin^{-1}(-1)]$$

$$=2\left(\frac{\pi}{2}-\left(-\frac{\pi}{2}\right)\right)\quad\Rightarrow\quad I=2\pi$$

OR

(b) Let $I=\int\frac{1}{x[(\log x)^2-3\log x-4]}\,dx$

Put $\log x=t\Rightarrow \frac{1}{x}dx=dt$

$$\therefore\quad I=\int\frac{dt}{t^2-3t-4}\quad\Rightarrow\quad I=\int\frac{dt}{(t+1)(t-4)}\qquad\text{...(i)}$$

Using partial fraction, we have

$$\frac{1}{(t+1)(t-4)}=\frac{A}{t+1}+\frac{B}{t-4}$$

$\Rightarrow\ 1=A(t-4)+B(t+1)$

On comparing terms of same coefficients, we get

$A+B=0$ ....(ii)

$-4A+B=1$ ....(iii)

Solving (ii) and (iii), we get $A=\frac{-1}{5},\ B=\frac{1}{5}$

$$\therefore\quad I=\int\frac{-1}{5(t+1)}dt+\int\frac{1}{5(t-4)}dt\qquad\text{(Using(i))}$$

$$=-\frac{1}{5}\log|t+1|+\frac{1}{5}\log|t-4|$$

$$=\frac{-1}{5}\log|\log x+1|+\frac{1}{5}\log|\log x-4|+C$$

$$\Rightarrow\quad I=\frac{1}{5}\log\left|\frac{\log x-4}{\log x+1}\right|+C$$

28. **(a)** The given differential equation is

$$2xy+y^2-2x^2\frac{dy}{dx}=0\ \Rightarrow\ \frac{dy}{dx}=\frac{2xy+y^2}{2x^2}$$

Put $y=vx\Rightarrow \frac{dy}{dx}=v+\frac{xdv}{dx}$.

$$\therefore\quad v+x\frac{dv}{dx}=\frac{2x\cdot vx+v^2x^2}{2x^2}$$

$$\Rightarrow\quad v+\frac{xdv}{dx}=\frac{2v+v^2}{2}\ \Rightarrow\ x\frac{dv}{dx}=\frac{2v+v^2}{2}-v$$

$$\Rightarrow\quad x\frac{dv}{dx}=\frac{v^2}{2}\ \Rightarrow\ \int\frac{dv}{v^2}=\frac{1}{2}\int\frac{dx}{x}$$

$$\Rightarrow\quad -\frac{1}{v}=\frac{1}{2}\log|x|+C\quad\Rightarrow\quad -\frac{x}{y}=\frac{1}{2}\log|x|+C$$

Put $x=1$ and $y=2$ in above equation, we get

$$-\frac{1}{2}=\frac{1}{2}\log|1|+C\Rightarrow C=\frac{-1}{2}$$

$\therefore$ Particular solution of given differential equation is

$$\frac{-x}{y}=\frac{1}{2}\log|x|-\frac{1}{2}$$

$$\Rightarrow\quad y=\frac{-2x}{\log|x|-1}\ \Rightarrow\ y=\frac{2x}{1-\log|x|}$$

OR

(b) The given differential equation is

$y\,dx=(x+2y^2)dy$

$\Rightarrow\ ydx=xdy+2y^2dy\Rightarrow ydx-xdy=2y^2dy$

$$\Rightarrow\quad \frac{ydx-xdy}{y^2}=2dy=d\left(\frac{x}{y}\right)=2dy$$

Integrating both sides, we get

$\int d\left(\frac{x}{y}\right)=2\int dy\ \Rightarrow\ \frac{x}{y}=2y+C,$ which is the required general solution.

29. We have,

Position vector of $A=2\hat{i}-\hat{j}+\hat{k}$

Position vector of $B=\hat{i}-3\hat{j}-5\hat{k}$

Position vector of $C=3\hat{i}-4\hat{j}-4\hat{k}$

Now, $\overrightarrow{AB}=\hat{i}-3\hat{j}-5\hat{k}-2\hat{i}+\hat{j}-\hat{k}$

$\Rightarrow \quad \overrightarrow{AB}=-\hat{i}-2\hat{j}-6\hat{k}$

Similarly, $\overrightarrow{BC}=3\hat{i}-4\hat{j}-4\hat{k}-\hat{i}+3\hat{j}+5\hat{k}=2\hat{i}-\hat{j}+\hat{k}$

and $\overrightarrow{AC}=3\hat{i}-4\hat{j}-4\hat{k}-2\hat{i}+\hat{j}-\hat{k}=\hat{i}-3\hat{j}-5\hat{k}$

Now, $\overrightarrow{AB}\cdot\overrightarrow{AC}=|\overrightarrow{AB}||\overrightarrow{AC}|\cos\angle BAC$

$$\Rightarrow \cos\angle BAC=\frac{(-\hat{i}-2\hat{j}-6\hat{k})\cdot(\hat{i}-3\hat{j}-5\hat{k})}{|-\hat{i}-2\hat{j}-6\hat{k}|\cdot|\hat{i}-3\hat{j}-5\hat{k}|}$$

$$=\frac{-1+6+30}{\sqrt{1+4+36}\sqrt{1+9+25}}=\frac{35}{\sqrt{41}\sqrt{35}}$$

$$\Rightarrow \quad \angle BAC=\cos^{-1}\left(\sqrt{\frac{35}{41}}\right)$$

Similarly, $\cos\angle ABC=\dfrac{\overrightarrow{BA}\cdot\overrightarrow{BC}}{|\overrightarrow{BA}||\overrightarrow{BC}|}$

$$\Rightarrow \quad \cos\angle ABC=\frac{(\hat{i}+2\hat{j}+6\hat{k})\cdot(2\hat{i}-\hat{j}+\hat{k})}{\sqrt{1+4+36}\sqrt{4+1+1}}$$

$$=\frac{2-2+6}{\sqrt{41}\sqrt{6}}=\frac{6}{\sqrt{41}\sqrt{6}} \Rightarrow \angle ABC=\cos^{-1}\left(\sqrt{\frac{6}{41}}\right)$$

And, $\cos\angle ACB=\dfrac{\overrightarrow{CA}\cdot\overrightarrow{CB}}{|\overrightarrow{CA}|\cdot|\overrightarrow{CB}|}$

$$=\frac{(-\hat{i}+3\hat{j}+5\hat{k})\cdot(-2\hat{i}+\hat{j}-\hat{k})}{\sqrt{1+9+25}\sqrt{4+1+1}}=0$$

$$\Rightarrow \quad \angle ACB=\cos^{-1}(0)=\frac{\pi}{2}$$

30. X can take values 0, 1, 2, 3, 4, 5

If X = 0, then the favourable outcomes are (1, 1), (2, 2), (3, 3) .... (6, 6), *i.e.*, 6 in number.

$$\therefore \quad P(X=0)=\frac{6}{36}=\frac{1}{6}$$

If X = 1, then the favourable outcomes are (1, 2), (2, 1), (3, 2), (2, 3), (3, 4), (4, 3), (5, 4), (4, 5), (5, 6), (6, 5)

$$\therefore \quad P(X=1)=\frac{10}{36}=\frac{5}{18}$$

If X = 2, then the favourable outcomes are (1, 3), (3, 1), (2, 4), (4, 2), (3, 5), (5, 3), (4, 6), (6, 4), *i.e.*, 8 in number

$$\therefore \quad P(X=2)=\frac{8}{36}=\frac{2}{9}$$

If X = 3, then the favourable outcomes are (1, 4), (4, 1), (2, 5), (5, 2), (3, 6), (6, 3) *i.e.*, 6 in number

$$\therefore \quad P(X=3)=\frac{6}{36}=\frac{1}{6}$$

If X = 4, then the favourable outcomes are (1, 5), (5, 1), (2, 6), (6, 2) *i.e.*, 4 in number

$$\therefore \quad P(X=4)=\frac{4}{36}=\frac{1}{9}$$

If X = 5, then the favourable outcomes are (1, 6), (6, 1) *i.e.*, 2 in number

$$\therefore \quad P(X=5)=\frac{2}{36}=\frac{1}{18}$$

So, required probability distribution of X is

X	0	1	2	3	4	5
$P(X)$	$\frac{1}{6}$	$\frac{5}{18}$	$\frac{2}{9}$	$\frac{1}{6}$	$\frac{1}{9}$	$\frac{1}{18}$

31. Let $I=\int x^2\cdot\sin^{-1}(x^{3/2})dx$

$$=\int x^{1/2}\cdot x^{3/2}\sin^{-1}(x^{3/2})dx$$

Put $x^{3/2}=t \Rightarrow \quad \frac{3}{2}x^{1/2}dx=dt$

$$\therefore \quad I=\frac{2}{3}\int \underset{\text{II}}{t}\cdot\underset{\text{I}}{\sin^{-1}t}\,dt$$

Using integration by parts, we get

$$I=\frac{2}{3}\left[\sin^{-1}t\cdot\frac{t^2}{2}-\int\frac{1}{\sqrt{1-t^2}}\times\frac{t^2}{2}dt\right]+C_1$$

$$=\frac{2}{3}\left[\frac{t^2}{2}\cdot\sin^{-1}t-\frac{1}{2}\int\frac{t^2}{\sqrt{1-t^2}}dt\right]+C_1$$

$$=\frac{t^2}{3}\sin^{-1}t+\frac{1}{3}\int\frac{-t^2}{\sqrt{1-t^2}}dt+C_1=\frac{t^2}{3}\sin^{-1}t+\frac{1}{3}I_1+C_1 \quad ...(i)$$

$$\text{Now,} \quad I_1=\int\frac{1-t^2-1}{\sqrt{1-t^2}}dt=\int\sqrt{1-t^2}\,dt-\int\frac{1}{\sqrt{1-t^2}}dt$$

$$=\frac{1}{2}\left[t\sqrt{1-t^2}+\sin^{-1}(t)\right]-\sin^{-1}t+C_2 \quad ...(ii)$$

Using (i) and (ii), we get

$$I=\frac{t^2}{3}\sin^{-1}t+\frac{1}{3}\times\frac{1}{2}\left[t\sqrt{1-t^2}+\sin^{-1}t\right]-\frac{1}{3}\sin^{-1}t+C$$

$$=\left(\frac{t^2}{3}-\frac{1}{6}\right)\sin^{-1}t+\frac{1}{6}t\sqrt{1-t^2}+C$$

$$=\left(\frac{2t^2-1}{6}\right)\sin^{-1}t+\frac{1}{6}\,t\sqrt{1-t^2}+C$$

32. (a) We have, $f(x)=\frac{2x}{1+x^2}$

For one-one : Let $f(x_1)=f(x_2) \Rightarrow \frac{2x_1}{1+x_1^2}=\frac{2x_2}{1+x_2^2}$

$\Rightarrow x_1+x_1x_2^2=x_2+x_1^2x_2 \Rightarrow (x_1-x_2)+x_1x_2(x_2-x_1)=0$

$\Rightarrow (x_1-x_2)(1-x_1x_2)=0 \Rightarrow x_1=x_2$ or $x_1x_2=1$, if $x_1\neq x_2$

$\Rightarrow f(x)$ is not one-one function.

For onto : Let $y=\frac{2x}{1+x^2}$

$\Rightarrow y+x^2y-2x=0 \Rightarrow yx^2-2x+y=0$

$\Rightarrow x=\frac{2\pm\sqrt{4-4y^2}}{2y} \Rightarrow x=\frac{1\pm\sqrt{1-y^2}}{y}$

Now, for $f(x)=2=y$, there exists no real value of $x\in R$.

$\therefore$ $f(x)$ is not onto.

We can see that, for $x\in R$, $\sqrt{1-y^2}\geq 0$

$\Rightarrow 1-y^2\geq 0 \Rightarrow y^2\leq 1 \Rightarrow -1\leq y\leq 1$

So, $f(x)$ is onto if $y\in[-1,1]$

$\therefore$ Required set $A=[-1,1]$.

OR

(b) For reflexive : Let $(a, b)\ R\ (a, b)$

$\Rightarrow a-a=b-b \Rightarrow 0=0$

$\therefore$ R is reflexive.

For symmetric : Let $(a, b)\ R\ (c, d)$

$\Rightarrow a-c=b-d \Rightarrow c-a=d-b$

$\Rightarrow (c, d)\ R\ (a, b)$ $\therefore$ R is symmetric.

For transitive : Let $(a, b)\ R\ (c, d)$ and $(c, d)\ R\ (e, f)$

$\Rightarrow a-c=b-d$ ...(i)

and $c-e=d-f$ ...(ii)

Adding (i) and (ii), we get

$a-c+c-e=b-d+d-f$

$\Rightarrow a-e=b-f \Rightarrow (a, b)\ R\ (e, f)$

$\therefore$ R is transitive.

So, R is an equivalence relation.

33. Let the required equation of the line be

$\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$ ...(i)

Now, line (i) passes through the mid-point of $A(2, 3, 4)$ and $B(4, 5, 8)$.

$\therefore$ $x_1=\frac{2+4}{2}=3,\ y_1=\frac{3+5}{2}=4,\ z_1=\frac{4+8}{2}=6$

$\Rightarrow (x_1, y_1, z_1)=(3, 4, 6)$ ...(ii)

Also, line (i) is perpendicular to the lines

$\frac{x-8}{3}=\frac{y+19}{-16}=\frac{z-10}{7}$ and $\frac{x-15}{3}=\frac{y-29}{8}=\frac{z-5}{-5}$

$\therefore$ $3a-16b+7c=0$ ...(iii)

$3a+8b-5c=0$ ...(iv)

Solving (iii) and (iv), we get

$$\frac{a}{16\times 5-7\times 8}=\frac{b}{7\times 3-3\times(-5)}=\frac{c}{3\times 8-3\times(-16)}$$

$\Rightarrow \frac{a}{80-56}=\frac{b}{21+15}=\frac{c}{24+48} \Rightarrow \frac{a}{24}=\frac{b}{36}=\frac{c}{72}$

$\Rightarrow \frac{a}{2}=\frac{b}{3}=\frac{c}{6}$

So, $a=2, b=3, c=6$ ...(v)

Using (i), (ii) and (v), we get the required equation of line is

$\frac{x-3}{2}=\frac{y-4}{3}=\frac{z-6}{6}$

34. (a) The given system of equation is

$\frac{2}{x}+\frac{3}{y}+\frac{10}{z}=4,\ \frac{4}{x}-\frac{6}{y}+\frac{5}{z}=1,\ \frac{6}{x}+\frac{9}{y}-\frac{20}{z}=2$

It can be rewritten as

$$\begin{bmatrix}2 & 3 & 10\\ 4 & -6 & 5\\ 6 & 9 & -20\end{bmatrix}\begin{bmatrix}1/x\\ 1/y\\ 1/z\end{bmatrix}=\begin{bmatrix}4\\ 1\\ 2\end{bmatrix}$$

$A\cdot X=B$ (say)

$\Rightarrow X=A^{-1}B$ ...(i)

where, $A=\begin{bmatrix}2 & 3 & 10\\ 4 & -6 & 5\\ 6 & 9 & -20\end{bmatrix}$, $X=\begin{bmatrix}1/x\\ 1/y\\ 1/z\end{bmatrix}$ and $B=\begin{bmatrix}4\\ 1\\ 2\end{bmatrix}$

Now, $|A|=\begin{vmatrix}2 & 3 & 10\\ 4 & -6 & 5\\ 6 & 9 & -20\end{vmatrix}$

$=2(120-45)-3(-80-30)+10(36+36)$

$=150+330+720=1200\neq 0$

$\therefore$ A^{-1} exists

So, the system of equation has a unique solution.

Now, $adj\ A=\begin{bmatrix}75 & 110 & 72\\ 150 & -100 & 0\\ 75 & 30 & -24\end{bmatrix}^T$

$\Rightarrow adj\ A=\begin{bmatrix}75 & 150 & 75\\ 110 & -100 & 30\\ 72 & 0 & -24\end{bmatrix}$

$$\Rightarrow A^{-1} = \frac{adj A}{|A|} = \frac{1}{1200}\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$$

Now, $$A^{-1}B = \frac{1}{1200}\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}\begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

$$= \frac{1}{1200}\begin{bmatrix} 75\times4+150\times1+75\times2 \\ 110\times4-100\times1+30\times2 \\ 72\times4+0\times1-24\times2 \end{bmatrix}$$

$$= \frac{1}{1200}\begin{bmatrix} 600 \\ 400 \\ 240 \end{bmatrix} = \begin{bmatrix} 1/2 \\ 1/3 \\ 1/5 \end{bmatrix}$$

Using (i), $X = A^{-1}B$

$$\Rightarrow \begin{bmatrix} 1/x \\ 1/y \\ 1/z \end{bmatrix} = \begin{bmatrix} 1/2 \\ 1/3 \\ 1/5 \end{bmatrix}$$

$\Rightarrow$ $x = 2, y = 3, z = 5$; which is the required solution.

OR

(b) We have, $A = \begin{bmatrix} 1 & \cot x \\ -\cot x & 1 \end{bmatrix}$

$$\Rightarrow A' = \begin{bmatrix} 1 & -\cot x \\ \cot x & 1 \end{bmatrix}$$

Also, $|A| = 1 + \cot^2 x = \text{cosec}^2 x \neq 0$ $\therefore$ A^{-1} exists

$$\Rightarrow adj\, A = \begin{bmatrix} 1 & \cot x \\ -\cot x & 1 \end{bmatrix}' = \begin{bmatrix} 1 & -\cot x \\ \cot x & 1 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \frac{1}{\text{cosec}^2 x}\begin{bmatrix} 1 & -\cot x \\ \cot x & 1 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \begin{bmatrix} \frac{1}{1+\cot^2 x} & \frac{-\cot x}{1+\cot^2 x} \\ \frac{\cot x}{1+\cot^2 x} & \frac{1}{1+\cot^2 x} \end{bmatrix}$$

Now, $$A'A^{-1} = \begin{bmatrix} 1 & -\cot x \\ \cot x & 1 \end{bmatrix}\begin{bmatrix} \frac{1}{1+\cot^2 x} & \frac{-\cot x}{1+\cot^2 x} \\ \frac{\cot x}{1+\cot^2 x} & \frac{1}{1+\cot^2 x} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1-\cot^2 x}{1+\cot^2 x} & \frac{-\cot x-\cot x}{1+\cot^2 x} \\ \frac{\cot x+\cot x}{1+\cot^2 x} & \frac{-\cot^2 x+1}{1+\cot^2 x} \end{bmatrix}$$

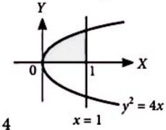
$$= \begin{bmatrix} \frac{\sin^2 x-\cos^2 x}{\sin^2 x+\cos^2 x} & \frac{-2\cos x\sin x}{\sin^2 x+\cos^2 x} \\ \frac{2\sin x\cos x}{\sin^2 x+\cos^2 x} & \frac{-\cos^2 x+\sin^2 x}{\cos^2 x+\sin^2 x} \end{bmatrix}$$

$$= \begin{bmatrix} -\cos 2x & -\sin 2x \\ \sin 2x & -\cos 2x \end{bmatrix}$$

Hence proved.

35. A_1 : Area bounded by $y^2 = 4x$, $x = 1$ in first quadrant

$$\therefore \quad A_1 = \int_0^1 \sqrt{4x}\,dx$$



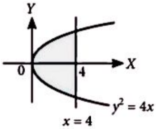
$$= 2\left[\frac{2x^{3/2}}{3}\right]_0^1 = \frac{4}{3} \text{ sq. units}$$

A_2 : Area bounded by $y^2 = 4x$, $x = 4$

$$\Rightarrow A_2 = 2\times\int_0^4 \sqrt{4x}\,dx = 2\times2\left[\frac{2x^{3/2}}{3}\right]_0^4$$

$$= \frac{8}{3}[4^{3/2}-0] = \frac{8}{3}\times8$$

$$= \frac{64}{3} \text{ sq. units}$$



So, $A_1 : A_2 = \frac{4}{3} : \frac{64}{3} = 1:16$

1− 6	20× 4	5	3÷ 2	3÷ 3	1
5	3÷ 1	3	6	24× 4	5+ 2
2− 4	2	12+ 1	5	6	3
1− 1	8+ 3	2	4	11+ 5	6
2	5	2− 6	3× 3	1	9+ 4
9+ 3	6	4	2× 1	2	5

36. (i) We have, $F=\frac{V^2}{500}-\frac{V}{4}+14$

Put $V = 40$ km/hr

$\therefore\ F=\frac{1600}{500}-\frac{40}{4}+14=3.2-10+14=7.2\,l/100\text{ km}$

(ii) $F=\frac{V^2}{500}-\frac{V}{4}+14$

$\therefore\ \frac{dF}{dV}=\frac{2V}{500}-\frac{1}{4}=\frac{V}{250}-\frac{1}{4}$

(iii) (a) For minimum, $\frac{dF}{dV}=0$

$\Rightarrow \frac{V}{250}-\frac{1}{4}=0 \Rightarrow V=\frac{250}{4}$

$\Rightarrow V = 62.5$ km/h

Now, $\frac{d^2F}{dV^2}=\frac{1}{250}>0$, for $V = 62.5$ km/h

So, required speed = 62.5 km/hr

OR

(iii) (b) We have, $\frac{dF}{dV}=\frac{V}{250}-\frac{1}{4}$

$\Rightarrow\ -0.01=\frac{V}{250}-\frac{1}{4}$

$\Rightarrow\ \frac{V}{250}=-0.01+\frac{1}{4}=0.24$

$\Rightarrow V = 250 \times 0.24 = 60$ km/h

So, $F=\frac{60^2}{500}-\frac{60}{4}+14=7.2-15+14=6.2\,l/100\text{ km}$

i.e., Fuel required for 100 km = 6.2 l

$\therefore$ Fuel required for 600 km = 6.2 × 6 = 37.2 litres

37. (i) Constraints are

$x + 2y \geq 10$, $x + y \geq 6$, $3x + y \geq 8$,

$4x + 5y > 28$ and $x \geq 0, y \geq 0$

(ii) Corner points of feasible region are

$A(10, 0)$, $B(2, 4)$, $C(1, 5)$ and $D(0, 8)$

Corner points	Value of $Z = 16x + 20y$
$A(10, 0)$	16 × 10 + 20 × 0 = 160
$B(2, 4)$	16 × 2 + 20 × 4 = 112 (minimum)
$C(1, 5)$	16 × 1 + 20 × 5 = 116
$D(0, 8)$	16 × 0 + 20 × 8 = 160

So, at $B(2, 4)$, the cost is minimum.

The minimum cost is ₹ 112.

38. (i) Probability of an airplane crash = $P(E_1)$ = 0.00001%

$\therefore$ Probability that the airplane will not crash = $P(E_2)$

= 1 – (Probability of an airplane crash)

= 1 – 0.00001% = 0.99999%

(ii) $P(A|E_1)$ = Probability of passengers surviving after the plane crash = 95%

$P(A|E_2)$ = Probability of passengers surviving when there is no crash = 100%

$\therefore\ P(A|E_1)+P(A|E_2)=\frac{95}{100}+\frac{100}{100}=\frac{195}{100}=1.95$

(iii) (a) $P(A)=P(E_1)P(A|E_1)+P(E_2)P(A|E_2)$

[$\because$ By theorem of total probability]

$=\frac{0.00001}{100}\times\frac{95}{100}+\frac{0.99999}{100}\times 1=\frac{99.99995}{10000}=0.0099$

OR

(iii) (b) $P(E_2|A)=\frac{P(E_2)P(A|E_2)}{P(E_1)P(A|E_1)+P(E_2)P(A|E_2)}$

[By Bayes' theorem]

$=\frac{\frac{0.99999}{100}\times 1}{\frac{0.00001}{100}\times\frac{95}{100}+\frac{0.99999}{100}\times 1}$

$=\frac{0.99999}{100}\times\frac{10000}{99.99995}=0.99\%$

SOLUTIONS TO MARCH 2024 QUIZ CLUB

1. –1
2. 5^6
3. $\frac{1}{81}$
4. 8
5. $\frac{2\sqrt{5}}{6}$
6. 13
7. 1
8. 6
9. 5
10. $\frac{1}{7}$
11. 8 !
12. $K^3|A|$
13. $\frac{7}{2}$
14. 0
15. 240 sq. cm/sec
16. $\frac{\sin^{-1}(2^x)}{\log 2}+C$
17. 2 sq. units
18. 3
19. 32
20. $\frac{1}{\sqrt{2}}$