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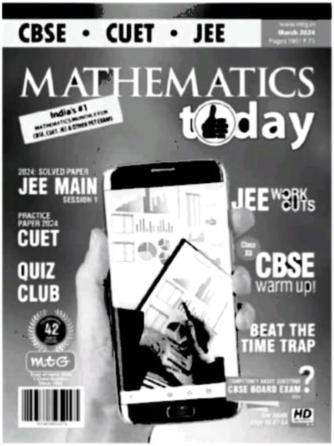

MATHEMATICS today

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Contents

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SOLVED PAPER

Held on 27th Jan, Morning Shift (Session - 1)

JEE MAIN 2024

SECTION-A (MULTIPLE CHOICE QUESTIONS)

1. If (a, b) be the orthocentre of the triangle whose vertices are (1, 2), (2, 3) and (3, 1), and $I_1 = \int_a^b x\sin(4x - x^2)dx$, $I_2 = \int_a^b \sin(4x - x^2)dx$, then $36\frac{I_1}{I_2}$ is equal to

(a) 66 (b) 88 (c) 72 (d) 80

2. If A denotes the sum of all the coefficients in the expansion of $(1 - 3x + 10x^2)^n$ and B denotes the sum of all the coefficients in the expansion of $(1 + x^2)^n$, then

(a) $B = A^3$ (b) $3A = B$ (c) $A = 3B$ (d) $A = B^3$

3. The distance of the point (7, –2, 11) from the line $\frac{x-6}{1} = \frac{y-4}{0} = \frac{z-8}{3}$ along the line $\frac{x-5}{2} = \frac{y-1}{-3} = \frac{z-5}{6}$ is

(a) 18 (b) 12 (c) 21 (d) 14

4. If $S = \{z \in \mathbf{C} : |z - i| = |z + i| = |z - 1|\}$, then $n(S)$ is

(a) 1 (b) 0 (c) 3 (d) 2

5. Let $a_1, a_2, \ldots, a_{10}$ be 10 observations such that $\sum_{k=1}^{10} a_k = 50$ and $\sum_{\forall k<j} a_k \cdot a_j = 1100$. Then the standard deviation of $a_1, a_2, \ldots, a_{10}$ is equal to

(a) 5 (b) $\sqrt{115}$ (c) $\sqrt{5}$ (d) 10

6. Four distinct points $(2k, 3k)$, (1, 0), (0, 1) and (0, 0) lie on a circle for k equal to

(a) $\frac{2}{13}$ (b) $\frac{5}{13}$ (c) $\frac{3}{13}$ (d) $\frac{1}{13}$

7. If the shortest distance between the lines $\frac{x-4}{1} = \frac{y+1}{2} = \frac{z}{-3}$ and $\frac{x-\lambda}{2} = \frac{y+1}{4} = \frac{z-2}{-5}$ is $\frac{6}{\sqrt{5}}$, then the sum of all possible values of λ is

(a) 5 (b) 7 (c) 8 (d) 10

8. Let $S = \{1, 2, 3, \ldots, 10\}$. Suppose M is the set of all the subsets of S, then the relation $R = \{(A, B) : A \cap B \neq \phi;\ A, B \in M\}$ is

(a) symmetric and transitive only
(b) reflexive only
(c) symmetric and reflexive only
(d) symmetric only

9. The function $f : N - \{1\} \to N$; defined by $f(n)$ = the highest prime factor of n, is

(a) one-one only
(b) both one-one and onto
(c) neither one-one nor onto
(d) onto only

10. Consider the matrix $f(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

Given below are two statements :

Statement I : $f(-x)$ is the inverse of the matrix $f(x)$.

Statement II : $f(x)\, f(y) = f(x + y)$.

In the light of the above statements, choose the correct answer from the options given below.

(a) Statement I is true but Statement II is false
(b) Both Statement I and Statement II are false
(c) Statement I is false but Statement II is true
(d) Both Statement I and Statement II are true

11. The length of the chord of the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$, whose mid point is $\left(1, \frac{2}{5}\right)$, is equal to

(a) $\frac{\sqrt{1541}}{5}$ (b) $\frac{\sqrt{1691}}{5}$ (c) $\frac{\sqrt{1741}}{5}$ (d) $\frac{\sqrt{2009}}{5}$

12. The number of common terms in the progressions 4, 9, 14, 19, ..., up to 25th term and 3, 6, 9, 12, ..., up to 37th term is

(a) 8 (b) 7 (c) 5 (d) 9

13. ${}^{n-1}C_r = (k^2 - 8)\,{}^nC_{r+1}$ if and only if

(a) $2\sqrt{2} < k \le 3$ (b) $2\sqrt{3} < k \le 3\sqrt{2}$

(c) $2\sqrt{2} < k < 2\sqrt{3}$ (d) $2\sqrt{3} < k < 3\sqrt{3}$

14. The portion of the line $4x + 5y = 20$ in the first quadrant is trisected by the lines L_1 and L_2 passing through the origin. The tangent of an angle between the lines L_1 and L_2 is

(a) $\frac{30}{41}$ (b) $\frac{25}{41}$ (c) $\frac{2}{5}$ (d) $\frac{8}{5}$

15. Let $x = x(t)$ and $y = y(t)$ be solutions of the differential equations $\frac{dx}{dt} + ax = 0$ and $\frac{dy}{dt} + by = 0$ respectively, $a, b \in \mathbf{R}$. Given that $x(0) = 2$; $y(0) = 1$ and $3y(1) = 2x(1)$, the value of t, for which $x(t) = y(t)$, is

(a) $\log_{\frac{4}{3}} 2$ (b) $\log_{\frac{2}{3}} 2$ (c) $\log_4 3$ (d) $\log_3 4$

16. If the shortest distance of the parabola $y^2 = 4x$ from the centre of the circle $x^2 + y^2 - 4x - 16y + 64 = 0$ is d, then d^2 is equal to

(a) 16 (b) 24 (c) 20 (d) 36

17. Consider the function,

$$f(x) = \begin{cases} \dfrac{a(7x - 12 - x^2)}{b|x^2 - 7x + 12|}, & x < 3 \\ 2^{\frac{\sin(x-3)}{x - [x]}}, & x > 3 \\ b, & x = 3 \end{cases}$$

where $[x]$ denotes the greatest integer less than or equal to x. If S denotes the set of all ordered pairs (a, b) such that $f(x)$ is continuous at $x = 3$, then the number of elements in S is

(a) 2 (b) Infinitely many

(c) 1 (d) 4

18. If $\int_0^1 \frac{1}{\sqrt{3+x} + \sqrt{1+x}}\,dx = a + b\sqrt{2} + c\sqrt{3}$, where a, b, c are rational numbers, then $2a + 3b - 4c$ is equal to

(a) 10 (b) 7 (c) 4 (d) 8

19. Let $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$, $\vec{b} = 3(\hat{i} - \hat{j} + \hat{k})$. Let $\vec{c}$ be the vector such that $\vec{a} \times \vec{c} = \vec{b}$ and $\vec{a} \cdot \vec{c} = 3$. Then $\vec{a} \cdot ((\vec{c} \times \vec{b}) - \vec{b} - \vec{c})$ is equal to

(a) 36 (b) 20 (c) 24 (d) 32

20. If $a = \lim_{x \to 0} \frac{\sqrt{1 + \sqrt{1 + x^4}} - \sqrt{2}}{x^4}$ and $b = \lim_{x \to 0} \frac{\sin^2 x}{\sqrt{2} - \sqrt{1 + \cos x}}$, then the value of ab^3 is

(a) 25 (b) 36 (c) 32 (d) 30

SECTION-B (NUMERICAL VALUE TYPE)

Attempt any 5 questions out of 10.

21. If $8 = 3 + \frac{1}{4}(3 + p) + \frac{1}{4^2}(3 + 2p) + \frac{1}{4^3}(3 + 3p) + \cdots \infty$, then the value of p is ______.

22. Let for a differentiable function $f : (0, \infty) \to R$, $f(x) - f(y) \ge \log_e\left(\frac{x}{y}\right) + x - y$, $\forall\, x, y \in (0, \infty)$. Then $\sum_{n=1}^{20} f'\left(\frac{1}{n^2}\right)$ is equal to ______.

23. If the solution of the differential equation $(2x + 3y - 2)\,dx + (4x + 6y - 7)\,dy = 0$, $y(0) = 3$, is $\alpha x + \beta y + 3\log_e|2x + 3y - \gamma| = 6$, then $\alpha + 2\beta + 3\gamma$ is equal to ______.

24. If α satisfies the equation $x^2 + x + 1 = 0$ and $(1 + \alpha)^7 = A + B\alpha + C\alpha^2$, $A, B, C \ge 0$, then $5(3A - 2B - C)$ is equal to ______.

25. Let $A = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$, $B = [B_1, B_2, B_3]$, where B_1, B_2, B_3 are column matrices, and $AB_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $AB_2 = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}$, $AB_3 = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$.

If $\alpha = |B|$ and β is the sum of all the diagonal elements of B, then $\alpha^3 + \beta^3$ is equal to _______.

26. Let the area of the region $\{(x, y) : x - 2y + 4 \ge 0, x + 2y^2 \ge 0, x + 4y^2 \le 8, y \ge 0\}$ be $\frac{m}{n}$, where m and n are coprime numbers. Then $m + n$ is equal to _____.

27. A fair die is tossed repeatedly until a six is obtained. Let X denote the number of tosses required and let $a = P(X = 3)$, $b = P(X \ge 3)$ and $c = P(X \ge 6 \mid X > 3)$. Then $\frac{b + c}{a}$ is equal to ______.

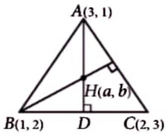
28. The least positive integral value of α, for which the angle between the vectors $\alpha\hat{i} - 2\hat{j} + 2\hat{k}$ and $\alpha\hat{i} + 2\alpha\hat{j} - 2\hat{k}$ is acute, is _______.

29. Let the set of all $a \in R$ such that the equation $\cos 2x + a \sin x = 2a - 7$ has a solution be $[p, q]$ and $r = \tan 9° - \tan 27° - \frac{1}{\cot 63°} + \tan 81°$, then pqr is equal to ______.

30. Let $f(x) = x^3 + x^2 f'(1) + x f''(2) + f'''(3)$, $x \in R$. Then $f'(10)$ is equal to ______.

SOLUTIONS

1. (c) : Let the vertices of the triangle are $A(3, 1)$, $B(1, 2)$ and $C(2, 3)$.



Slope of $BC = \frac{3-2}{2-1} = 1$

Since, AD is perpendicular to BC.

Slope of $AD = -1$

Equation of line AD is

$y - 1 = -1(x - 3)$

Since, (a, b) lies on AD.

$\therefore \ b - 1 = -1(a - 3) \Rightarrow a + b = 4$

Now, $I_1 = \int_a^b x\sin[(4-x)x]dx$

$I_1 = 4\int_a^b \sin[(4-x)x]\,dx - \int_a^b x\sin[(4-x)x]\,dx$

$I_1 = 4I_2 - I_1 \qquad \left[\because \int_a^b \sin[(4-x)x]\,dx = I_2\right]$

$\Rightarrow 2I_1 = 4I_2 \Rightarrow \frac{I_1}{I_2} = 2 \ \therefore \ \frac{36I_1}{I_2} = 36\times 2 = 72$

2. (d) : Putting $x = 1$ in $(1 - 3x + 10x^2)^n$, we get

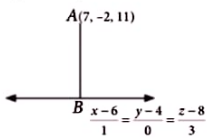
$A = (1 - 3 + 10)^n = 8^n$ ...(i)

Putting $x = 1$ in $(1 + x^2)^n$, we get

$B = (1 + 1)^n = 2^n$ ...(ii)

From (i) and (ii), we get $A = B^3$

3. (d) : Let A be the point $(7, -2, 11)$.



Equation of line AB is

$\frac{x-7}{2} = \frac{y+2}{-3} = \frac{z-11}{6} = \lambda$

Point B is of the form $(2\lambda + 7, -3\lambda - 2, 6\lambda + 11)$.

Now, point B lies on the given line.

$\therefore \ \frac{2\lambda+7-6}{1} = \frac{-3\lambda-2-4}{0} = \frac{6\lambda+11-8}{3}$

$\Rightarrow -3\lambda - 6 = 0 \Rightarrow \lambda = -2$

Thus point B is $(3, 4, -1)$.

$\therefore$ Required distance, $AB = \sqrt{4^2 + (-6)^2 + 12^2} = 14$ units

4. (a) : Given, $|z - i| = |z + i| = |z - 1|$ ...(i)

Putting $z = x + iy$ in (i), we get

$|x + iy - i| = |x + iy + i| = |x + iy - 1|$

Now, $|x + i(y - 1)| = |x - 1 + iy|$

$\Rightarrow x^2 + (y - 1)^2 = (x - 1)^2 + y^2 \Rightarrow x - y = 0$ ...(ii)

Again, $|x + i(y + 1)| = |x - 1 + iy|$

$\Rightarrow x^2 + (y + 1)^2 = (x - 1)^2 + y^2 \Rightarrow x + y = 0$ ...(iii)

From (ii) and (iii), we get $x = 0$ and $y = 0$

Thus, $z = 0 + i0 \ \therefore \ n(S) = 1$

5. (c) : Given, $\sum_{k=1}^{10} a_k = 50$ and $\sum_{\forall k<j} a_k \cdot a_j = 1100$

$\sigma^2 = \frac{\sum a_k^2}{10} - (\text{Mean})^2 = \frac{\sum a_k^2}{10} - \left(\frac{\sum a_k}{10}\right)^2 = \frac{\sum a_k^2}{10} - 25$ ...(i)

Also, $\left(\sum a_k\right)^2 = \sum a_k^2 + 2\sum_{\forall k<j} a_k a_j$

$\Rightarrow \ \sum a_k^2 = 2500 - 2200 = 300$

From (i), we get $\sigma^2 = \frac{300}{10} - 25 = 30 - 25 = 5 \Rightarrow \sigma = \sqrt{5}$

6. (b)

7. (c): Here, $x_1 = 4, y_1 = -1, z_1 = 0$ and $x_2 = \lambda, y_2 = -1, z_2 = 2$

$a_1 = 1, b_1 = 2, c_1 = -3$ and $a_2 = 2, b_2 = 4, c_2 = -5$

$$\because \ \frac{6}{\sqrt{5}} = \left|\frac{\begin{vmatrix} \lambda-4 & 0 & 2 \\ 1 & 2 & -3 \\ 2 & 4 & -5 \end{vmatrix}}{\sqrt{(-10+12)^2 + (-6+5)^2 + (4-4)^2}}\right|$$

$\Rightarrow \ \frac{6}{\sqrt{5}} = \left|\frac{2\lambda - 8}{\sqrt{5}}\right| \Rightarrow \lambda = 7$ or 1

8. (d) : Given, $A \in M \Rightarrow A \cap A = \phi$, if $A = \phi$

So, R is not reflexive.

If $(A, B) \in R \Rightarrow A \cap B \neq \phi \Rightarrow B \cap A \neq \phi \Rightarrow (B, A) \in R$

So, R is symmetric.

Form IV

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I, Mahabir Singh, authorised signatory for MTG Learning Media Pvt. Ltd. hereby declare that particulars given above are true to the best of my knowledge and belief.

For MTG Learning Media Pvt. Ltd.
Mahabir Singh
Director

If (A, B) and $(B, C) \in R \Rightarrow A \cap B \neq \phi$ and $B \cap C \neq \phi$
$\Rightarrow A \cap C$ is not necessarily non-empty set.
Hence, $A \cap C$ not necessarily belongs to R.
So, R is not a transitive relation.
$\therefore$ Given relation is symmetric only.

9. (c) : Given, $f: N - \{1\} \to N$
$f(n)$ = the highest prime factor of n
For one-one : If $n = 4, f(n) = 2$
If $n = 8$, $f(n) = 2$ $\quad \therefore f$ is not one-one.
For onto : Range = All prime numbers
Co-domain = Set of natural numbers
$\Rightarrow$ Range $\neq$ Co-domain $\quad \therefore f$ is not onto

10. (d)

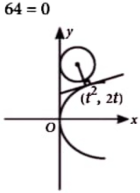
11. (b) : Ellipse : $\dfrac{x^2}{25} + \dfrac{y^2}{16} = 1$ ...(i)
Equation of chord having mid point (x_1, y_1) is
$$\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = \frac{x_1^2}{a^2} + \frac{y_1^2}{b^2} \Rightarrow \frac{x}{25} + \frac{y}{40} = \frac{1}{25} + \frac{1}{100}$$
$\Rightarrow 8x = 5(2 - y)$ ...(ii)
From (i) & (ii), we get
$$x = 1 \pm \frac{\sqrt{19}}{2} \text{ and } y = \frac{2}{5}(1 \mp 2\sqrt{19})$$
$\therefore$ Length of chord $= \dfrac{\sqrt{1691}}{5}$

12. (b) : 4, 9, 14, 19, ... are in A.P. with $d_1 = 5, n_1 = 25$
3, 6, 9, 12, ... are in A.P. with $d_2 = 3, n_2 = 37$
L.C.M. $(d_1, d_2) = 15$
Common terms = 9, 24, 39, 54, 69, 84, 99

13. (a) : We have, ${}^{n-1}C_r = (k^2 - 8)\,{}^nC_{r+1}$
$$\Rightarrow k^2 - 8 = \frac{r+1}{n} \leq 1 \qquad [\because n \geq r + 1]$$
$\Rightarrow k^2 - 8 \leq 1 \Rightarrow k^2 - 9 \leq 0 \Rightarrow -3 \leq k \leq 3$ ...(i)
But $k^2 > 8$, as ${}^{n-1}C_r$ can't be negative and 0.
$\Rightarrow k^2 - 8 > 0 \Rightarrow (k - 2\sqrt{2})(k + 2\sqrt{2}) > 0$
$\Rightarrow k < -2\sqrt{2}$ or $k > 2\sqrt{2}$ ...(ii)
From (i) and (ii), we get $2\sqrt{2} < k \leq 3$ or $-3 \leq k < -2\sqrt{2}$

14. (a) **15. (a)**

16. (c) : Circle : $x^2 + y^2 - 4x - 16y + 64 = 0$
Centre $\equiv (2, 8)$, Radius = 2 units
Parabola : $y^2 = 4x$; $a = 1$
Equation of normal at $(t^2, 2t)$ of parabola : $xt + y = t^3 + 2t$ ...(i)
Since, normal passes through centre, then $2t + 8 = t^3 + 2t$ ...(ii)
$\Rightarrow t^3 - 8 = 0$
$\Rightarrow (t - 2)(t^2 + 4 + 2t) = 0$
$\Rightarrow t = 2 \qquad (\because t \geq 0)$



d = distance between (2, 8) and (4, 4) $= \sqrt{4+16} = \sqrt{20}$
$\Rightarrow d^2 = 20$

17. (c) : As $f(x)$ is continuous at $x = 3$.
So, L.H.L. = R.H.L. = $f(a)$
$$\Rightarrow \lim_{x \to 3^-} \frac{a(7x - 12 - x^2)}{b|x^2 - 7x + 12|} = \lim_{x \to 3^+} 2^{\frac{\sin(x-3)}{x - [x]}} = b$$
$$\Rightarrow \frac{-a}{b} = 2^1 = b \qquad \left[\because \lim_{n \to 0} \frac{\sin n}{n} = 1\right]$$
$\Rightarrow b = 2, a = -4 \quad \therefore$ Number of elements in $S = 1$

18. (d) : $\displaystyle\int_0^1 \frac{1}{\sqrt{3+x} + \sqrt{1+x}}\, dx$
$$= \int_0^1 \frac{\sqrt{3+x} - \sqrt{1+x}}{(3+x) - (1+x)}\, dx = \frac{1}{2}\int_0^1 \left(\sqrt{3+x} - \sqrt{1+x}\right) dx$$
$$= 3 - \frac{2\sqrt{2}}{3} - \sqrt{3} = a + b\sqrt{2} + c\sqrt{3} \Rightarrow a = 3, b = \frac{-2}{3}, c = -1$$
$$\therefore 2a + 3b - 4c = 2 \times 3 + 3 \times \left(\frac{-2}{3}\right) - 4(-1) = 8$$

19. (c) : $\vec{a} \cdot ((\vec{c} \times \vec{b}) - \vec{b} - \vec{c}) = \vec{a} \cdot (\vec{c} \times \vec{b}) - \vec{a} \cdot \vec{b} - \vec{a} \cdot \vec{c}$
[Using distributive property of multiplication]
$= [\vec{a}\ \vec{c}\ \vec{b}] - \vec{a} \cdot \vec{b} - \vec{a} \cdot \vec{c}$ ...(i)
Now, $\vec{a} \times \vec{c} = \vec{b} \Rightarrow (\vec{a} \times \vec{c}) \cdot \vec{b} = \vec{b} \cdot \vec{b}$
$\Rightarrow [\vec{a}\ \vec{c}\ \vec{b}] = |\vec{b}|^2 = \left(\sqrt{(3)^2 + (-3)^2 + (3)^2}\right)^2 = 27$
$\Rightarrow [\vec{a}\ \vec{c}\ \vec{b}] = 27$ ...(ii)
$\vec{a} \cdot \vec{b} = 3(\hat{i} + 2\hat{j} + \hat{k})(\hat{i} - \hat{j} + \hat{k}) = 3(1 - 2 + 1) = 0$ ...(iii)
and $\vec{a} \cdot \vec{c} = 3$ ...(iv)
Using (ii), (iii) and (iv) in (i), we get
$\vec{a} \cdot ((\vec{c} \times \vec{b}) - \vec{b} - \vec{c}) = 27 - 0 - 3 = 24$

20. (c) : $a = \displaystyle\lim_{x \to 0} \frac{\sqrt{1 + \sqrt{1 + x^4}} - \sqrt{2}}{x^4}$
Rationalising numerator, we get
$$a = \lim_{x \to 0} \frac{\sqrt{1 + x^4} - 1}{x^4\left(\sqrt{1 + \sqrt{1 + x^4}} + \sqrt{2}\right)}$$
Again rationalising numerator, we get
$$a = \lim_{x \to 0} \frac{1}{\left(\sqrt{1 + x^4} + 1\right)\left(\sqrt{1 + \sqrt{1 + x^4}} + \sqrt{2}\right)} = \frac{1}{2 \times 2\sqrt{2}} = \frac{1}{4\sqrt{2}}$$
Now, $b = \displaystyle\lim_{x \to 0} \frac{\sin^2 x}{\sqrt{2} - \sqrt{1 + \cos x}}$
$$= \lim_{x \to 0} \frac{(1 - \cos^2 x)(\sqrt{2} + \sqrt{1 + \cos x})}{[2 - (1 + \cos x)]}$$
[By rationalising denominator]

$$= \lim_{x\to 0}(1+\cos x)(\sqrt{2}+\sqrt{1+\cos x}) = 4\sqrt{2}$$

$$\therefore \quad ab^3 = \frac{1}{4\sqrt{2}} \times (4\sqrt{2})^3 = 32$$

21. (9) :

$$8=3+\frac{1}{(4)}(3+p)+\frac{1}{(4)^2}(3+2p)+\frac{1}{(4)^3}(3+3p)+\ldots\infty \quad \ldots(i)$$

Multiplying both sides by 1/4, we get

$$2=\frac{3}{4}+\frac{3+p}{(4)^2}+\frac{3+2p}{(4)^3}+\ldots\ldots+\infty \quad \ldots(ii)$$

Subtracting (ii) from (i), we get

$$3=p\left[\frac{1}{4}+\frac{1}{(4)^2}+\frac{1}{(4)^3}+\ldots\ldots+\infty\right]$$

$$\Rightarrow \; 3=p\left[\frac{\frac{1}{4}}{1-\frac{1}{4}}\right] \left[\because S_\infty = \frac{a}{1-r} \text{ for infinite geometric series}\right]$$

$$\Rightarrow \; 3=p\left[\frac{1}{4}\times\frac{4}{3}\right] \quad \Rightarrow p = 9$$

22. (2890) :

$$f(x)-f(y) \ge \log_e\left(\frac{x}{y}\right)+x-y \,\forall (x,y)\in(0,\infty) \quad \ldots(i)$$

Now, $f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\log_e \frac{\left(\frac{x+h}{x}\right)+h}{h}$

$$= \lim_{h\to 0}\left[\frac{\log_e\left(1+\frac{h}{x}\right)}{h}+1\right] = \frac{1}{x}+1 \quad \ldots(ii)$$

Now, $\sum_{n=1}^{20} f'\left(\frac{1}{n^2}\right) = \sum_{n=1}^{20}\frac{1}{\frac{1}{n^2}}+1$ [Using (i)]

$$= \sum_{n=1}^{20}(n^2+1) = \frac{20\times 21\times 41}{6}+20 = 2890$$

23. (29) : $(2x + 3y - 2)\, dx + (4x + 6y - 7)\, dy = 0$

$$\frac{dy}{dx} = \frac{-(2x+3y-2)}{(4x+6y-7)} \quad \ldots(i)$$

Let $2x + 3y = t$

$$\Rightarrow \; 2+\frac{3dy}{dx} = \frac{dt}{dx} \;\Rightarrow\; \frac{dy}{dx} = \frac{1}{3}\left[\frac{dt}{dx}-2\right]$$

Putting in (i), we get

$$\frac{1}{3}\left[\frac{dt}{dx}\right]-\frac{2}{3} = \frac{-(t-2)}{2t-7}$$

$$\Rightarrow \frac{1}{3}\left[\frac{dt}{dx}\right] = \frac{2}{3}-\frac{t-2}{2t-7} = \frac{4t-14-3t+6}{3(2t-7)} \;\Rightarrow dx = \frac{2t-7}{t-8}dt$$

$$\Rightarrow \; \int dx + c = \int 2dt + 9\int\frac{dt}{t-8} \Rightarrow x + c = 2t + 9\ln|t-8|$$

$$\Rightarrow \; 2(2x + 3y) + 9 \ln |2x + 3y - 8| = x + c \quad \ldots(i)$$

which is the solution of given differential equation.

Now, $y(0) = 3 \;\Rightarrow\; 2(9) + 9 \ln |1| = c \Rightarrow c = 18$

Putting the value of c in (i), we get

$$4x + 6y + 9 \ln |2x + 3y - 8| = x + 18$$

$$\Rightarrow \; 3x + 6y + 9 \ln |2x + 3y - 8| = 18$$

$$\Rightarrow \; x + 2y + 3 \ln |2x + 3y - 8| = 6$$

$\therefore \; \alpha = 1, \beta = 2, \gamma = 8$

Therefore, $\alpha + 2\beta + 3\gamma = 1 + 2(2) + 3(8) = 1 + 4 + 24 = 29$

24. (5) : As α satisfies $x^2 + x + 1 = 0 \Rightarrow \alpha = \omega$ or ω^2

Now, $(1 + \alpha)^7 = (1 + \omega)^7 = A + B\omega + C\omega^2$

$$\Rightarrow \; -\omega^2 = A + B\omega + C\omega^2 \quad [\because 1 + \omega + \omega^2 = 0 \text{ and } \omega^3 = 1]$$

$$\Rightarrow \; A + B\omega + (C + 1)\omega^2 = 0$$

As $\omega = -\frac{1}{2}+\frac{\sqrt{3}}{2}i$ & $\omega^2 = -\frac{1}{2}-\frac{\sqrt{3}}{2}i$

$$\Rightarrow \; A+B\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)+(C+1)\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)=0$$

$$\Rightarrow \; A-\frac{B}{2}-\frac{C+1}{2}=0 \;\&\; \frac{\sqrt{3}}{2}B-\frac{\sqrt{3}}{2}(C+1)=0$$

$$\Rightarrow \; 2A - B - C - 1 = 0 \;\;\&\;\; B - C = 1$$

On solving, we get $A = B$ and $B - C = 1$

$\therefore \; 5(3A - 2B - C) = 5(3B - 2B - C) = 5(B - C) = 5 \times 1 = 5$

25. (28) : Let $B_1 = \begin{bmatrix} a_1 \\ b_1 \\ c_1 \end{bmatrix}$, $AB_1 = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}\begin{bmatrix} a_1 \\ b_1 \\ c_1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$$\Rightarrow \; 2a_1 + c_1 = 1,\; a_1 + b_1 = 0,\; a_1 + c_1 = 0$$

On solving, we get $a_1 = 1, b_1 = -1, c_1 = -1$ ...(i)

Similarly, let $B_2 = \begin{bmatrix} a_2 \\ b_2 \\ c_2 \end{bmatrix}$, $AB_2 = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}\begin{bmatrix} a_2 \\ b_2 \\ c_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}$

$$\Rightarrow \; 2a_2 + c_2 = 2,\; a_2 + b_2 = 3 \text{ and } a_2 + c_2 = 0$$

On solving, we get $a_2 = 2, b_2 = 1$ and $c_2 = -2$ ...(ii)

Similarly, let $B_3 = \begin{bmatrix} a_3 \\ b_3 \\ c_3 \end{bmatrix}$, $AB_3 = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}\begin{bmatrix} a_3 \\ b_3 \\ c_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$

$$\Rightarrow \; 2a_3 + c_3 = 3,\; a_3 + b_3 = 2 \text{ and } a_3 + c_3 = 1$$

$$\Rightarrow \; a_3 = 2,\; b_3 = 0 \text{ and } c_3 = -1 \quad \ldots(iii)$$

MONTHLY TEST DRIVE CLASS XII ANSWER KEY

1. (a)	2. (a)	3. (a)	4. (b)	5. (b)
6. (a)	7. (a,b)	8. (a,d)	9. (c,d)	10. (a,b)
11. (c)	12. (a,b,d)	13. (c)	14. (b)	15. (c)
16. (a)	17. (27)	18. (3)	19. (164)	20. (4)

Thus, $B=\begin{bmatrix}1 & 2 & 2\\ -1 & 1 & 0\\ -1 & -2 & -1\end{bmatrix}$ [By using (i), (ii) & (iii)]

$\alpha = |B| = 1(-1+0) - 2(1-0) + 2(2+1) = -1 - 2 + 6 = 3$

and β = sum of diagonal elements of $B = 1 + 1 + (-1) = 1$

Hence, $\alpha^3 + \beta^3 = (3)^3 + (1)^3 = 28$

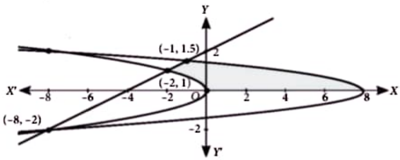
26. (119) : Given, $x - 2y + 4 = 0$ ...(i)

$x + 2y^2 = 0$ ...(ii)

$x + 4y^2 = 8$ ...(iii)

The points of intersection of (i) and (iii) are $(-1, 1.5)$ and $(-8, -2)$.

The points of intersection of (i) and (ii) are $(-2, 1)$ and $(-8, -2)$.



$\therefore$ Required area

$$\int_0^1\left[(8-4y^2)-(-2y^2)\right]dy+\int_1^{3/2}\left[(8-4y^2)-(2y-4)\right]dy$$

$$=\left[8y-\frac{2y^3}{3}\right]_0^1+\left[12y-y^2-\frac{4}{3}y^3\right]_1^{3/2}$$

$$=\frac{22}{3}+18-\frac{9}{4}-\frac{9}{2}-\frac{29}{3}=\frac{107}{12}=\frac{m}{n}\quad\therefore\ m+n=119$$

27. (12) : Probability of getting six = 1/6

Probability of not getting a six = 5/6

Let X denote the number of tosses required. Then,

$$a=P(X=3)=\frac{5}{6}\times\frac{5}{6}\times\frac{1}{6}=\frac{5^2}{6^3}=\frac{25}{216}$$

$$b=P(X\geq 3)=1-[P(X=1)+P(X=2)]$$

$$=1-\left[\frac{1}{6}+\frac{5}{6}\times\frac{1}{6}\right]=\frac{25}{36}$$

$$c=P(X\geq 6\,|\,X>3)=\frac{P[(X\geq 6)\cap(X>3)]}{P(X>3)}=\frac{P(X\geq 6)}{P(X>3)}$$

$$=\frac{\left(\frac{5}{6}\right)^5\times\frac{1}{6}+\left(\frac{5}{6}\right)^6\times\frac{1}{6}+\ldots\ldots\ldots}{1-\frac{1}{6}-\left(\frac{5}{6}\right)\times\left(\frac{1}{6}\right)-\left(\frac{5}{6}\right)^2\times\left(\frac{1}{6}\right)}$$

$$=\frac{\left(\frac{5}{6}\right)^5\times\frac{1}{6}\left[1+\frac{5}{6}+\left(\frac{5}{6}\right)^2+\ldots\ldots\right]}{\frac{25}{36}-\frac{25}{216}}=\frac{25}{36}$$

$$\therefore\quad \frac{b+c}{a}=\frac{\frac{25}{36}+\frac{25}{36}}{\frac{25}{216}}=12$$

28. (5) : $\cos\theta=\dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$

As θ is acute, so $\cos\theta > 0$.

$$\frac{(\alpha\hat{i}-2\hat{j}+2\hat{k})\cdot(\alpha\hat{i}+2\alpha\hat{j}-2\hat{k})}{\sqrt{\alpha^2+(-2)^2+(2)^2}\times\sqrt{\alpha^2+(2\alpha)^2+(-2)^2}}>0$$

$$\Rightarrow\quad\frac{\alpha^2-4\alpha-4}{\sqrt{\alpha^2+8}\times\sqrt{5\alpha^2+4}}>0$$

$\Rightarrow\ \alpha^2 - 4\alpha - 4 > 0 \Rightarrow \alpha^2 - 4\alpha + 4 - 4 - 4 > 0$

$\Rightarrow\ (\alpha-2)^2 - 8 > 0 \Rightarrow (\alpha-2)^2 > 8$

$\Rightarrow\ \alpha - 2 > 2\sqrt{2}$ or $\alpha - 2 < -2\sqrt{2}$ (Neglect)

$\Rightarrow\ \alpha > 2 + 2\sqrt{2} = 4.83$

$\Rightarrow$ Least positive integral value of $\alpha = 5$

29. (48) : $\cos 2x + a\sin x = 2a - 7$

$1 - 2\sin^2 x + a\sin x = 2a - 7$

Let $t = \sin x\quad\therefore\ 2t^2 - at + 2a - 8 = 0$

$\Rightarrow\ (t-2)(2t+4-a) = 0 \Rightarrow t = 2$ or $t=\dfrac{a-4}{2}$

$\therefore\ \sin x = 2$ (not possible) or $2\sin x = a - 4$

$\Rightarrow\ a = 2\sin x + 4 \in [2, 6]$ $[\because \sin x \in [-1, 1]]$

So, $p = 2$, $q = 6$

Now, $r=\tan 9^\circ-\tan 27^\circ-\dfrac{1}{\cot 63^\circ}+\tan 81^\circ$

$= (\tan 9^\circ + \cot 9^\circ) - (\tan 27^\circ + \cot 27^\circ)$

$= 2\operatorname{cosec}18^\circ - 2\operatorname{cosec}54^\circ$ $[\because \tan\theta + \cot\theta = 2\operatorname{cosec}2\theta]$

$$=2\left[\frac{4}{\sqrt{5}-1}-\frac{4}{\sqrt{5}+1}\right]=4\Rightarrow r=4$$

$\therefore\ pqr = 2\times 6\times 4 = 48$

30. (202) : $f(x) = x^3 + x^2 f'(1) + x f''(2) + f'''(3),\ x\in R$

$f'(x) = 3x^2 + 2x f'(1) + f''(2)$,

$f''(x) = 6x + 2f'(1)$

$f'''(x) = 6$

$\Rightarrow\ f'''(3) = 6$

$f''(2) = 6(2) + 2f'(1) = 12 + 2f'(1)$,

$f'(1) = 3(1)^2 + 2(1)f'(1) + f''(2) = 3 + 2f'(1) + f''(2)$

$\Rightarrow\ f'(1) = 3 + 2f'(1) + 12 + 2f'(1)$

$\Rightarrow\ 3f'(1) = -15 \Rightarrow f'(1) = -5$

$\therefore\ f''(2) = 12 + 2(-5) = 2$

Now, $f'(10) = 3(10)^2 + 2(10)f'(1) + f''(2)$

$= 300 + 20(-5) + 2 = 202$

JEE WORK OUTS

SINGLE OPTION CORRECT TYPE

1. Consider the set A of all determinants of order 3 with entries 0 or 1 only. Let B be the subset of A consisting of all determinants with value 1. Let C be the subset of A consisting of all determinants with value -1. Then
   (a) C is empty
   (b) B has as many elements as C
   (c) $A = B \cup C$
   (d) B has twice as many elements as C

2. Let $f(x) = \begin{cases} (x-1)^2 \sin\left(\dfrac{1}{x-1}\right) - |x|, & x \neq 1 \\ -1, & x = 1 \end{cases}$.
   Then $f(x)$ is differentiable for
   (a) all x (b) all x except 0
   (c) all x except 0 and 1 (d) all x except 1

3. Let LL' be the latus rectum of the parabola $y^2 = 4x$ and PP' is a double ordinate between the vertex and LL'. If the area of the trapezium is maximum, then the distance of the vertex from PP' is
   (a) $\frac{1}{3}$ (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) $\frac{1}{9}$

4. Let $I = \displaystyle\int_a^{a+\pi} \frac{|\sin 2x|}{|\sin x| + |\cos x|}\,dx$, $a \in R$ and $J = \displaystyle\int_0^{\pi/4} \frac{1}{\sin x + \cos x}\,dx$, then the value of $I + 4J$ is equal to
   (a) 16 (b) 8 (c) 4 (d) 2

5. Find the area (in sq. units) of the region bounded by the curves $y = x^2$, $y = |2 - x^2|$ and $y = 2$, which lies to the right of the line $x = 1$.
   (a) $\frac{4}{3}(5+3\sqrt{2})$ (b) $\frac{4}{3}(5-3\sqrt{2})$
   (c) $5+3\sqrt{2}$ (d) $5-3\sqrt{2}$

6. The number of points in $(-2\pi, 2\pi)$, where $f(x) = \cos^{-1}(\cos x)$ is not differentiable is
   (a) 0 (b) 1 (c) 2 (d) 3

7. Let G be the greatest value and M be the maximum value of $f(x) = x(x-1)^2$, $0 \le x \le 2$. Then the ratio $\frac{G}{M} =$
   (a) 13 (b) $\frac{25}{2}$ (c) $\frac{27}{2}$ (d) 14

8. If $A(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 \end{pmatrix}$, then $A(\theta)^3$ will be a null matrix if and only if
   (a) $\theta = (2k+1)\pi/3$, $k \in I$
   (b) $\theta = (4k-1)\pi/3$ $k \in I$
   (c) $\theta = (3k-1)\pi/4$, $k \in I$
   (d) None of these

9. The point on the line $\frac{x+2}{2} = \frac{y+6}{3} = \frac{z-34}{-10}$ which is nearest to the line $\frac{x+6}{4} = \frac{y-7}{-3} = \frac{z-7}{-2}$ is (a, b, c), where $a + b + c =$
   (a) 9 (b) 10 (c) 11 (d) 12

10. If $\vec{a}, \vec{b}$ and $\vec{c}$ are non-coplanar vectors such that $\vec{\alpha} = \vec{b} \times \vec{c}$, $\vec{\beta} = \vec{c} \times \vec{a}$ and $\vec{\gamma} = \vec{a} \times \vec{b}$, then
    $\begin{vmatrix} \vec{\alpha}\cdot\vec{a} & \vec{\alpha}\cdot\vec{b} & \vec{\alpha}\cdot\vec{c} \\ \vec{\beta}\cdot\vec{a} & \vec{\beta}\cdot\vec{b} & \vec{\beta}\cdot\vec{c} \\ \vec{\gamma}\cdot\vec{a} & \vec{\gamma}\cdot\vec{b} & \vec{\gamma}\cdot\vec{c} \end{vmatrix} =$

(a) 0 (b) 1

(c) $[\vec{\alpha}\ \vec{\beta}\ \vec{\gamma}][\vec{a}\ \vec{b}\ \vec{c}]^2$ (d) $[\vec{a}\,\vec{b}\,\vec{c}]^3$

11. If random variable X : waiting time in minutes for bus and p.d.f. of X is given by $f(x)=\begin{cases}\frac{1}{5}, & 0\le x\le 5\\ 0, & \text{otherwise,}\end{cases}$ then probability of waiting time not more than 4 minutes is

(a) 0.3 (b) 0.8 (c) 0.2 (d) 0.5

12. For $x\in R$, two real valued functions $f(x)$ and $g(x)$ are such that, $g(x)=\sqrt{x}+1$ and $fog(x)=x+3-\sqrt{x}$. Then $f(0)$ is equal to

(a) 5 (b) 1 (c) 0 (d) –3

13. Among the relations

$S=\{(a,b): a, b\in R-\{0\},\ 2+\frac{a}{b}>0\}$ and

$T=\{(a,b): a, b\in R, a^2-b^2\in Z\}$

(a) S is transitive but T is not
(b) both S and T are symmetric
(c) neither S nor T is transitive
(d) T is symmetric but S is not

14. If the inverse trigonometric functions take principal values, then

$\cos^{-1}\left(\frac{3}{10}\cos\left(\tan^{-1}\left(\frac{4}{3}\right)\right)+\frac{2}{5}\sin\left(\tan^{-1}\left(\frac{4}{3}\right)\right)\right)$ is equal to

(a) 0 (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{6}$

15. If f be a function such that $f(9)=9$ and $f'(9)=3$, then $\lim_{x\to 9}\frac{\sqrt{f(x)}-3}{\sqrt{x}-3}$ is equal to

(a) 9 (b) 3

(c) 1 (d) None of these

16. Let $0<\alpha<\frac{\pi}{2}$ be a fixed angle. If $P=(\cos\theta, \sin\theta)$ and $Q=(\cos(\alpha-\theta), \sin(\alpha-\theta))$, then Q is obtained from P by

(a) clockwise rotation around origin through angle α
(b) anticlockwise rotation around origin through angle α
(c) reflection in the line through the origin with slope $\tan\alpha$
(d) reflection in the line through the origin with slope $\tan\alpha/2$

17. If the circles $(x-1)^2+(y-3)^2=r^2$ and $x^2+y^2-8x+2y+8=0$ intersect in two distinct points, then

(a) $2<r<8$ (b) $r=2$

(c) $r<2$ (d) $r>2$

18. The shortest distance between the line $y-x=1$ and the curve $x=y^2$ is

(a) $\frac{3\sqrt{2}}{8}$ (b) $\frac{2\sqrt{3}}{8}$ (c) $\frac{3\sqrt{2}}{5}$ (d) $\frac{\sqrt{3}}{4}$

19. An ellipse intersects the hyperbola $2(x^2-y^2)=1$ orthogonally. The eccentricity of the ellipse is reciprocal of that of the hyperbola. If the axes of the hyperbola are the coordinate axes, then the equation of ellipse is

(a) $x^2+2y^2=2$ (b) $2x^2+y^2=2$

(c) $x^2+2y^2=4$ (d) $2x^2+y^2=4$

20. The area (in sq. units) of the quadrilateral formed by the tangents at the end points of the latus rectum to the ellipse $\frac{x^2}{9}+\frac{y^2}{5}=1$, is

(a) $\frac{27}{2}$ (b) 27 (c) $\frac{27}{4}$ (d) 18

ONE OR MORE THAN ONE OPTION(S) CORRECT TYPE

21. If $f_n(\theta)=\tan\frac{\theta}{2}(1+\sec\theta)(1+\sec 2\theta)\ldots(1+\sec 2^n\theta)$, then

(a) $f_2\left(\frac{\pi}{16}\right)=1$ (b) $f_3\left(\frac{\pi}{32}\right)=1$

(c) $f_4\left(\frac{\pi}{64}\right)=1$ (d) $f_5\left(\frac{\pi}{128}\right)=1$

22. In a triangle ABC, $a=8$, $c=3b$. Its maximum area is divisible by

(a) 2 (b) 3 (c) 5 (d) 7

23. Number of ways in which the letters of the word "TOMATO" can be arranged if two alike vowels are separated, is not equal to

(a) number of ways in which letters of the word "KARNATAKA" can be arranged if no two alike letters are separated.
(b) number of 3 digit numbers with at least one 3 and at least one 2.
(c) number of ways in which Ram and Rama can exchange their maps if Ram has 3 and Rama has 7 maps, all maps being different, maintaining their original number of maps at the end.
(d) number of way in which 2 alike apples and 4 alike oranges can be distributed in three children if each child get none, one or more fruits.

24. Let $a\bar{z}+z\bar{a}+b=0$ represent a straight line in argand plane, given that $a \in Z$ and $b \in R$. Let z_1 be a point in the argand plane and let z_2 be the image of z_1 in the line $a\bar{z}+z\bar{a}+b=0$, then

(a) $(a\bar{z}_1+z_1\bar{a}+b)+(a\bar{z}_2+z_2\bar{a}+b)=0$

(b) $\frac{a}{\bar{a}}+\frac{(z_1-z_2)}{(\bar{z}_1-\bar{z}_2)}=0$ (c) $\frac{a}{\bar{a}}+\frac{(z_1-z_2)}{(\bar{z}_1-\bar{z}_2)}=1$

(d) None of these

25. Let a_1, a_2, a_3 ... and b_1, b_2, b_3 ... be arithmetic progressions such that $a_1 = 25$, $b_1 = 75$ and $a_{100} + b_{100} = 100$, then

(a) the difference between successive terms in progression 'a' is opposite in sign of the difference in progression 'b'.

(b) $a_n + b_n = 100$ for any n.

(c) $(a_1 + b_1), (a_2 + b_2), (a_3 + b_3), \ldots$ are in A.P.

(d) $\sum_{r=1}^{100}(a_r+b_r)=10000$

26. Let $f(x)=\begin{cases} x\sin\frac{1}{x}, & x\neq 0 \\ 0, & x=0 \end{cases}$ and $g(x) = x f(x)$.

Then at $x = 0$,

(a) g is differentiable, g' is not continuous

(b) g is differentiable and f is not

(c) both g and f are differentiable

(d) g is differentiable and g' is continuous

27. Let $\frac{dy}{dx}+y=f(x)$, where y is a continuous function of x with $y(0) = 1$ and $f(x)=\begin{cases} e^{-x} & \text{if } 0\le x\le 2 \\ e^{-2} & \text{if } x>2 \end{cases}$.

Which of the following hold(s) good?

(a) $y(1) = 2e^{-1}$ (b) $y'(1) = -e^{-1}$

(c) $y(3) = -2e^{-3}$ (d) $y'(3) = -2e^{-3}$

28. Let $\vec{a}=\hat{i}+2\hat{j}+\hat{k}, \vec{b}=\hat{i}-\hat{j}+\hat{k}$ and $\vec{c}=\hat{i}+\hat{j}-\hat{k}$. A vector in the plane of $\vec{a}$ and $\vec{b}$, whose projection on $\vec{c}$ is $\frac{1}{\sqrt{3}}$, is

(a) $4\hat{i}-\hat{j}+4\hat{k}$ (b) $3\hat{i}+\hat{j}-3\hat{k}$

(c) $2\hat{i}+\hat{j}+2\hat{k}$ (d) $4\hat{i}+\hat{j}-4\hat{k}$

29. If the lines $\frac{x-1}{k}=\frac{y-2}{2}=\frac{z-3}{3}$ and $\frac{x-2}{3}=\frac{y-3}{k}=\frac{z-1}{2}$ intersect at a point, then k is

(a) -2 (b) $5/2$ (c) -5 (d) 2

30. In a survey of population of 450 people, it is found that 205 can speak English (E), 210 can speak Hindi (H) and 120 people can speak Tamil (T). If 100 people can speak both E and H, 80 can speak both E and T, 35 can speak both H and T, and 20 can speak all the three languages. Then

(a) $n(E' \cap H' \cap T') = 110$

(b) $n(E \cap H' \cap T') = 45$

(c) $n(E' \cap H' \cap T') = 340$

(d) None of these

COMPREHENSION TYPE

Paragraph for Q. No. 31 and 32

The parametric equation of given curve is $x = a(2\cos t + \cos 2t)$, $y = a(2\sin t - \sin 2t)$.

31. The equation of tangent at any point 't' is

(a) $x\sin\left(\frac{t}{2}\right)+y\cos\left(\frac{t}{2}\right)=a\sin\left(\frac{3t}{2}\right)$

(b) $x\cos\left(\frac{t}{2}\right)-y\sin\left(\frac{t}{2}\right)=a\sin\left(\frac{3t}{2}\right)$

(c) $x\sin\left(\frac{t}{2}\right)+y\cos\left(\frac{t}{2}\right)=3a\sin\left(\frac{3t}{2}\right)$

(d) $x\cos\left(\frac{t}{2}\right)-y\sin\left(\frac{t}{2}\right)=3a\sin\left(\frac{3t}{2}\right)$

32. The equation of normal at any point 't' is

(a) $x\cos\left(\frac{t}{2}\right)+y\sin\left(\frac{t}{2}\right)=3a\cos\left(\frac{3t}{2}\right)$

(b) $x\cos\left(\frac{t}{2}\right)-y\sin\left(\frac{t}{2}\right)=3a\cos\left(\frac{3t}{2}\right)$

(c) $x\cos\left(\frac{t}{2}\right)+y\sin\left(\frac{t}{2}\right)=3a\sin\left(\frac{3t}{2}\right)$

(d) $x\cos\left(\frac{t}{2}\right)-y\sin\left(\frac{t}{2}\right)=3a\sin\left(\frac{3t}{2}\right)$

Paragraph for Q. No. 33 and 34

Consider $\omega=\cos\left(\frac{2\pi}{7}\right)+i\sin\left(\frac{2\pi}{7}\right)$ and $\alpha=\omega+\omega^2+\omega^4$ and $\beta = \omega^3 + \omega^5 + \omega^6$.

33. $\alpha + \beta$ equals

(a) 0 (b) -1 (c) -2 (d) 1

34. α and β are the roots of the equation

(a) $x^2 + x + 1 = 0$ (b) $x^2 + x + 2 = 0$

(c) $x^2 + 3x + 5 = 0$ (d) None of these

Paragraph for Q. No. 35 to 37

Consider $C: x^2 + y^2 = 9, E: \frac{x^2}{9} + \frac{y^2}{4} = 1, L: y = 2x$

35. P is a point on the circle C, the perpendicular PQ to the major axis of the ellipse E meets the ellipse at M, then $\frac{MQ}{PQ}$ is equal to

(a) 1/3 (b) 2/3

(c) 1/2 (d) None of these

36. If L represents the line joining the point P on C to its centre O, then equation of the tangent at M to the ellipse E is

(a) $x + 3y = 3\sqrt{5}$ (b) $4x + 3y = \sqrt{5}$

(c) $x + 3y + 3\sqrt{6} = 0$ (d) $4x + 3y + \sqrt{5} = 0$

37. Equation of the diameter of the ellipse E conjugate to the diameter represented by L is

(a) $9x + 2y = 0$ (b) $2x + 9y = 0$

(c) $4x + 9y = 0$ (d) $4x - 9y = 0$

MATRIX MATCH TYPE

38. Match Column-I with Column-II and select the correct answer using the options given below.

	Column-I		Column-II
(P)	If $\lvert\vec{a}\rvert = \lvert\vec{b}\rvert = \lvert\vec{c}\rvert$ such that angle between each pair of vectors is $\frac{\pi}{3}$ and $\lvert\vec{a}+\vec{b}+\vec{c}\rvert = \sqrt{6}$, then $2\lvert\vec{a}\rvert =$	(1)	3
(Q)	If $\vec{a}$ is perpendicular to $\vec{b}+\vec{c}$, $\vec{b}$ is perpendicular to $\vec{c}+\vec{a}$, $\vec{c}$ is perpendicular to $\vec{a}+\vec{b}$, $\lvert\vec{a}\rvert = 2, \lvert\vec{b}\rvert = 3$ and $\lvert\vec{c}\rvert = 6$, then $\lvert\vec{a}+\vec{b}+\vec{c}\rvert - 2 =$	(2)	2
(R)	If $\vec{a} = 2\hat{i}+3\hat{j}-\hat{k}, \vec{b} = -\hat{i}+2\hat{j}-4\hat{i}$, $\vec{c} = \hat{i}+\hat{j}+\hat{k}$ and $\vec{d} = 3\hat{i}+2\hat{j}+\hat{k}$, then $\frac{1}{7}(\vec{a}\times\vec{b})\cdot(\vec{c}\times\vec{d}) =$	(3)	4
(S)	If $\lvert\vec{a}\rvert = \lvert\vec{b}\rvert = \lvert\vec{c}\rvert = 2$ and $\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{c} = \vec{c}\cdot\vec{a} = 2$, then $[\vec{a}\ \vec{b}\ \vec{c}]\cos 45° =$	(4)	5

	P	Q	R	S
(a)	2	3	4	1
(b)	3	1	2	4
(c)	2	4	1	3
(d)	3	1	4	2

39. Match Column-I with Column-II and select the correct answer using the options given below.

	Column - I		Column - II
(P)	$\int \frac{e^{2x}-1}{e^{2x}+1}dx$ is equal to	(1)	$x - \log\left[1+\sqrt{1-e^{2x}}\right] + C$
(Q)	$\int \frac{1}{(e^x+e^{-x})^2}dx$ is equal to	(2)	$\log(e^x+1) - x - e^{-x} + C$
(R)	$\int \frac{e^{-x}}{1+e^x}dx$ is equal to	(3)	$\log(e^{2x}+1) - x + C$
(S)	$\int \frac{1}{\sqrt{1-e^{2x}}}dx$ is equal to	(4)	$-\frac{1}{2\left(e^{2x}+1\right)} + C$

	P	Q	R	S
(a)	1	3	4	2
(b)	3	1	4	2
(c)	3	1	2	4
(d)	3	4	2	1

40. Match Column-I with Column-II and select the correct answer using the options given below.

	Column-I		Column-II
(P)	If $f(x) = \cos x - 1 + \frac{x^2}{2!} - \frac{x^3}{3!}$, then	(1)	minimum value is -4
(Q)	If $f(x) = \cos x - 1 + \frac{x^2}{2!}$, then	(2)	there is no extremum at $x = 0$
(R)	If $f(x) = x^4 e^{-x^2}$, then	(3)	the function reaches minimum at $x = 0$
(S)	If $f(x) = \sin 3x - 3 \sin x$, then	(4)	the function reaches maximum at $x = \sqrt{2}$

	P	Q	R	S
(a)	2	3	2,4	1,2
(b)	1	2	3	1,4
(c)	3	3	2	1,3
(d)	4	4	1	3,4

NUMERICAL ANSWER TYPE

41. The number of values of x in $[0, 4\pi]$ satisfying the inequation $|\sqrt{3}\cos x - \sin x| \ge 2$, is ____ .

42. If the sixth term in the expansion of $\left(\frac{1}{x^{8/3}} + x^2 \log_{10} x\right)^8$ is 5600. The value of x is ____ .

43. The number of arrangements of the letters of the word FORTUNE when the order of vowels is unaltered is x, when the order of consonants is unaltered is y and when the order of vowels and consonants is unaltered is z. Then $\frac{x+y}{5z}$ equals ____ .

44. If the straight lines joining the origin to the points of intersection of the straight line $69x + 25y = 3450$ and the circle $(x - 25)^2 + (y - 69)^2 = c^2$ are at right angle, then c^2 is equal to ____ .

45. If $\lim_{x\to 0} \frac{3\tan 3x - 4\tan 2x - \tan x}{x^2 \tan x} = L$, then the sum of the digits of L is ____ .

46. Let k be a positive real number and

$$A = \begin{bmatrix} 2k-1 & 2\sqrt{k} & 2\sqrt{k} \\ 2\sqrt{k} & 1 & -2k \\ -2\sqrt{k} & 2k & -1 \end{bmatrix}, B = \begin{bmatrix} 0 & 2k-1 & \sqrt{k} \\ 1-2k & 0 & 2\sqrt{k} \\ -\sqrt{k} & -2\sqrt{k} & 0 \end{bmatrix}$$

If det (adj A) + det (adj B) = 10^6, then $[k]$ is ____ .

47. If the curve C in the xy plane has the equation $x^2 + xy + y^2 = 1$, then fourth power of the greatest distance of a point on C from the origin, is ____ .

48. The equation of a curve whose slope at any point is thrice its abscissa and which passes through $(-1, -3)$ is $2y = \lambda(x^2 - 3)$, then the value of λ must be equal to ____ .

49. If the straight lines joining the origin to the points of intersection of $2x + 3y = k$ and $3x^2 - xy + 3y^2 + 2x - 3y - 4 = 0$ are at right angle, then $6k - \frac{52}{k}$ is ____.

50. In a test an examine either guesses or copies or knows the answer to a multiple choice question with m choice out of which exactly one is correct. The probability that he makes a guess is 1/3 and the probability that he copies the answer is 1/6. The probability that his answer is correct given that he copied to the question given that he correctly answered it is 120/141. Then m is equal to _____.

SOLUTIONS

1. (b): If we interchange any 2 rows of a determinant in the set B, its value becomes -1 and hence it is in C. Likewise, for every determinant in C, there is a corresponding determinant in B.

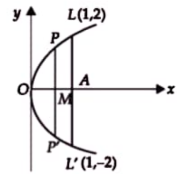
$\therefore$ B and C have the same number of elements.

2. (b): $|x|$ is not differentiable at $x = 0$ only.

$(x-1)^2 \sin\left(\frac{1}{x-1}\right)$ is differentiable for all x with derivative 0 at $x = 1$.

Hence, $f(x)$ is differentiable for all x except $x = 0$.

3. (d): $P = (t^2, 2t)$, $L = (1, 2)$



Area of $PLL'P' = f(t)$

$= \frac{1}{2}(LL' + PP')(OA - OM)$

$= (2 + 2t)(1 - t^2)$

$= 2(1 + t - t^2 - t^3)$

Now, $f'(t) = 1 - 2t - 3t^2 = 0$

$\Rightarrow \ t = \frac{1}{3}$. So, $OM = t^2 = \frac{1}{9}$.

4. (c) : $I = \int_a^{a+\pi} \frac{|\sin 2x|}{|\sin x| + |\cos x|} dx$, $a \in R$

Now, $\frac{|\sin 2x|}{|\sin x| + |\cos x|}$ is periodic with period $\frac{\pi}{2}$.

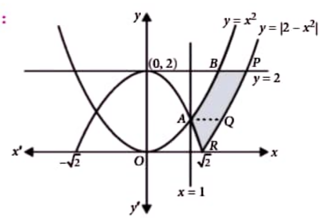
$\therefore \ I = 2\int_0^{\pi/2} \frac{|\sin 2x|}{|\sin x| + |\cos x|} dx = 4\int_0^{\pi/4} \frac{|\sin 2x|}{|\sin x| + |\cos x|} dx$

and $J = \int_0^{\pi/4} \frac{1}{\sin x + \cos x} dx$

$\Rightarrow I + 4J = 4\int_0^{\pi/4} \frac{1 + \sin 2x}{\sin x + \cos x} dx = 4\int_0^{\pi/4} \frac{(\sin x + \cos x)^2}{\sin x + \cos x} dx$

$= 4\int_0^{\pi/4} (\sin x + \cos x) dx = 4[-\cos x + \sin x]_0^{\pi/4} = 4$

5. (b):



The shaded region is the required region.
Curves $y = x^2$, $y = |2 - x^2|$ meet at $A(1, 1)$.

Area $ABPQ = \int_1^2 \sqrt{2+y}\,dy - \int_1^2 \sqrt{y}\,dy$

$$= \left[\frac{2}{3}(2+y)^{3/2} - \frac{2}{3}y^{3/2}\right]_1^2 = \frac{2}{3}(9 - 2\sqrt{2} - 3\sqrt{3}) \quad ...(i)$$

Area $AQR = \int_0^1 (\sqrt{2+y} - \sqrt{2-y})\,dy$

$$= \frac{2}{3}\left[(2+y)^{3/2} + (2-y)^{3/2}\right]_0^1 \quad ...(ii)$$

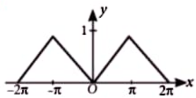
$$= \frac{2}{3}[3\sqrt{3} + 1 - 4\sqrt{2}]$$

Hence, required area of the region is (i) + (ii)

$$= \frac{2}{3}[9 - 2\sqrt{2} - 3\sqrt{3} + 3\sqrt{3} + 1 - 4\sqrt{2}]$$

$$= \frac{4}{3}(5 - 3\sqrt{2}) \text{ sq. units}$$

6. (d) :



$f(x)$ is not differentiable at $x = 0$, and $x = \pm\pi$

7. (c) : $f'(x) = (x - 1)^2 + 2x(x - 1) = 3x^2 - 4x + 1$

Now, $f'(x) = 0 \Rightarrow 3x^2 - 4x + 1 = 0 \quad \Rightarrow x = 1, \frac{1}{3}$

$\min f = f(1) = 0$, $\max f = f\left(\frac{1}{3}\right) = \frac{1}{3}\left(\frac{1}{3} - 1\right)^2 = \frac{4}{27}$

$\Rightarrow M = \frac{4}{27}$, $f(2) = 2 = G \quad \therefore \frac{G}{M} = \frac{27}{2}$

8. (d): $A(\theta)^3 = A(3\theta) = \begin{bmatrix} \cos 3\theta & -\sin 3\theta & 0 \\ \sin 3\theta & \cos 3\theta & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Now, $\begin{bmatrix} \cos 3\theta & -\sin 3\theta & 0 \\ \sin 3\theta & \cos 3\theta & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$\Rightarrow \cos 3\theta = 0$ and $\sin 3\theta = 0$

$\Rightarrow \theta = (2k + 1)\frac{\pi}{6}$ and $\theta = \frac{n\pi}{3}$, $k, n \in I$

9. (c) : Let P and Q be point on the lines, then
$P(-2 + 2s, -6 + 3s, 34 - 10s)$, $Q(-6 + 4t, 7 - 3t, 7 - 2t)$
The $d.r$'s of PQ are $(-4 + 4t - 2s, 13 - 3t - 3s, -27 - 2t + 10s)$

PQ is perpendicular to the two lines with $d.r$'s 2, 3, –10 and 4, –3, –2.

$\therefore \; 2(-4 + 4t - 2s) + 3(13 - 3t - 3s) - 10(-27 - 2t + 10s) = 0$
and $4(-4 + 4t - 2s) - 3(13 - 3t - 3s) - 2(-27 - 2t + 10s) = 0$
$\Rightarrow 113s - 19t = 301,\; 29t - 19s = 1$
Solving above equations,we get $s = 3$, $t = 2$,
So, $P = (4, 3, 4) = (a, b, c) \Rightarrow a + b + c = 11$

10. (d): We have, $\begin{vmatrix} \vec{\alpha}\cdot\vec{a} & \vec{\alpha}\cdot\vec{b} & \vec{\alpha}\cdot\vec{c} \\ \vec{\beta}\cdot\vec{a} & \vec{\beta}\cdot\vec{b} & \vec{\beta}\cdot\vec{c} \\ \vec{\gamma}\cdot\vec{a} & \vec{\gamma}\cdot\vec{b} & \vec{\gamma}\cdot\vec{c} \end{vmatrix}$

$$= \begin{vmatrix} [\vec{a}\;\vec{b}\;\vec{c}] & 0 & 0 \\ 0 & [\vec{a}\;\vec{b}\;\vec{c}] & 0 \\ 0 & 0 & [\vec{a}\;\vec{b}\;\vec{c}] \end{vmatrix} = [\vec{a}\;\vec{b}\;\vec{c}]^3$$

11. (b): We have, $P(X \le 4) = P(0 \le x \le 4)$

$$= \int_0^4 f(x)\,dx = \int_0^4 \frac{1}{5}\,dx = \frac{1}{5}[x]_0^4 = \frac{1}{5}[4 - 0] = \frac{4}{5} = 0.8$$

12. (a) : For $x \in R$, $f(x)$ and $g(x)$ are real valued functions
$g(x) = \sqrt{x} + 1$ and $fog(x) = x + 3 - \sqrt{x}$

$fog(x) = x + 3 - \sqrt{x} \Rightarrow f(g(x)) = x + 3 - \sqrt{x}$

$= (\sqrt{x} + 1)^2 - 3(\sqrt{x} + 1) + 5$

$= [g(x)]^2 - 3[g(x)] + 5 \qquad (\because g(x) = \sqrt{x} + 1)$

So, $f(x) = x^2 - 3x + 5 \quad \therefore f(0) = 0 - 3 \times 0 + 5 = 5$

Note: But if we consider the domain of the composite function $fog(x)$, then $f(0)$ will not be defined as $g(x)$ can't be equal to zero.

13. (d): For relation $T = a^2 - b^2 \in I$
Then, (b, a) on relation $R \Rightarrow b^2 - a^2 \in I$

$\therefore \; T$ is symmetric, $S = \left\{(a, b) : a, b \in R - \{0\}, 2 + \frac{a}{b} > 0\right\}$

$2 + \frac{a}{b} > 0 \Rightarrow \frac{a}{b} > -2 \Rightarrow \frac{b}{a} > \frac{-1}{2}$

If $(b, a) \in S$, then $2 + \frac{b}{a}$ is not necessarily positive.

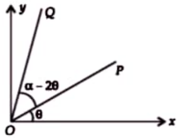
$\therefore \; S$ is not symmetric.

14. (c)

15. (b): $\lim_{x \to 9} \frac{\sqrt{f(x)} - 3}{\sqrt{x} - 3} = \lim_{x \to 9} \frac{f(x) - 9}{x - 9} \times \frac{\sqrt{x} + 3}{\sqrt{f(x)} + 3}$

$$= f'(9) \times \left(\frac{\sqrt{9} + 3}{\sqrt{f(9)} + 3}\right) = f'(9) \times \frac{3 + 3}{3 + 3} = f'(9) = 3$$

16. (d): OP is inclined at angle θ with x-axis OQ is inclined at angle $(\alpha - \theta)$ with x-axis.



The bisector of angle POQ is inclined at angle $\frac{\theta + \alpha - \theta}{2} = \frac{\alpha}{2}$ with x-axis.

$\therefore$ Q is obtained from P by reflection in the line through origin with slope $\tan\frac{\alpha}{2}$.

17. (a): Centres : $C_1(1, 3)$, $C_2(4, -1)$,

Radii : $r_1 = r$, $r_2 = \sqrt{16 + 1 - 8} = 3$

$C_1C_2 = \sqrt{9 + 16} = 5$, $|r_1 - r_2| < C_1C_2 < r_1 + r_2$

$\Rightarrow |r - 3| < 5 < r + 3$

$\therefore r > 2, |r - 3| < 5 \Rightarrow r - 3 < 5 \Rightarrow r < 8$

$\therefore 2 < r < 8$

18. (a): The tangent at (x_1, y_1) on $y^2 - x = 0$ is $S_1 = 0$

$\Rightarrow yy_1 - \frac{1}{2}(x + x_1) = 0$, its slope is $\frac{1}{2y_1}$.

For shortest distance, the tangent is parallel to $y - x = 1$

$\therefore \frac{1}{2y_1} = 1 \Rightarrow y_1 = \frac{1}{2}, x_1 = \frac{1}{4}$

Shortest distance is the distance of $\left(\frac{1}{4}, \frac{1}{2}\right)$ from the line $y - x - 1 = 0$

$\therefore$ It is $\frac{\left|\frac{1}{2} - \frac{1}{4} - 1\right|}{\sqrt{2}} = \frac{3}{4\sqrt{2}} = \frac{3\sqrt{2}}{8}$.

19. (a): The eccentricity of the hyperbola is $\sqrt{2}$.

The eccenticity of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $\frac{1}{\sqrt{2}}$.

$\Rightarrow b^2 = a^2(1 - e^2) = \frac{a^2}{2}$

$\therefore$ The ellipse is $x^2 + 2y^2 = a^2$

The foci of hyperbola are $(\pm 1, 0)$. The confocal conices are orthogonal.

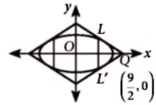
$\therefore$ The foci of the ellipse are $(\pm 1, 0)$

$\therefore \sqrt{a^2 - b^2} = 1, \sqrt{\frac{a^2}{2}} = 1, a = \sqrt{2}$

$\therefore$ The ellipse is $x^2 + 2y^2 = 2$.

20. (b): In $\frac{x^2}{9} + \frac{y^2}{5} = 1$, wer have $a = 3$, $b = \sqrt{5}$

$\Rightarrow e = \sqrt{1 - \frac{5}{9}} = \frac{2}{3}$



$\therefore$ Extermities of one of latus rectum are $\left(2, \frac{5}{3}\right)$ and $\left(2, -\frac{5}{3}\right)$.

So, equation of tangent at $\left(2, \frac{5}{3}\right)$ is $\frac{x(2)}{9} + \frac{y\left(\frac{5}{3}\right)}{5} = 1$

$\Rightarrow 2x + 3y = 9$

Since, this tangent intersects coordinates axes

$\Rightarrow x$ – intercept $= 9/2$, y – intercept $= 3$

$\therefore$ Area of quadrilateral $= 4 \times$ Area of ΔPOQ

$= 4 \times \frac{1}{2} \times \frac{9}{2} \times 3 = 27$ sq. units.

21. (a, b, c, d): $f_0(\theta) = \tan\frac{\theta}{2}(1 + \sec\theta)$

$$= \frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}\left(1 + \frac{1}{\cos\theta}\right) = \frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}} \frac{(\cos\theta + 1)}{\cos\theta}$$

$$= \frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{\cos\theta} = \frac{\sin\theta}{\cos\theta} = \tan\theta$$

$f_1(\theta) = \tan\theta\,(1 + \sec\theta)(1 + \sec 2\theta)$

$$= \frac{\sin\theta}{\cos\theta} \frac{(\cos 2\theta + 1)}{\cos 2\theta} = \tan 2\theta$$

Likewise $f_n(\theta) = \tan(2^n\theta)$

$f_2\left(\frac{\pi}{16}\right) = \tan\left(2^2 \cdot \frac{\pi}{16}\right) = 1,$

$f_3\left(\frac{\pi}{32}\right) = \tan\left(2^3 \cdot \frac{\pi}{32}\right) = 1,$

$f_4\left(\frac{\pi}{64}\right) = \tan\left(2^4 \cdot \frac{\pi}{64}\right) = 1,$

$f_5\left(\frac{\pi}{128}\right) = \tan\left(2^5 \cdot \frac{\pi}{128}\right) = 1$

22. (a, b): $16\Delta^2 = (a + b + c)(b + c - a)(c + a - b)(a + b - c)$

$= (8 + 4b)(4b - 8)(8 + 2b)(8 - 2b)$

$\Rightarrow \frac{\Delta^2}{4} = (b^2 - 4)(16 - b^2) = -b^4 + 20b^2 - 64$

$= -(b^2 - 10)^2 + 100 - 64$

$\therefore \Delta_{max}^2 = 4 \times 36 \Rightarrow \Delta_{max} = 2 \times 6 = 12$

23. (b, c, d) : Arrangement should be × T × M × A × T ×

Number of ways = $^5C_2 \times \frac{4!}{2!} = 10 \times 12 = 120$

(a) The given word is KARNATAKA
K = 2, A = 4, R = 1, N = 1, T = 1

[K K] [A A A A] [R] [N] [T]

$\therefore$ Number of ways = 5! = 120

(b) Possible ways are 2, 3, 3 or 3, 2, 2 or 2, 3, non-zero digit or 2, 3, 0

$\therefore$ Number of ways = $\frac{3!}{2!} + \frac{3!}{2!} + (^7C_1 \times 3!) + (2 \times 2)$
$= 3 + 3 + 42 + 4 = 52$

(c) $M_1, M_2, ..., M_7 \,|N_1, N_2, N_3$

Total ways $= 3(^7C_1) + 3(^7C_2) + 1(^7C_3) = 21 + 63 + 35 = 119$

(d) Let x, y and z be the no. of fruits obtained by three children in which 2 alike apples and 4 alike oranges are distributed.

$\Rightarrow x + y + z = 6$

Now, no. of ways in which each child get none, one or more fruits = $(6 + 3 - 1)\, C_{3-1} = 28$

24. (a, b) : If z_2 is image of z_1 in given line, then $\frac{z_1 + z_2}{2}$ must lie on line.

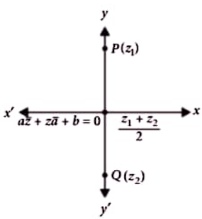
$\therefore \bar{a}\left(\frac{z_1 + z_2}{2}\right) + a\left(\overline{\frac{z_1 + z_2}{2}}\right) + b = 0$

$\Rightarrow (a\bar{z}_1 + z_1\bar{a} + b) + (a\bar{z}_2 + z_2\bar{a} + b) = 0$

We know that complex slope of two perpendiculars is related by expression $\omega_1 + \omega_2 = 0$

where ω_1 = complex slope of 1st line

ω_2 = complex slope of 2nd line



$\therefore$ Complex slope of $a\bar{z} + z\bar{a} + b = 0$ is $\frac{a}{\bar{a}}$.

and that of line joining PQ is $\frac{z_1 - z_2}{\bar{z}_1 - \bar{z}_2}$

$\therefore \frac{a}{\bar{a}} + \frac{(z_1 - z_2)}{(\bar{z}_1 - \bar{z}_2)} = 0$

25. (a, b, c, d) : Here $a_1, (a_1 + d), (a_1 + 2d), ...$ and $b_1, (b_1 + d_1), (b_1 + 2d_1), ...$ are in A.P.

$\therefore a_{100} = a_1 + 99d$ ...(i), $b_{100} = b_1 + 99d_1$ ...(ii)

Adding (i) and (ii), we get $a_{100} + b_{100} = 100 + 99\,(d + d_1)$

Hence, $d + d_1 = 0 \Rightarrow d = -d_1 \Rightarrow$ options (a), (b) and (c) are obviously true.

(d) $\sum_{r=1}^{100}(a_r + b_r) = \frac{100}{2}[(a_1 + b_1) + (a_{100} + b_{100})]$

$= \frac{100 \times 200}{2} = 10^4$

26. (a, b) : f is continuous at $x = 0$

$\because \lim_{x \to 0} \sin\frac{1}{x}$ does not exist.

$\therefore$ f is not differentiable at $x = 0$.

g is differentiable at $x = 0$ $\left[\text{Since, } \lim_{x \to 0} x\sin\frac{1}{x} = 0\right]$

$\therefore g'(0) = 0$

For $x \neq 0, g'(x) = 2x\sin\frac{1}{x} - \cos\frac{1}{x}$

$\lim_{x \to 0} g'(x)$ does not exist. $\therefore g'$ is not continuous.

27. (a, b, d) : $\frac{dy}{dx} + y = f(x)$ is linear differential equation.

I.F. = e^x

$\therefore$ Solution is $ye^x = \int e^x f(x)dx + C$

Now if $0 \le x \le 2$, then $ye^x = \int e^x \times e^{-x}\,dx + C$

$\Rightarrow ye^x = x + C,\ y(0) = 1 \Rightarrow C = 1$

$\therefore \quad y = \frac{x+1}{e^x} \Rightarrow y(1) = \frac{2}{e}$

Also, $y' = \frac{e^x - (x+1)e^x}{e^{2x}} \Rightarrow y'(1) = \frac{e - 2e}{e^2} = \frac{-e}{e^2} = -\frac{1}{e}$

If $x > 2$, $ye^x = \int e^{x-2}\,dx \quad \therefore ye^x = e^{x-2} + C$

$\therefore \quad y = e^{-2} + Ce^{-x}$

As y is continuous

$\therefore \lim_{x \to 2} \frac{x+1}{e^x} = \lim_{x \to 2}(e^{-2} + Ce^{-x})$

$\therefore 3e^{-2} = e^{-2} + Ce^{-2} \Rightarrow C = 2$

$\therefore$ For $x \ge 2$, $y = e^{-2} + 2e^{-x} \Rightarrow y(3) = 2e^{-3} + e^{-2}$

$\Rightarrow y' = -2e^{-x} \Rightarrow y'(3) = -2e^{-3}$

28. (a, c) : A vector in the plane of $\vec{a}$ and $\vec{b}$ is

$\vec{a} + \lambda\vec{b} = (1 + \lambda)\hat{i} + (2 - \lambda)\hat{j} + (1 + \lambda)\hat{k}$

Its projection on $\vec{c}$ is $\frac{1}{\sqrt{3}} \Rightarrow (\vec{a} + \lambda\,\vec{b})\cdot\frac{\vec{c}}{|\vec{c}|} = \pm\frac{1}{\sqrt{3}}$

$\Rightarrow 1 + \lambda + 2 - \lambda - 1 - \lambda = \pm 1$

$\Rightarrow \lambda = 2 \pm 1 \Rightarrow \lambda = 1$ or 3

If $\lambda = 1$, $\vec{a} + \lambda\vec{b} = 2\hat{i} + \hat{j} + 2\hat{k}$,

and if $\lambda = 3$, $\vec{a} + \lambda\vec{b} = 4\hat{i} - \hat{j} + 4\hat{k}$

29. (b, c) : The lines $\vec{r} = \vec{a} + s\vec{b}, \vec{r} = \vec{c} + t\,\vec{d}$ intersect if $\left[(\vec{c} - \vec{a})\ \vec{b}\ \ \vec{d}\right] = 0$

$$\therefore \begin{vmatrix} 2-1 & 3-2 & 1-3 \\ k & 2 & 3 \\ 3 & k & 2 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} 1 & 1 & -2 \\ k & 2 & 3 \\ 3 & k & 2 \end{vmatrix} = 0$$

$\Rightarrow 4 - 3k + 9 - 2k - 2(k^2 - 6) = 0 \Rightarrow 2k^2 + 5k - 25 = 0$

$\Rightarrow k = -5, \frac{5}{2}$

30. (a, b) : $n(U) = 450, n(E) = 205, n(H) = 210, n(T) = 120$
$n(E \cap H) = 100,\ n(E \cap T) = 80,\ n(H \cap T) = 35$ and $n(E \cap H \cap T) = 20$

Now, $n(E' \cap H' \cap T') = n(E \cup H \cup T)'$
$= n(U) - n(E \cup H \cup T)$ ...(i)

$n(E \cup H \cup T) = n(E) + n(H) + n(T) - [n(E \cap H) + n(E \cap T) + n(H \cap T)] + n(E \cap H \cap T)$
$= (205 + 210 + 120) - (100 + 80 + 35) + 20$
$= 535 - 215 + 20 = 555 - 215 = 340$

Hence, from (i)
$n(E' \cap H' \cap T') = 450 - 340 = 110$

Now, we have to find
$n(E \cap H' \cap T') = n\{E \cap (H \cup T)'\}$
$= n(E) - n(E \cap (H \cup T))$
$= n(E) - n\{(E \cap H) \cup (E \cap T)\}$
$= n(E) - [n(E \cap H) + n(E \cap T) - n\{(E \cap H) \cap (E \cap T)\}]$
$= n(E) - n(E \cap H) - n(E \cap T) + n(E \cap H \cap T)$
$= 205 - 100 - 80 + 20 = 225 - 180 = 45$

31. (a) : Equation of tangent at 't' is

$y - a(2\sin t - \sin 2t) = -\tan\left(\frac{t}{2}\right)[x - a(2\cos t + \cos 2t)]$

$\Rightarrow y\cos\left(\frac{t}{2}\right) - a\left(2\sin t\cos\frac{t}{2} - \sin 2t\cos\frac{t}{2}\right)$

$= -x\sin\frac{t}{2} + a\left(2\sin\frac{t}{2}\cos t + \sin\frac{t}{2}\cos 2t\right)$

$\Rightarrow x\sin\left(\frac{t}{2}\right) + y\cos\left(\frac{t}{2}\right) + a\sin(2t - t/2) - 2a\sin(t + t/2) = 0$

$\Rightarrow x\sin\left(\frac{t}{2}\right) + y\cos\left(\frac{t}{2}\right) = a\sin\left(\frac{3t}{2}\right)$

32. (b): Equation of normal at 't' is

$y - a(2\sin t - \sin 2t) = \cot\left(\frac{t}{2}\right)[x - a(2\cos t + \cos 2t)]$

$\Rightarrow y\sin\left(\frac{t}{2}\right) - a\left(2\sin t\sin\left(\frac{t}{2}\right) - \sin 2t\sin\left(\frac{t}{2}\right)\right)$

$= x\cos\left(\frac{t}{2}\right) - a\left(2\cos t\cos\left(\frac{t}{2}\right) + \cos 2t\cos\left(\frac{t}{2}\right)\right)$

$\Rightarrow y\sin\left(\frac{t}{2}\right) + 2a\cos\left(t + \frac{t}{2}\right) + a\cos\left(2t - \frac{t}{2}\right) = x\cos\left(\frac{t}{2}\right)$

$\Rightarrow x\cos\left(\frac{t}{2}\right) - y\sin\left(\frac{t}{2}\right) = 3a\cos\left(\frac{3t}{2}\right)$

33. (b) : As given, $\omega = \cos\left(\frac{2\pi}{7}\right) + i\sin\left(\frac{2\pi}{7}\right)$

$\therefore \omega^7 = 1$

So, ω is the 7th roots of unity.

$\therefore 1 + \omega + \omega^2 + \omega^3 + \omega^4 + \omega^5 + \omega^6 = 0$

$\Rightarrow \omega + \omega^2 + \omega^3 + \omega^4 + \omega^5 + \omega^6 = -1$ ...(i)

$\therefore \alpha + \beta = -1$

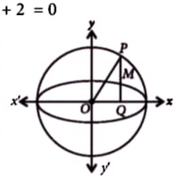
34. (b) : Required equation is

$x^2 - (\alpha + \beta)x + \alpha\beta = 0$

Now, $\alpha\beta = (\omega + \omega^2 + \omega^4)(\omega^3 + \omega^5 + \omega^6)$
$= \omega^4 + \omega^6 + \omega^7 + \omega^5 + \omega^7 + \omega^8 + \omega^7 + \omega^9 + \omega^{10}$
$= 3 + \omega^4 + \omega^5 + \omega^6 + \omega^8 + \omega^9 + \omega^{10}$
$= 3 + \omega^4 + \omega^5 + \omega^6 + \omega + \omega^2 + \omega^3$
$= 3 + (-1) = 2$

$\therefore$ Required equation is $x^2 + x + 2 = 0$

35. (b) : Let the coordinates of P be $(3\cos\theta, 3\sin\theta)$, then the eccentric angle of M, the point where the ordinate PQ through P meets the ellipse is θ and the coordinates of M are $(3\cos\theta, 2\sin\theta)$.



$\therefore PQ = 3\sin\theta$ and $MQ = 2\sin\theta \Rightarrow \frac{MQ}{PQ} = \frac{2}{3}$

36. (a) : Line $L: y = 2x$ meets the circle $C: x^2 + y^2 = 9$ at points for which $x^2 + 4x^2 = 9 \Rightarrow x = \pm\frac{3}{\sqrt{5}}$

$\therefore$ Coordinates of P are $\left(\pm\frac{3}{\sqrt{5}}, \pm\frac{6}{\sqrt{5}}\right)$,

$\Rightarrow$ Coordinates of M are $\left(\pm\frac{3}{\sqrt{5}}, \pm\frac{4}{\sqrt{5}}\right)$

Equation of the tangent at M to the ellipse E is

$\frac{x(\pm 3)}{9\sqrt{5}} + \frac{y(\pm 4)}{4\sqrt{5}} = 1 \Rightarrow x + 3y = \pm 3\sqrt{5}$

37. (b): Let $y = mx$ be the diameter conjugate to the diameter $L: y = 2x$ of the ellipse E then

$2m = -4/9$ $(\because\ mm' = -b^2/a^2)$

$\Rightarrow m = -2/9$ and the equation of the conjugate diameter is $y = (-2/9)x$ or $2x + 9y = 0$.

38. (c) : (P) $|\vec{a} + \vec{b} + \vec{c}| = \sqrt{6}$

$\Rightarrow \vec{a}^2 + \vec{b}^2 + \vec{c}^2 + 2\vec{a}\cdot\vec{b} + 2\vec{b}\cdot\vec{c} + 2\vec{c}\cdot\vec{a} = 6$

$\Rightarrow\ |\vec{a}|=1 \quad \therefore \quad 2|\vec{a}|=2$

(Q) $\vec{a}$ is perpendicular to $\vec{b}+\vec{c} \Rightarrow \vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c}=0$ ...(i)

$\vec{b}$ is perpendicular to $\vec{a}+\vec{c} \Rightarrow \vec{b}\cdot\vec{a}+\vec{b}\cdot\vec{c}=0$ ...(ii)

$\vec{c}$ is perpendicular to $\vec{a}+\vec{b} \Rightarrow \vec{c}\cdot\vec{a}+\vec{c}\cdot\vec{b}=0$ ...(iii)

From (i), (ii) and (iii), we get $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}=0$

$\Rightarrow\ |\vec{a}+\vec{b}+\vec{c}|=7 \quad \therefore \quad |\vec{a}+\vec{b}+\vec{c}|-2=5$

(R) $(\vec{a}\times\vec{b})\cdot(\vec{c}\times\vec{d})=(\vec{a}\cdot\vec{c})(\vec{b}\cdot\vec{d})-(\vec{b}\cdot\vec{c})(\vec{a}\cdot\vec{d})=21$

$\therefore\ \frac{1}{7}(\vec{a}\times\vec{b})\cdot(\vec{c}\times\vec{d})=3$

(S) We know that $[\vec{a}\times\vec{b}\ \ \vec{b}\times\vec{c}\ \ \vec{c}\times\vec{a}]=[\vec{a}\ \vec{b}\ \vec{c}]^2$

and $[\vec{a}\ \vec{b}\ \vec{c}]^2=\begin{vmatrix}\vec{a}\cdot\vec{a} & \vec{a}\cdot\vec{b} & \vec{a}\cdot\vec{c}\\ \vec{b}\cdot\vec{a} & \vec{b}\cdot\vec{b} & \vec{b}\cdot\vec{c}\\ \vec{c}\cdot\vec{a} & \vec{c}\cdot\vec{b} & \vec{c}\cdot\vec{c}\end{vmatrix}=\begin{vmatrix}4 & 2 & 2\\ 2 & 4 & 2\\ 2 & 2 & 4\end{vmatrix}=32$

$\therefore\ [\vec{a}\ \vec{b}\ \vec{c}]=4\sqrt{2} \Rightarrow [\vec{a}\ \vec{b}\ \vec{c}]\cos 45^\circ=4$

39. (d): (P) $\int\frac{e^{2x}-1}{e^{2x}+1}dx=\int\frac{e^x-e^{-x}}{e^x+e^{-x}}dx$

$=\int\frac{d(e^x+e^{-x})}{e^x+e^{-x}}=\log(e^x+e^{-x})+C=\log(e^{2x}+1)-x+C$

(Q) Let $I=\int\frac{1}{(e^x+e^{-x})^2}dx=\int\frac{e^{2x}}{(e^{2x}+1)^2}dx$

Put $e^{2x}+1=t \Rightarrow 2e^{2x}\,dx=dt$

$\therefore\ I=\frac{1}{2}\int\frac{1}{t^2}dt=-\frac{1}{2}\cdot\frac{1}{t}+C=-\frac{1}{2(e^{2x}+1)}+C$

(R) Let $I=\int\frac{e^{-x}}{1+e^x}dx=\int\frac{e^{-x}e^{-x}}{e^{-x}+1}dx$

Put $e^{-x}+1=t \Rightarrow -e^{-x}\,dx=dt$

$\therefore\ I=-\int\frac{(t-1)}{t}dt=\int\left(\frac{1}{t}-1\right)dt$

$=\log t-t+C_1=\log(e^{-x}+1)-(e^{-x}+1)+C_1$

$=\log(e^x+1)-x-e^{-x}-1+C_1$

$=\log(e^x+1)-x-e^{-x}+C \qquad [\because\ C=C_1-1]$

(S) Let $I=\int\frac{1}{\sqrt{1-e^{2x}}}dx=\int\frac{e^{-x}}{\sqrt{e^{-2x}-1}}dx$

Put $e^{-x}=t$ or $-e^{-x}\,dx=dt$

$\therefore\ I=-\int\frac{1}{\sqrt{t^2-1}}dt$

$=-\log\left[t+\sqrt{t^2-1}\right]+C=-\log\left[e^{-x}+\sqrt{e^{-2x}-1}\right]+C$

$=-\log\left[\frac{1}{e^x}+\frac{\sqrt{1-e^{2x}}}{e^x}\right]+C$

$=-\log\left[1+\sqrt{1-e^{2x}}\right]+\log e^x+C$

$=x-\log\left[1+\sqrt{1-e^{2x}}\right]+C$

40. (a): (P) We have, $f(x)=\cos x-1+\frac{x^2}{2!}-\frac{x^3}{3!}$

$\Rightarrow\ f'(x)=-\sin x+x-\frac{x^2}{2}\Rightarrow f'(0)=0$

$f''(x)=-\cos x+1-x \Rightarrow f''(0)=0$

$f'''(x)=\sin x-1 \Rightarrow f'''(0)=-1\neq 0$

Thus, $f(x)$ has no extremum at $x=0$

(Q) We have, $f(x)=\cos x-1+\frac{x^2}{2!}$

$\Rightarrow\ f'(x)=-\sin x+x\Rightarrow f'(0)=0$

$\Rightarrow\ f''(x)=-\cos x+1\Rightarrow f''(0)=0$

$\Rightarrow\ f'''(x)=\sin x\Rightarrow f'''(0)=0$

$\Rightarrow\ f''''(x)=\cos x\Rightarrow f''''(0)=1\neq 0$

Thus, $f(x)$ has a minima at $x=0$

(R) We have, $f(x)=x^4e^{-x^2}$

$\Rightarrow\ f'(x)=4x^3e^{-x^2}+x^4e^{-x^2}(-2x)=x^3e^{-x^2}[4-2x^2]$

Now, $f'(x)=0\Rightarrow x=0,\ x^2=2\Rightarrow x=\pm\sqrt{2}$

Now, $f''(x)=x^3e^{-x^2}(-4x)+(4-2x^2)$
$[3x^2e^{-x^2}+x^3e^{-x^2}(-2x)]$

$=e^{-x^2}[-4x^4+(3x^2-2x^4)(4-2x^2)]$

$f''(0)=0,\ f''(\sqrt{2})=\frac{-16}{e^2}<0,\ f''(-\sqrt{2})=\frac{-16}{e^2}<0$

Thus, $f(x)$ has maxima at $x=\pm\sqrt{2}$ and no extremum at $x=0$

(S) We have, $f(x)=\sin 3x-3\sin x=3\sin x-4\sin^3x-3\sin x$
$=-4\sin^3x$

$\Rightarrow\ f'(x)=-12\sin^2x\cos x\Rightarrow f'(0)=0$

$\Rightarrow\ f''(x)=-24\sin x\cos^2x+12\sin^3x \Rightarrow f''(0)=0$

$\Rightarrow\ f'''(x)=-24\cos^3x+48\sin^2x\cos x+36\sin^2x\cos x$

$\Rightarrow\ f'''(0)=-24\neq 0$

Thus, $f(x)$ has no extremum at $x=0$.

Also, $f(x)=-4\sin^3x \Rightarrow -4\le f(x)\le 4 \quad (\because\ -1\le\sin x\le 1)$

41. (4): We know that, $|\sqrt{3}\cos x-\sin x|\le\sqrt{3+1}=2$

But as given $|\sqrt{3}\cos x-\sin x|\ge 2$

$\Rightarrow|\sqrt{3}\cos x-\sin x|=2\Rightarrow\left|\frac{\sqrt{3}}{2}\cos x-\frac{1}{2}\sin x\right|=1$

$\Rightarrow\left|\cos\left(x+\frac{\pi}{6}\right)\right|=1\Rightarrow\cos\left(x+\frac{\pi}{6}\right)=\pm 1$

$\Rightarrow x+\frac{\pi}{6}=2n\pi,(2n+1)\pi,n\in I$

$$\Rightarrow x+\frac{\pi}{6}=0,\pi,2\pi,3\pi,4\pi,5\pi,..,$$

$$\Rightarrow x=\frac{5\pi}{6},\frac{11\pi}{6},\frac{17\pi}{6},\frac{23\pi}{6} \qquad (\text{As } x\in[0,4\pi])$$

Hence, there are 4 values.

42. (10) : $T_6={}^8C_5(x^{-8/3})^3(x^2\log_{10}x)^5=5600$

$\Rightarrow x^2(\log_{10}x)^5=100$. Only $x=10$ satisfies this given condition.

43. (6) : $x=\frac{7!}{3!}, y=\frac{7!}{4!}, z=\frac{7!}{3!\,4!} \Rightarrow x+y=30z$

$\therefore \frac{x+y}{5z}=6$

44. (5386) : Making the equation of the circle homogeneous with the help of the equation of the line, we get

$$x^2+y^2-2(25x+69y)\frac{(69x+25y)}{2\times69\times25}+[(69)^2+(25)^2-c^2]\left[\frac{69x+25y}{2\times69\times25}\right]^2=0$$

$\Rightarrow (2\times69\times25)^2(x^2+y^2)-4\times69\times(25)^2x(69x+25y)-4\times(69)^2\times25y(69x+25y)+[(69)^2+(25)^2-c^2]\times[(69)^2x^2+(25)^2y^2+2\times69\times25xy]=0$

which is the equation of the required lines.

Since, they are at right angles.

$\therefore$ Coefficient of x^2 + coefficient of $y^2=0$

$\Rightarrow [(69)^2+(25)^2-c^2]\,[(69)^2+(25)^2]=0$

$\Rightarrow c^2=(69)^2+(25)^2=4761+625=5386$

45. (7) : Using $\tan x=x+\frac{x^3}{3}+.....$, we find

$$L=\lim_{x\to0}\frac{3(3x+9x^3)-4\left(2x+\frac{8x^3}{3}\right)-\left(x+\frac{x^3}{3}\right)+..}{x^3+\frac{x^5}{3}+....}$$

$=27-\frac{32}{3}-\frac{1}{3}=16$

46. (4) : $\det A=(2k-1)(4k^2-1)+2\sqrt{k}\,(4k\sqrt{k}+2\sqrt{k})+2\sqrt{k}\,(4k\sqrt{k}+2\sqrt{k})$

$=(2k-1)(4k^2-1)+8(2k+1)k$

$=(2k+1)((2k-1)^2+8k)=(2k+1)^3$

$\det(\text{adj } A)=(\det A)^2=(2k+1)^6$

$\det B=0$ since B is skew symmetric matrix of order 3.

$\det(\text{adj } B)=(\det B)^2=0 \;\therefore\; (2k+1)^6=10^6$

$\Rightarrow 2k+1=10 \Rightarrow k=4.5$

So, $[k]=[4.5]=4$

47. (4) : Let $x=r\cos\theta, y=r\sin\theta$

$\therefore r^2(1+\cos\theta\sin\theta)=1$

$\Rightarrow r^2=\frac{2}{2+\sin2\theta} \Rightarrow r^2_{max}=\frac{2}{1} \Rightarrow r^4_{max}=4.$

48. (3) : $\because \frac{dy}{dx}=3x \Rightarrow dy=3xdx$

On integrating, we get $y=\frac{3x^2}{2}+c$

Since, it passes through $(-1,-3)$, then $-3=\frac{3}{2}+c$

$\therefore c=-\frac{9}{2} \;\therefore\; y=\frac{3x^2}{2}-\frac{9}{2} \Rightarrow 2y=3(x^2-3)$

$\therefore \lambda=3$

49. (5) : The pair of lines is

$$3x^2-xy+3y^2+(2x-3y)\frac{(2x+3y)}{k}-4\frac{(2x+3y)^2}{k^2}=0,$$

Now, coefficient of x^2 + coefficient of $y^2=0$

$\Rightarrow 3+3+\frac{4-9}{k}-\frac{4(4+9)}{k^2}=0 \Rightarrow 6k-\frac{52}{k}=5$

50. (5) : Let G denote the event the examinee guesses, C the event that the examinee copies and K the event that the examinee knows. Let R denote the event that the answer is right. We have

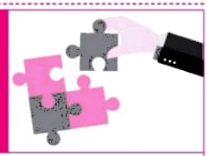
$P(G)=\frac{1}{3}$, $P(C)=\frac{1}{6}$ and $P(K)=1-\frac{1}{3}-\frac{1}{6}=\frac{1}{2}$

Also, $P(R/G)=\frac{1}{m}, P(R/C)=\frac{1}{8}, P(R/K)=1$

By Bayes' rule, $P(K/R)=24m/(16+25m)=120/141$

$\Rightarrow m=5$

PUZZLE CORNER



ANSWER - FEBRUARY 2024

2– 3	5	24× 1	4	6	2÷ 2
3÷ 6	2	3– 5	17+ 3	1– 4	1
6× 1	3	2	6	5	24× 4
2	3– 1	4 4	5	3	6
1– 5	4	4– 6	2	1– 1	2– 3
4	2÷ 6	3	1 1	2	5

beat the TIME TRAP

Duration : 30 minutes

SECTION-I

Single Option Correct Type

1. If $f(x) = 2x^2 + bx + c$ and $f(0) = 3$ and $f(2) = 1$, then $f(1)$ is equal to
   (a) 1 (b) 2
   (c) 0 (d) 1/2

2. The roots of $ax^2 + x + 1 = 0$, where $a \neq 0$, are in the ratio 1 : 1. Then a is equal to
   (a) $\frac{1}{4}$ (b) $\frac{1}{2}$
   (c) $\frac{3}{4}$ (d) 1

3. The number of integers greater than 6,000 that can be formed, using the digits 3, 5, 6, 7 and 8 without repetition, is
   (a) 120 (b) 72
   (c) 216 (d) 192

4. The sum of first 9 terms of the series
   $\frac{1^3}{1}+\frac{1^3+2^3}{1+3}+\frac{1^3+2^3+3^3}{1+3+5}+ ....$ is
   (a) 142 (b) 192 (c) 71 (d) 96

5. The equation of the hyperbola whose foci are (6, 5), (– 4, 5) and eccentricity is 5/4, is
   (a) $\frac{(x-1)^2}{16}-\frac{(y-5)^2}{9}=1$
   (b) $\frac{x^2}{16}-\frac{y^2}{9}=1$
   (c) $\frac{(x-1)^2}{16}-\frac{(y-5)^2}{9}=-1$
   (d) None of these

6. If $f(x) = (ax^2 + b)^3$, then the function g such that $f(g(x)) = g(f(x))$ is given by
   (a) $g(x)=\left(\frac{b-x^{1/3}}{a}\right)$
   (b) $g(x)=\frac{1}{(ax^2+b)^3}$
   (c) $g(x) = (ax^2 + b)^{1/3}$
   (d) $g(x)=\left(\frac{x^{1/3}-b}{a}\right)^{1/2}$

7. If A and B are 2 × 2 matrices, then which of the following is true?
   (a) $(A + B)^2 = A^2 + B^2 + 2AB$
   (b) $(A - B)^2 = A^2 + B^2 - 2AB$
   (c) $(A - B)(A + B) = A^2 + AB - BA - B^2$
   (d) $(A + B)(A - B) = A^2 - B^2$

8. The minimum value of the function
   $f(x)=\frac{1}{\sin x+\cos x}$ in the interval $\left[0,\frac{\pi}{2}\right]$, is
   (a) $\frac{\sqrt{2}}{2}$ (b) $-\frac{\sqrt{2}}{2}$
   (c) $\frac{2}{\sqrt{3}+1}$ (d) $-\frac{2}{\sqrt{3}+1}$

9. $\int_2^4 \log[x]\,dx$ is equal to
   (a) log 2 (b) log 3
   (c) log 5 (d) None of these

10. The area of the region bounded by the curves $y = x^2$ and $y = 4x - x^2$ is
    (a) $\frac{16}{3}$ sq. units (b) $\frac{8}{3}$ sq. units
    (c) $\frac{4}{3}$ sq. units (d) $\frac{2}{3}$ sq. unit

SECTION-II

Numerical Answer Type

11. An integrating factor of the differential equation $\sin x \dfrac{dy}{dx} + 2y\cos x = 1$ is $(\sin x)^{\lambda}$, where $\lambda =$ ______.

12. If in a triangle ABC, $\overrightarrow{BC} = \dfrac{\vec{u}}{|\vec{u}|} - \dfrac{\vec{v}}{|\vec{v}|}$ an $\overrightarrow{AC} = \dfrac{2\vec{u}}{|\vec{u}|}$, where $|\vec{u}| \neq |\vec{v}|$, then the value of $|\cos 2A + \cos 2B + \cos 2C|$ is ______.

13. Let $f: R \to R$ be the function defined by $f(x) = x^3 + 5$. Then $f^{-1}(32)$ is ______.

14. Let X and Y be two events such that $P(X) = \dfrac{1}{3}, P(Y) = \dfrac{4}{15}$ and $P(Y|X) = \dfrac{2}{5}$. Then $P(X'|Y) =$ ______.

15. In a triangle ABC, let $\angle A = \dfrac{\pi}{2}$ and $(a + b + c)\cdot(b + c - a) = \lambda bc$, then λ equals ______.

SOLUTIONS

1. **(c):** $f(x) = 2x^2 + bx + c$, $f(0) = 3 \Rightarrow 3 = c$

$f(2) = 1 \Rightarrow 1 = 8 + 2b + c$

$\therefore\ 2b + c = -7 \Rightarrow 2b + 3 = -7 \Rightarrow b = -5$

$\therefore\ f(1) = 2 \times 1^2 + (-5) \times 1 + 3 = 2 - 5 + 3 = 0$

2. **(a):** Let x_1 and x_2 be the roots of $ax^2 + x + 1 = 0$, then

$x_1 + x_2 = \dfrac{-1}{a}$ ...(i)

and $x_1x_2 = \dfrac{1}{a}$ ...(ii)

Also, $x_1 : x_2 = 1 : 1 \Rightarrow x_1 = x_2$ ...(iii)

Using (iii) in (i), we get

$2x_1 = \dfrac{-1}{a} \Rightarrow x_1 = \dfrac{-1}{2a}$

Using (iii) in (ii), we get

$x_1^2 = \dfrac{1}{a}\quad \therefore \dfrac{1}{4a^2} = \dfrac{1}{a}$

$\Rightarrow 4a = 1 \Rightarrow a = \dfrac{1}{4}$

3. **(d):** Numbers having 5 digits = 5! = 120

Numbers having 4 digits = (3)(4)(3)(2) = 72

As the first digit can be filled in 3 ways, *i.e.*, 6, 7, and 8 and as repetition is not allowed, the other choices are 4, 3 and 2 in that order.

$\therefore$ Required number of ways = 192

4. **(d):** The n^{th} term, t_n is

$$\frac{1^3 + 2^3 + ... + n^3}{1 + 3 + ... + (2n-1)} = \frac{\frac{n^2(n+1)^2}{4}}{n^2} = \frac{(n+1)^2}{4}$$

$$\therefore\ S_n = \sum_{n=1}^{9} t_n = \sum_{n=1}^{9} \frac{(n+1)^2}{4} = \frac{1}{4}\left[\sum_{n=1}^{9} n^2 + 2\sum_{n=1}^{9} n + \sum_{n=1}^{9} 1\right]$$

$$= \frac{1}{4}\left[\frac{9(10)(19)}{6} + \frac{2\cdot 9\cdot 10}{2} + 9\right] = \frac{9}{4}\left[\frac{95}{3} + 10 + 1\right]$$

$$= \frac{3}{4}(128) = 96$$

5. **(a):** Centre of the hyperbola is the mid point of the line joining the two foci, therefore, the coordinates of the centre are (1, 5). Now distance between the foci = 10

$\Rightarrow\ 2ae = 10 \Rightarrow ae = 5 \Rightarrow a = 4$ $[\because e = 5/4]$

Now, $b^2 = a^2(e^2 - 1) \Rightarrow b = 3$

Hence, the equation of the hyperbola is

$$\frac{(x-1)^2}{16} - \frac{(y-5)^2}{9} = 1$$

6. **(d):** Let $y = f(x) = (ax^2 + b)^3 \Rightarrow ax^2 + b = y^{1/3}$

$$\Rightarrow x^2 = \frac{y^{1/3} - b}{a} \Rightarrow x = \left(\frac{y^{1/3} - b}{a}\right)^{1/2} = f^{-1}(y)$$

Since, $f(g(x)) = g(f(x))$ $\therefore$ We have, $g(x) = \left(\dfrac{x^{1/3} - b}{a}\right)^{1/2}$

7. **(c):** Given that, A and B are 2×2 matrices.

$\therefore\ (A - B) \times (A + B) = A \times A + A \times B - B \times A - B \times B$

$= A^2 + AB - BA - B^2$

8. **(a)**

MONTHLY TEST DRIVE CLASS XI ANSWER KEY

1. (b)	2. (b)	3. (b)	4. (a)	5. (d)
6. (d)	7. (a,c)	8. (a)	9. (a,d)	10. (a,b,c,d)
11. (a,b)	12. (a,b)	13. (b,c)	14. (b)	15. (a)
16. (b)	17. (5)	18. (120)	19. (4)	20. (2.8)

9. (d): We have,

$$\int_2^4 \log[x]dx = \int_2^3 \log[x]dx + \int_3^4 \log[x]dx$$

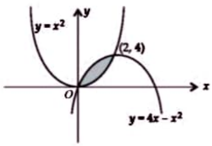
$$= \int_2^3 \log 2\,dx + \int_3^4 \log 3\,dx$$

= (log2)(3 – 2) + (log3) (4 – 3) = log3 + log2 = log 6

10. (b): We have, $y = x^2$ ...(i)

$y = 4x - x^2$ ...(ii)

Their intersection point can be given by $x = 0, 2$ and $y = 0, 4$



∴ Required area

$$= \int_0^2 (4x - x^2)dx - \int_0^2 x^2 dx$$

$$= \left[\frac{4x^2}{2} - \frac{x^3}{3}\right]_0^2 - \left[\frac{x^3}{3}\right]_0^2 = 2(4) - \frac{8}{3} - \frac{8}{3}$$

$$= 8 - \frac{8}{3} - \frac{8}{3} = 8 - \frac{16}{3} = \frac{8}{3} \text{ sq. units}$$

11. (2): The given differential equation can be written as $\frac{dy}{dx} + 2y\cot x = \operatorname{cosec} x$

$$\text{I.F.} = e^{\int 2\cot x\,dx} = e^{2\log|\sin x|} = (\sin x)^2$$

But given, I.F. = $(\sin x)^\lambda$

∴ $\lambda = 2$

12. (1): Given, $\overrightarrow{BC} = \frac{\vec{u}}{|\vec{u}|} - \frac{\vec{v}}{|\vec{v}|}, \overrightarrow{AC} = \frac{2\vec{u}}{|\vec{u}|}$

$$\Rightarrow \overrightarrow{AB} = \overrightarrow{AC} - \overrightarrow{BC} = \frac{\vec{u}}{|\vec{u}|} + \frac{\vec{v}}{|\vec{v}|}$$

Now, $\overrightarrow{AB} \cdot \overrightarrow{BC} = \left(\frac{\vec{u}}{|\vec{u}|} + \frac{\vec{v}}{|\vec{v}|}\right) \cdot \left(\frac{\vec{u}}{|\vec{u}|} - \frac{\vec{v}}{|\vec{v}|}\right)$

$$= \left(\frac{\vec{u}\cdot\vec{u}}{|\vec{u}|^2} - \frac{\vec{v}\cdot\vec{v}}{|\vec{v}|^2}\right) = \frac{|\vec{u}|^2}{|\vec{u}|^2} - \frac{|\vec{v}|^2}{|\vec{v}|^2} = 1 - 1 = 0$$

$$\Rightarrow B = \frac{\pi}{2}$$

Consider, cos 2*A* + cos 2*B* + cos 2*C*

= –1 – 4 cos *A* cos *B* cos *C* = –1 – 0 = –1 [∵ ∠*B* = π/2]

∴ |cos 2*A* + cos 2*B* + cos 2*C*| = |–1| = 1

13. (3): Let $y = f(x) = x^3 + 5 \Rightarrow x^3 = y - 5$

$\Rightarrow x = (y - 5)^{1/3}$ ∴ $f^{-1}(x) = (x - 5)^{1/3}$

So, $f^{-1}(32) = (32 - 5)^{1/3} = 3$

14. (0.5) : Since, $P(Y|X) = \frac{P(X \cap Y)}{P(X)}$, so we have

$$P(X \cap Y) = P(Y|X) \cdot P(X) = \frac{2}{5} \cdot \frac{1}{3} = \frac{2}{15}$$

Now, $P(X|Y) = \frac{P(X \cap Y)}{P(Y)} = \frac{\frac{2}{15}}{\frac{4}{15}} = \frac{1}{2}$

$$\therefore \quad P(X'|Y) = (1 - P(X|Y)) = 1 - \frac{1}{2} = \frac{1}{2}$$

15. (2): Since, $\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{[(b+c)^2 - a^2] - 2bc}{2bc}$

$$\therefore \quad \cos\frac{\pi}{2} = \frac{(b+c-a)(b+c+a) - 2bc}{2bc}$$

$\Rightarrow 0 = \frac{\lambda bc - 2bc}{2bc}$ [Given $(a + b + c)(b + c - a) = \lambda bc$]

$$\Rightarrow 0 = \frac{(\lambda - 2)bc}{2bc} \Rightarrow \lambda = 2$$

❖❖

SAMURAI SUDOKU

ANSWER · FEBRUARY 2024

2	8	6	7	3	5	4	9	1				3	2	5	1	7	9	4	8	6
4	5	9	2	6	1	7	3	8				8	1	9	3	6	4	2	7	5
1	7	3	8	9	4	2	6	5				6	7	4	5	2	8	9	1	3
6	1	7	5	2	3	9	8	4				7	6	1	2	3	5	8	4	9
5	4	2	1	8	9	6	7	3				4	3	2	8	9	7	6	5	1
9	3	8	4	7	6	1	5	2				9	5	8	4	1	6	3	2	7
7	6	4	3	1	8	5	2	9	6	4	7	1	8	3	9	5	2	7	6	4
8	2	5	9	4	7	3	1	6	8	9	2	5	4	7	6	8	3	1	9	2
3	9	1	6	5	2	8	4	7	5	1	3	2	9	6	7	4	1	5	3	8
						6	8	5	3	2	4	9	7	1						
						1	3	2	7	8	9	6	5	4						
						7	9	4	1	5	6	3	2	8						
7	6	8	1	2	9	4	5	3	9	7	1	8	6	2	9	1	7	3	4	5
2	4	3	5	8	6	9	7	1	2	6	8	4	3	5	8	2	6	7	1	9
5	1	9	3	4	7	2	6	8	4	3	5	7	1	9	3	4	5	8	6	2
1	2	5	9	3	8	7	4	6				6	4	8	7	5	2	9	3	1
6	8	4	2	7	5	1	3	9				5	9	3	6	8	1	4	2	7
9	3	7	4	6	1	5	8	2				1	2	7	4	3	9	6	5	8
3	5	6	7	1	2	8	9	4				9	5	4	2	6	8	1	7	3
8	7	1	6	9	4	3	2	5				3	7	1	5	9	4	2	8	6
4	9	2	8	5	3	6	1	7				2	8	6	1	7	3	5	9	4

PRACTICE PAPER 2024

CUET (UG)

Exam on Between 15th and 31st May 2024

General Instructions

This practice paper contains two sections i.e. Section A and Section B [B1 and B2].
Section A has 15 questions covering both i.e. Mathematics/Applied Mathematics which is compulsory for all candidates.
Section B1 has 30 questions from Mathematics out of which 20 questions need to be attempted.
Section B2 has 30 questions purely from Applied Mathematics out of which 20 question need to be attempted.

SECTION A

1. Consider the following statements and choose the correct option.

   I. If A is any square matrix, then $\frac{A+A'}{2}$ is always skew-symmetric.
   II. If A and B are invertible matrices of the same order, then $A + B$ is also invertible.
   III. If A, B, C are three matrices such that both AB and AC are defined and are equal, then B and C are equal matrices.
   IV. For any matrix A, AA' is always defined and is a square matrix.
   V. For any square matrix A with real entries, $A-A'$ is a skew symmetric matrix.

   (a) Only IV and V are correct
   (b) Only II, III and IV are correct
   (c) Only I, III and IV are correct
   (d) All are correct

2. **Statement-I :** The area of ellipse $2x^2 + 3y^2 = 6$ will be more than the area bounded by $2|x| + 3|y| \leq 6$.
   Statement-II : The length of major axis of the ellipse $2x^2 + 3y^2 = 6$ is less than the distance between the points of $2|x| + 3|y| \leq 6$ on X-axis.
   In the light of above statements, choose the correct answer from the options given below.

   (a) Both statement I and statement II are true
   (b) Both statement I and statement II are false
   (c) Statement I is true but statement II is false
   (d) Statement I is false but statement II is true

3. Consider the following statements and choose the correct option.

   I. $\int \frac{\sin x + \cos x}{\sqrt{1+\sin 2x}}\,dx = x + C$
   II. $\int \frac{dx}{1+\cos x} = \tan\frac{x}{2} + C$
   III. $\int \left(\frac{x^2+2}{x+1}\right)dx = \frac{x^2}{2} - x + \log|x+1| + C$
   IV. $\int \frac{dx}{\sqrt{16-9x^2}} = \frac{1}{4}\sin^{-1}\left(\frac{3x}{4}\right) + C$
   V. $\int \frac{x+3}{(x+4)^2} e^x\,dx = e^x\left(\frac{1}{x+4}\right) + C$

   (a) II, III and IV are correct
   (b) II, IV and V are correct
   (c) I, II and V are correct
   (d) All are correct.

4. If $x\begin{bmatrix}3\\2\end{bmatrix} + y\begin{bmatrix}1\\-1\end{bmatrix} = \begin{bmatrix}15\\5\end{bmatrix}$, then the value of x and y are

   (a) $x = -4, y = -3$
   (b) $x = 4, y = 3$
   (c) $x = -4, y = 3$
   (d) $x = 4, y = -3$

5. Let a solution $y = y(x)$ of the differential equation $x\sqrt{x^2-1}\,dy - y\sqrt{y^2-1}\,dx = 0$ satisfy $y(2) = \frac{2}{\sqrt{3}}$.
   Statement-I : $y(x) = \sec\left(\sec^{-1}x - \frac{\pi}{6}\right)$.
   Statement-II : $y(x)$ is given by $\frac{1}{y} = \frac{2\sqrt{3}}{x} - \sqrt{1-\frac{1}{x^2}}$
   In the light of above statements, choose the correct answer from the options given below.

   (a) Both Statement I and Statement II are true
   (b) Both Statement I and Statement II are false
   (c) Statement I is true but Statement II is false
   (d) Statement I is false but Statement II is true

6. If $x = \cos\theta$ and $y = \sin 5\theta$, then $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} =$

(a) $-5y$ (b) $5y$ (c) 25 (d) $-25y$

7. A company manufactures two types of chemicals A and B. Each chemical requires the same type of raw materials P and Q. The table below shows number of units of P and Q required to manufacture one unit of A and one unit of B and the atmost availability of P and Q.

Chemical → / Raw material ↓	A	B	Availability
P	3	2	120
Q	2	5	160

The company gets profits of ₹ 350 and ₹ 400 by selling one unit of chemical A and one unit of chemical B respectively. If the entire production of A and B is sold, then formulate the problem as LPP.

(a) Maximize $z = 350x + 400y$ subject to
$3x + 2y \ge 120$, $2x + 5y \le 160$, $x \ge 0$, $y \ge 0$

(b) Maximize $z = 350x + 400y$ subject to
$3x + 2y \le 120$, $2x + 5y \le 160$, $x \ge 0$, $y \ge 0$

(c) Maximize $z = 350x + 400y$ subject to
$3x + 2y \ge 120$, $2x + 5y \ge 160$, $x \ge 0$, $y \ge 0$

(d) Maximize $z = 350x + 400y$ subject to
$3x + 2y \le 120$, $2x + 5y \ge 160$, $x \ge 0$, $y \ge 0$

8. A rod 108 metres long is bent to form a rectangle. Find its dimensions, if its area is maximum.

(a) 27 metres, 27 metres (b) 35 metres, 19 metres
(c) 31 metres, 23 metres (d) 25 metres, 21 metres

9. If the matrix $\begin{bmatrix} 2 & 3 \\ 5 & -1 \end{bmatrix} = A + B$, where A is symmetric and B is skew symmetric, then $B =$

(a) $\begin{bmatrix} 2 & 4 \\ 4 & -1 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

10. If $\int \frac{f(x)}{\log \cos x} dx = -\log(\log \cos x) + C$, then $f(x)$ is equal to

(a) $\tan x$ (b) $-\sin x$
(c) $-\cos x$ (d) $-\tan x$

11. A fair dice is thrown two times. The probability that first throw gives a multiple of 2 and second throw gives a multiple of 3 is

(a) $\frac{1}{24}$ (b) $\frac{1}{18}$ (c) $\frac{1}{6}$ (d) $\frac{1}{12}$

12. If ω is a complex cube root of unity, then a root of the equation $\begin{vmatrix} x+1 & \omega & \omega^2 \\ \omega & x+\omega^2 & 1 \\ \omega^2 & 1 & x+\omega \end{vmatrix} = 0$ is

(a) $x = 0$ (b) $x = 1$
(c) $x = \omega$ (d) $x = \omega^2$

13. Consider the following statements and choose the correct option.

I. At (3, 2) and (–1, 2) on the curve $x^2 + y^2 - 2x - 4y + 1 = 0$, the tangents are parallel to y-axis.

II. $f(x) = \tan^{-1}(\sin x + \cos x)$ is an increasing function on $(0, \pi/4)$.

III. $f(x) = \sin x + \sqrt{3}\cos x$ has maximum value at $\frac{\pi}{3}$.

IV. $f(x) = \tan^{-1}(\sin x + \cos x)$ is an increasing function on $(0, \pi)$.

V. $f(x) = \sin x + \sqrt{3}\cos x$ has maximum value at $\frac{\pi}{6}$.

(a) II, III and IV are correct
(b) I, II and V are correct
(c) III, IV and V are correct
(d) All are correct.

14. Match column-I with column-II and select the correct answer using the options given below :

Column-I		Column-II	
(P)	The solution of $\frac{dy}{dx} = 2^{y-x}$ is	(1)	$x^2(y+3)^3 = e^{y+2}$
(Q)	The solution of $\frac{dy}{dx} = 1 + x + y^2 + xy^2$, when $y = 0$, $x = 0$ is	(2)	$2^{-x} - 2^{-y} = k$
(R)	Solution of $x^2\frac{dy}{dx} = x^2 + xy + y^2$ is	(3)	$y = \tan\left(x + \frac{x^2}{2}\right)$

(S)	The solution of $2(y+3)-xy\frac{dy}{dx}=0$, given that $y(1)=-2$ is	(4)	$\tan^{-1}\left(\frac{y}{x}\right)=\log\|x\|+C$

(a) (P) → (1), (Q) → (2), (R) → (3), (S) → (4)
(b) (P) → (1), (Q) → (3), (R) → (4), (S) → (2)
(c) (P) → (2), (Q) → (1), (R) → (3), (S) → (4)
(d) (P) → (2), (Q) → (3), (R) → (4), (S) → (1)

15. Consider the following statements and choose the correct option.

I. Matrix addition is commutative.
II. Matrix addition is associative.
III. Matrix multiplication is commutative.
IV. Matrix multiplication is not commutative.

(a) Only I is correct
(b) Only I, II and IV are correct
(c) Only IV is correct
(d) All are correct

SECTION B1 (MATHEMATICS)

16. Let $f: R \to R$ be defined as $f(x)=\frac{\sin\pi\{x\}}{x^2-x+1}\ \forall x\in R$, where $\{x\}$ is fractional part function. Then

(a) f is either even or odd function
(b) f is a zero function
(c) f is one-one function
(d) None of these

17. If $\cot(\sin^{-1}x)=\cos(\tan^{-1}\sqrt{3})$, then $x=$

(a) 0 (b) $\frac{2}{\sqrt{3}}$ (c) 2 (d) $\frac{2}{\sqrt{5}}$

18. Match column-I with column-II and select the correct answer using the options given below :

	Column-I		Column-II
(P)	The domain of the function $f(x)=\sqrt{4x-3}+\sqrt{2x-6}$ is	(1)	$\left[0,\frac{1}{2}\right]$
(Q)	The range of the function $y=\frac{x^2}{1+x^4}$ is	(2)	$[3,\infty)$
(R)	The domain of $f(x)=\frac{\log_2(x+3)}{x^2+3x+2}$ is	(3)	$\left(-\infty,\frac{2}{3}\right]\cup(1,\infty)$
(S)	The range of $f(x)=\frac{x^2-2}{x^2-3}$ is	(4)	$(-3,\infty)-\{-1,-2\}$

(a) (P) → (1), (Q) → (2), (R) → (4), (S) → (3)
(b) (P) → (3), (Q) → (4), (R) → (2), (S) → (1)
(c) (P) → (4), (Q) → (1), (R) → (2), (S) → (3)
(d) (P) → (2), (Q) → (1), (R) → (4), (S) → (3)

19. Let $f(x)=\begin{cases}\frac{x^3+x^2-16x+20}{(x-2)^2}, & \text{if } x\neq 2\\ b, & \text{if } x=2\end{cases}$

If $f(x)$ is continuous for all x, then b is equal to

(a) 7 (b) 3 (c) 2 (d) 5

20. Match column-I with column-II and select the correct answer using the options given below :

	Column-I		Column-II
(P)	The function $f(x)=\frac{x}{(1+x^2)}$ decreases in the interval	(1)	$(-\infty,\infty)$
(Q)	The function $f(x)=\tan^{-1}x-x$ decreases in the interval	(2)	$(-\infty,0)$
(R)	The function $f(x)=x-e^x+\tan\left(\frac{2\pi}{7}\right)$ increases in the interval	(3)	$(0,\infty)$
		(4)	$(-\infty,-1)\cup(1,\infty)$

(a) (P) → (4), (Q) → (1), (R) → (2)
(b) (P) → (2), (Q) → (3), (R) → (4)
(c) (P) → (1), (Q) → (2), (R) → (3)
(d) (P) → (4), (Q) → (2), (R) → (3)

21. If $x=e^t\sin t$, $y=e^t\cos t$, then $\frac{d^2y}{dx^2}$ at $x=\pi$ is

(a) $2e^{\pi}$ (b) $\frac{1}{2}e^{\pi}$ (c) $\frac{1}{2e^{\pi}}$ (d) $\frac{2}{e^{\pi}}$

22. The value of $\cot^{-1}\left[\frac{\sqrt{1-\sin x}+\sqrt{1+\sin x}}{\sqrt{1-\sin x}-\sqrt{1+\sin x}}\right]$, where $x\in\left(0,\frac{\pi}{4}\right)$ is

(a) $\pi-\frac{x}{3}$ (b) $\frac{x}{2}$ (c) $\pi-\frac{x}{2}$ (d) $\frac{x}{2}-\pi$

23. The function $f: R\to R$ defined by $f(x)=(x-1)(x-2)(x-3)$ is

(a) one-one but not onto
(b) onto but not one-one
(c) both one-one and onto
(d) neither one-one nor onto

24. If $x = 3\cos t - 2\cos^3 t$, $y = 3\sin t - 2\sin^3 t$, then $\frac{d^2y}{dx^2}$ at $t = \frac{\pi}{6}$ is

(a) $\frac{16}{2\sqrt{3}}$ (b) $-\frac{16}{3}$ (c) $\frac{16}{3}$ (d) $\frac{-16}{2\sqrt{3}}$

25. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors, then $|\vec{a}-\vec{b}|^2 + |\vec{b}-\vec{c}|^2 + |\vec{c}-\vec{a}|^2$ does not exceed

(a) 4 (b) 9 (c) 8 (d) 6

26. Statement-I : If the cartesian equation of a line is $\frac{x-5}{3} = \frac{y+4}{7} = \frac{z-6}{2}$, then its vector form is $\vec{r} = 5\hat{i} - 4\hat{j} + 6\hat{k} + \lambda(3\hat{i} + 7\hat{j} + 2\hat{k})$.

Statement-II : The cartesian equation of the line which passes through the point (–2, 4, –5) and parallel to the line given by $\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}$ is $\frac{x+3}{-2} = \frac{y-4}{4} = \frac{z+8}{-5}$.

In the light of above statements, choose the correct answer from the options given below:

(a) Both Statement I and Statement II are true
(b) Both Statement I and Statement II are false
(c) Statement I is true but Statement II is false
(d) Statement I is false but Statement II is true

27. In solving the LPP :
"minimize $f = 6x + 10y$ subject to constraints $x \geq 6$, $y \geq 2$, $2x + y \geq 10$, $x \geq 0$, $y \geq 0$" redundant constraints are

(a) $x \geq 6, y \geq 2$
(b) $2x + y \geq 10, x \geq 0, y \geq 0$
(c) $x \geq 6$
(d) None of these

28. A random variable X has the following probability distribution :

X	0	1	2	3	4	5	6	7
$P(X)$	a	$4a$	$3a$	$7a$	$8a$	$10a$	$6a$	$9a$

Find $P(X < 3)$, $P(X \geq 4)$, $P(0 < X < 5)$ respectively.

(a) $\frac{1}{6}, \frac{11}{24}, \frac{33}{48}$ (b) $\frac{1}{6}, \frac{33}{48}, \frac{11}{24}$

(c) $\frac{1}{4}, \frac{11}{26}, \frac{21}{44}$ (d) $\frac{11}{26}, \frac{1}{4}, \frac{21}{44}$

29. Given $f(x) = \log\left(\frac{1+x}{1-x}\right)$ and $g(x) = \frac{3x + x^3}{1+3x^2}$, then $fog(x)$ equals

(a) $-f(x)$ (b) $3f(x)$
(c) $[f(x)]^3$ (d) None of these

30. If the equations $2x + 3y + z = 0$, $3x + y - 2z = 0$ and $ax + 2y - bz = 0$ has non-trivial solution, then

(a) $a - b = 2$ (b) $a + b + 1 = 0$
(c) $a + b = 3$ (d) $a - b - 8 = 0$

31. Find λ if the vectors $\hat{i} - \hat{j} + \hat{k}$, $3\hat{i} + \hat{j} + 2\hat{k}$ and $\hat{i} + \lambda\hat{j} - 3\hat{k}$ are coplanar.

(a) 5 (b) 12 (c) 15 (d) 8

32. Match column-I with column-II and select the correct answer using the options given below :

Column-I		Column-II	
(P)	The equation of the plane through the points (2, 1, 0), (3, –2, –2) and (3, 1, 7) is	(1)	$x + y + 2z - 19 = 0$
(Q)	The equation of a plane which bisects perpendicularly the line joining the points $A(2, 3, 4)$ and $B(4, 5, 8)$ is	(2)	$3x - 2y + 6z - 27 = 0$
(R)	The equation of the plane through the points (2, 1, –1), (–1, 3, 4) and perpendicular to the plane $x - 2y + 4z = 10$ is	(3)	$7x + 3y - z - 17 = 0$
(S)	If the line drawn from the point (–2, –1, –3) meets a plane at right angle at the point (1, –3, 3), then the equation of the plane is	(4)	$18x + 17y + 4z - 49 = 0$

(a) (P) → (2), (Q) → (3), (R) → (4), (S) → (1)
(b) (P) → (3), (Q) → (1), (R) → (4), (S) → (2)
(c) (P) → (1), (Q) → (2), (R) → (3), (S) → (4)
(d) (P) → (3), (Q) → (1), (R) → (2), (S) → (4)

33. Statement-I : $f : N \to Y$ be a function defined as $f(x) = 4x + 3$, where $Y = \{y \in N : y = 4x + 3$ for some $x \in N\}$ is invertible.

Statement-II : If a function is one-one and onto, then it is a invertible function.

In the light of above statements, choose the correct answer from the options given below.

(a) Both statement I and statement II are true
(b) Both statement I and statement II are false
(c) Statement I is true but statement II is false
(d) Statement I is false but statement II is true

34. The solution of differential equation $(e^y + 1)\cos x\,dx + e^y \sin x\,dy = 0$ is

(a) $(e^y + 1)\sin x = c$ (b) $e^x \sin x = c$
(c) $(e^x + 1)\cos x = c$ (d) None of these

35. The area of the region bounded by the curve $y = \sqrt{4 - x^2}$ and x-axis is

(a) 8π sq. units (b) 2π sq. units
(c) 16π sq. units (d) 6π sq. units

Case Based MCQs

Case I : Read the following passage and answer the questions from 36 to 40.

Integration is the process of finding the anti-derivative of a function. In this process, we are provided with the derivative of a function and asked to find out the function (*i.e.*, Primitive).

Integration is the inverse process of differentiation.

Let $f(x)$ be a function of x. If there is a function $g(x)$, such that $\frac{d}{dx}(g(x)) = f(x)$, then $g(x)$ is called an integral of $f(x)$ w.r.t. x and is denoted by $\int f(x)dx = g(x) + c$, where c is constant of integration.

36. Evaluate : $\int (3x + 4)^3 dx$

(a) $\frac{(3x+4)^4}{12} + c$ (b) $\frac{(3x+4)^5}{12} + c$
(c) $\frac{(3x+5)^4}{12} + c$ (d) $\frac{(3x+4)^4}{4} + c$

37. Evaluate : $\int \frac{(x+1)^2}{x(x^2+1)} dx$

(a) $\log|x| + \tan^{-1} x + c$ (b) $\log|x| + 2\tan^{-1} x + c$
(c) $\log|x| + 2\sin^{-1} x + c$ (d) $\log|x| + \sin^{-1} x + c$

38. Evaluate : $\int \sin^2 x\,dx$

(a) $\frac{x}{2} + \frac{\sin 2x}{4} + c$ (b) $\frac{x}{2} + \sin 2x + c$
(c) $x - \sin 2x + c$ (d) $\frac{x}{2} - \frac{\sin 2x}{4} + c$

39. Evaluate : $\int \tan^2 x\,dx$

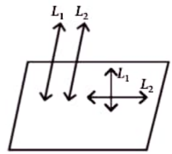
(a) $\tan x + c$ (b) $\tan x + x + c$
(c) $\tan x - x + c$ (d) $\tan x + x^2 + c$

40. Evaluate : $\int \frac{dx}{\sin^2 x \cos^2 x}$

(a) $-2\cot 2x + c$ (b) $2\cot 2x + c$
(c) $\cot 2x + c$ (d) $\cot x + c$

Case II : Read the following passage and answer the questions from 41 to 45.

If a_1, b_1, c_1 and a_2, b_2, c_2 are direction ratios of two lines say L_1 and L_2 respectively. Then $L_1 \parallel L_2$ iff $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ and $L_1 \perp L_2$ iff $a_1a_2 + b_1b_2 + c_1c_2 = 0$.



41. If l_1, m_1, n_1 and l_2, m_2, n_2 are the direction cosines of L_1 and L_2 respectively, then L_1 will be perpendicular to L_2, iff

(a) $l_1l_2 + m_1m_2 + n_1n_2 = 0$ (b) $l_1m_2 + m_1l_2 + n_1n_2 = 0$
(c) $\frac{l_1}{l_2} = \frac{m_1}{m_2} = \frac{n_1}{n_2}$ (d) None of these

42. If l_1, m_1, n_1 and l_2, m_2, n_2 are direction cosines of L_1 and L_2 respectively, then L_1 will be parallel to L_2, iff

(a) $l_1l_2 + m_1m_2 + n_1n_2 = 0$ (b) $l_1m_2 + m_1l_2 + n_1n_2 = 0$
(c) $\frac{l_1}{l_2} = \frac{m_1}{m_2} = \frac{n_1}{n_2}$ (d) $m_1n_2 + m_2n_2 + l_1l_2 = 0$

43. The coordinates of the foot of the perpendicular drawn from the point $A(1, 2, 1)$ to the line joining $B(1, 4, 6)$ and $C(5, 4, 4)$, are

(a) (1, 2, 1) (b) (2, 4, 5) (c) (3, 4, 5) (d) (4, 3, 5)

44. The direction ratios of the line which is perpendicular to the lines with direction ratios proportional to (1, –2, –2) and (0, 2, 1) are

(a) $< 1, 2, 1 >$ (b) $< 2, -1, 2 >$
(c) $< -1, 2, 2 >$ (d) None of these

45. The lines $\frac{x-2}{3} = \frac{y+1}{-2} = \frac{z-2}{0}$ and $\frac{x-1}{1} = \frac{y+3/2}{3/2} = \frac{z+5}{2}$ are

(a) parallel (b) perpendicular
(c) skew lines (d) non-intersecting

SECTION B2 (APPLIED MATHEMATICS)

16. Two vessels A and B contain milk and water in the ratio 7 : 5 and 17 : 7 respectively. In what ratio mixtures from two vessels should be mixed to get a new mixture containing milk and water in the ratio 5 : 3?

(a) 2 : 1 (b) 1 : 2 (c) 3 : 1 (d) 1 : 3

17. Find the absolute maximum of
$f(x) = 2x^3 - 9x^2 + 12x - 5$ in $[0, 3]$.
(a) 4 (b) –5 (c) 3 (d) 0

18. If $\begin{bmatrix} x+y & 2x+z \\ x-y & 2z+w \end{bmatrix} = \begin{bmatrix} 4 & 7 \\ 0 & 10 \end{bmatrix}$, then the values of x, y, z and w respectively are
(a) 2, 2, 3, 4 (b) 2, 3, 1, 2
(c) 3, 3, 0, 1 (d) None of these

19. Mr. Sharma borrowed ₹ 1000000 from a bank to purchase a house and decided to repay the loan by equal monthly installments in 10 years. If bank charges interest at 9% p.a. compounded monthly, calculate the EMI. (Given $(1.0075)^{120} = 2.4514$)
(a) ₹ 13667 (b) ₹ 12667.42
(c) ₹ 11667.45 (d) ₹ 10667.45

20. The random variable X can take only the values 0, 1, 2. Given that, $P(X = 0) = P(X = 1) = p$ and that $E(X^2) = E(X)$, find the value of p.
(a) 1/5 (b) 3/10 (c) 2/5 (d) 1/2

21. Find the last digit of 12^{12}.
(a) 6 (b) 8 (c) 4 (d) 2

22. If $\frac{3x-4}{2} \geq \frac{x+1}{4} - 1$, then $x \in$
(a) $[1, \infty)$ (b) $(1, \infty)$ (c) $(-5, 5)$ (d) $[-5, 5]$

23. If the total cost function is given by $C = \frac{x^3}{4} + 9x^2 - 7$, then find the average cost.
(a) $\frac{x^4}{4} + 9x^3 - 7x$ (b) $\frac{3x^2}{4} + 18x$
(c) $\frac{x^2}{4} + 9x - 7$ (d) $\frac{x^2}{4} + 9x - \frac{7}{x}$

24. Consider the following statements:
Statement-I : Fisher's ideal index number satisfied both time reversal and factor reversal test.
Statement-II : The time reversal test is satisfied if $P_{01} \times P_{10} \neq 1$.
In the light of above statements, choose the correct answer from the options given below.
(a) Both Statement I and Statement II are true
(b) Both Statement I and Statement II are false.
(c) Statement I is true but Statement II is false.
(d) Statement I is false but Statement II is true.

25. A bond of face value ₹ 1000 matures in 5 years. Interest is paid semi-annually and bond is priced to yield 8% p.a. If the present value of bond is ₹ 800, find the annual coupon rate. (Use $(1.04)^{-10} = 0.6761$)
(a) 2.05% (b) 3.05% (c) 3.06% (d) 4.05%

26. A swimmer whose speed in swimming pool is 4 km/h, swims between two points in a river and returns back to the starting point. He took 10 minutes more to cover the distance upstream than downstream. If the speed of the stream is 2 km/h, find the distance between two points.
(a) 2.0 km (b) 0.5 km (c) 1.5 km (d) 2.5 km

27. $Z = 7x + y$, subject to $5x + y \geq 5$, $x + y \geq 3$, $x \geq 0$, $y \geq 0$. The minimum value of Z occurs at
(a) (3, 0) (b) $\left(\frac{1}{2}, \frac{5}{2}\right)$ (c) (7, 0) (d) (0, 5)

28. The function $f(x) = 1 - x^3 - x^5$ is decreasing for
(a) $1 \leq x \leq 5$ (b) $x \leq 1$
(c) $x \geq 1$ (d) all values of x

29. A man buys ₹ 50 shares of a company which pays ₹ 12% dividend. He buys the shares at such a price that his profit is 15% on his investment. At what price did he buy the shares?
(a) ₹ 40 (b) ₹ 30 (c) ₹ 20 (d) ₹ 50

30. $\frac{d^2}{dx^2}(\cos^{-1}(1-x))$ is equal to
(a) $\frac{x-1}{(2x-x^2)^{3/2}}$ (b) $\frac{1-x}{(2x-x^2)^{3/2}}$
(c) $\frac{1}{2(2x-x^2)^{3/2}}$ (d) None of these

31. Calculate the difference between first four years and first three years moving average for the following series of observations.

Year	2015	2016	2017	2018	2019
Demand	55	62	54	69	72

(a) 4.66 (b) 4.25 (c) 3.76 (d) 3

32. Find the probability distribution of the number of successes in two tosses of a die where a success is defined as 'a number greater than 4'.

(a)

X	0	1	2
P(X)	4/9	4/9	1/9

(b)

X	0	1	2
P(X)	1/9	4/9	4/9

(c)

X	0	1	2
P(X)	4/9	1/9	4/9

(d)

X	0	1	2
P(X)	4/9	1/3	1/3

33. If A and B are square matrices of same order and A' denotes the transpose of A, then

(a) $(AB)' = B'A'$

(b) $(AB)' = A'B'$

(c) $AB = O \Rightarrow |A| = 0$ and $|B| = 0$

(d) $AB = O \Rightarrow A = O$ or $B = O$

34. The rise in demand before Diwali is an example of

(a) Seasonal trend

(b) Cyclical trend

(c) Secular trend

(d) Irregular trend

35. Consider the following statements

I. The general L.P.P. calls for optimizing a linear function for variables called the constraints or restrictions.

II. Linear scale is involved in L.P.P.

III. A feasible solution to the linear programming problem should satisfy the problem constraints.

IV. If no feasible region is obtained *via* plotting the constraints, then the linear programming problem has infeasible region.

Which of the above statements are correct?

(a) I, II, III, IV (b) I, II, IV

(c) I, IV (d) IV only

36. A pipe can fill the tank in 3 hours and another pipe can empty the full tank in 4 hours. If both the pipes are opened together, then find how much time will they take to fill the tank?

(a) 12 hours (b) 14 hours

(c) 10 hours (d) None of these

37. Given that the total cost function for x units of a commodity is $C(x) = \frac{x^3}{3} + 3x^2 - 7x + 16$, then average cost function is

(a) $x^2 + 3x - 7$ (b) $\frac{1}{3}x^2 + 3x - 7 + \frac{16}{x}$

(c) $\frac{1}{3}x^2 - 3x + 7$ (d) None of these

38. A man has 20 debentures of a company and receives an interest of ₹ 30 per quarter. If the return on his investment is 8% per annum, find the par value of a debenture.

(a) ₹ 55 (b) ₹ 60 (c) ₹ 75 (d) ₹ 85

39. Match column-I with column-II and select the correct answer using the options given below :

	Column-I		Column-II
(P)	If $A = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$, then A^2 is equal to	(1)	$\begin{bmatrix} \cos(x+y) & \sin(x+y) \\ -\sin(x+y) & \cos(x+y) \end{bmatrix}$
(Q)	If $P(x) = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$, then $P(x)\,P(y)$ is equal to	(2)	$\begin{bmatrix} -2 & 0 \\ -3 & 0 \end{bmatrix}$
(R)	If $\begin{bmatrix} d & a & 3 \\ 2 & b & -1 \\ c & 1 & 0 \end{bmatrix}$ is a skew symmetric matrix, then $\begin{bmatrix} a & b \\ c & d \end{bmatrix} =$	(3)	$m \times m$ or $n \times n\,(m = n)$
(S)	If A is of order $m \times n$ and both $A + B$ and AB are defined, then B is of order	(4)	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(a) (P) → (3), (Q) → (2), (R) → (1), (S) → (4)

(b) (P) → (1), (Q) → (2), (R) → (3), (S) → (4)

(c) (P) → (4), (Q) → (1), (R) → (2), (S) → (3)

(d) (P) → (2), (Q) → (3), (R) → (4), (S) → (1)

40. Match column-I with column-II and select the correct answer using the options given below :

	Column-I		Column-II
(P)	If $y = \log\left(\frac{1-x^2}{1+x^2}\right)$, then $\frac{dy}{dx} =$	(1)	$\frac{1}{\sqrt{x^2+1}}$
(Q)	$\frac{d}{dx}\{\log(x + \sqrt{x^2+1})\} =$	(2)	0
(R)	If $y = \frac{e^x - e^{-x}}{e^x + e^{-x}}$, then $\frac{dy}{dx} =$	(3)	$\frac{-4x}{1-x^4}$
(S)	If $y = \tan^{-1}\left(\frac{2x}{1-x^2}\right) - \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ and $x \in (-1, 0)$, then $\frac{dy}{dx} =$	(4)	$\frac{4e^{2x}}{(e^{2x}+1)^2}$

(a) (P) → (3), (Q) → (2), (R) → (4), (S) → (1)
(b) (P) → (3), (Q) → (1), (R) → (4), (S) → (2)
(c) (P) → (1), (Q) → (3), (R) → (1), (S) → (4)
(d) (P) → (2), (Q) → (4), (R) → (2), (S) → (3)

41. Consider the following statements:
Statement-I : A sample from the population does not have to share the same characteristics as the population.
Statement-II : A method of using sample to estimate population parameters is known as statistical inference.
In the light of above statements, choose the correct answer from the options given below :
(a) Both Statement I and Statement II are true
(b) Both Statement I and Statement II are false.
(c) Statement I is true but Statement II is false.
(d) Statement I is false but Statement II is true.

42. Find the maximum profit that a company can make, if the profit function is given by $P(x) = 41 + 24x - 18x^2$.
(a) 25 (b) 43 (c) 62 (d) 49

43. Find the last two digits of the product 2345 × 6789.
(a) 05 (b) 50 (c) 10 (d) 01

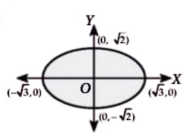
44. Statement-I : The points on the curve $y^2 = x + \sin x$ at which the tangent is parallel to x-axis lies on a straight line.
Statement-II : For a function $y = f(x) = 0$, if tangent is parallel to x-axis, then
$$\frac{dy}{dx} = 0 \text{ or } \frac{dx}{dy} = \infty.$$
In the light of above statements, choose the correct answer from the options given below
(a) Both Statement I and Statement II are true
(b) Both Statement I and Statement II are false.
(c) Statement I is true but Statement II is false.
(d) Statement I is false but Statement II is true.

45. The rate of interest used to discounts the bond's cash flow is called.
(a) Discount (b) Loan
(c) Sinking fund (d) None of these

SOLUTIONS

1. (a) : From definition only IV and V are correct statements.

2. (d) :

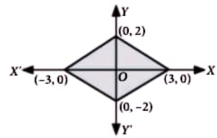


Area of ellipse $\frac{x^2}{3} + \frac{y^2}{2} = 1$ is given by

$$4\int_0^{\sqrt{3}} \sqrt{2\left(1 - \frac{x^2}{3}\right)}\,dx = 4\sqrt{\frac{2}{3}}\int_0^{\sqrt{3}} \sqrt{3 - x^2}\,dx$$
$$= 4\sqrt{\frac{2}{3}}\left[\frac{x}{2}\sqrt{3 - x^2} + \frac{3}{2}\sin^{-1}\frac{x}{\sqrt{3}}\right]_0^{\sqrt{3}}$$
$$= 4\sqrt{\frac{2}{3}} \times \frac{3}{2} \times \frac{\pi}{2} = \sqrt{\frac{2}{3}} \times 3\pi = \sqrt{6}\,\pi \text{ sq. units}$$
$$= 2\sqrt{6}\left(\frac{\pi}{2}\right) = 7.69 \text{ (approx.)}$$

Area bounded by $2|x| + 3|y| \le 6$

$$= 4\int_0^3 \left(\frac{6 - 2x}{3}\right)dx$$
$$= 4\left[2x - \frac{x^2}{3}\right]_0^3$$
$= 4\,[6 - 3 - 0]$
$= 12$ sq. units



and length of major axis of the ellipse $= 2\sqrt{3} < 3 + 3$

3. (c) : I. Let $I = \int \frac{\sin x + \cos x}{\sqrt{1 + \sin 2x}}\,dx$
$$\Rightarrow I = \int \frac{(\sin x + \cos x)}{\sqrt{\sin^2 x + \cos^2 x + 2\sin x \cos x}}\,dx$$
$$= \int \frac{\sin x + \cos x}{\sqrt{(\sin x + \cos x)^2}}\,dx = \int 1 \cdot dx = x + C$$

II. Let $I = \int \frac{dx}{1 + \cos x} = \frac{1}{2}\int \frac{1}{\cos^2 \frac{x}{2}}\,dx$
$$= \frac{1}{2}\int \sec^2\frac{x}{2}\,dx = \frac{1}{2}\tan\frac{x}{2} \cdot 2 + C = \tan\frac{x}{2} + C$$

III. Let $I = \int \frac{x^2 + 2}{x + 1}\,dx = \int\left(x - 1 + \frac{3}{x + 1}\right)dx$
$$= \frac{x^2}{2} - x + 3\log|x + 1| + C$$

IV. Let $I = \int \frac{dx}{\sqrt{16 - 9x^2}}$
$$\Rightarrow I = \int \frac{dx}{\sqrt{(4)^2 - (3x)^2}} = \frac{1}{3}\sin^{-1}\left(\frac{3x}{4}\right) + C$$

V. Let $I = \int \frac{x + 3}{(x + 4)^2}\,e^x\,dx$
$$= \int e^x\left(\frac{1}{(x + 4)} - \frac{1}{(x + 4)^2}\right)dx = e^x\left(\frac{1}{x + 4}\right) + C$$
$$\left[\text{Using } \int e^x\{f(x) + f'(x)\}dx = e^x f(x) + C\right]$$

4. (b): We have, $x\begin{bmatrix}3\\2\end{bmatrix}+y\begin{bmatrix}1\\-1\end{bmatrix}=\begin{bmatrix}15\\5\end{bmatrix}$

$\Rightarrow \begin{bmatrix}3x\\2x\end{bmatrix}+\begin{bmatrix}y\\-y\end{bmatrix}=\begin{bmatrix}15\\5\end{bmatrix} \Rightarrow \begin{bmatrix}3x+y\\2x-y\end{bmatrix}=\begin{bmatrix}15\\5\end{bmatrix}$

$\Rightarrow$ $3x + y = 15$ ...(i) $2x - y = 5$ ...(ii)

Solving equation (i) and (ii), we get $x = 4$ and $y = 3$

5. (c): Given, $\frac{dy}{dx}=\frac{y\sqrt{y^2-1}}{x\sqrt{x^2-1}}$

$\Rightarrow \int\frac{dy}{y\sqrt{y^2-1}}=\int\frac{dx}{x\sqrt{x^2-1}} \Rightarrow \sec^{-1}y=\sec^{-1}x+C$

At $x = 2$, $y=\frac{2}{\sqrt{3}}$

$\Rightarrow \frac{\pi}{6}=\frac{\pi}{3}+C \Rightarrow C=-\frac{\pi}{6}$

Now, $y=\sec\left(\sec^{-1}x-\frac{\pi}{6}\right)$

$\Rightarrow \frac{1}{y}=\cos\left(\cos^{-1}\frac{1}{x}-\cos^{-1}\frac{\sqrt{3}}{2}\right)$

$=\cos\left[\cos^{-1}\left(\frac{\sqrt{3}}{2x}+\sqrt{1-\frac{1}{x^2}}.\sqrt{1-\frac{3}{4}}\right)\right]$

$\Rightarrow \frac{1}{y}=\frac{\sqrt{3}}{2x}+\frac{1}{2}\sqrt{1-\frac{1}{x^2}}$

6. (d)

7. (b): Let x units of chemical A and y units of chemical B are being manufactured to maximize the profit.

The total profit z on x units of A and y units of B is

$z = 350x + 400y$

Hence, the LPP is formulated as

Maximize, $z = 350x + 400y$

Subject to, $3x + 2y \le 120$, $2x + 5y \le 160$, $x \ge 0$, $y \ge 0$

8. (a): Let x be the length and y be the breadth of the rectangle

$\therefore$ $2x + 2y = 108 \Rightarrow y = 54 - x$

Now, area of rectangle $= xy = x(54 - x)$

Let $f(x) = 54x - x^2$

$\therefore$ $f'(x) = 54 - 2x$ and $f''(x) = -2$

For critical point $f'(x) = 0$

$\Rightarrow$ $54 - 2x = 0 \Rightarrow x = 27$; $f''(27) = -2 < 0$

Hence, length and breadth both are equal to 27 m.

9. (d): $B=\frac{1}{2}\left\{\begin{bmatrix}2&3\\5&-1\end{bmatrix}-\begin{bmatrix}2&5\\3&-1\end{bmatrix}\right\}$

$=\frac{1}{2}\begin{bmatrix}0&-2\\2&0\end{bmatrix}=\begin{bmatrix}0&-1\\1&0\end{bmatrix}$

10. (a): We have, $\int\frac{f(x)}{\log\cos x}dx=-\log(\log\cos x)+C$

Differentiating both sides w.r.t. x, we get

$\frac{f(x)}{\log\cos x}=\frac{-1}{\log\cos x}\times\frac{1}{\cos x}\times(-\sin x)$

$\Rightarrow \frac{f(x)}{\log\cos x}=\frac{\tan x}{\log\cos x} \Rightarrow f(x)=\tan x$

11. (c)

12. (a): $\begin{vmatrix}x+1&\omega&\omega^2\\\omega&x+\omega^2&1\\\omega^2&1&x+\omega\end{vmatrix}=0$

Expanding along R_1, we get

$(x + 1)[x^2 + x\omega + x\omega^2 + \omega^3 - 1] - \omega[x\omega + \omega^2 - \omega^2]$
$+ \omega^2[\omega - x\omega^2 - \omega^4] = 0$

$\Rightarrow$ $x^3 + x^2\omega + x^2\omega^2 + x^2 = 0$

$\Rightarrow$ $x^3 + x^2(1 + \omega + \omega^2) = 0$

$\Rightarrow$ $x^3 = 0 \Rightarrow x = 0$ $[\because 1 + \omega + \omega^2 = 0]$

Therefore, $x = 0$ is a root of the given equation.

13. (b): I. We have, $x^2 + y^2 - 2x - 4y + 1 = 0$ ...(1)

let $P(x_1, y_1)$ be a required point on the given curve where the tangent is parallel to y-axis, then

Differentiating (1) w.r.t. x, we get

$2x+2y\frac{dy}{dx}-2-4\frac{dy}{dx}+0=0 \Rightarrow \frac{dy}{dx}=\frac{2(1-x)}{2(y-2)}=\frac{1-x}{y-2}$

$\Rightarrow$ Slope of tangent at $P(x_1, y_1) = \frac{1-x_1}{y_1-2}$

$\because$ The tangent is parallel to Y-axis,

$\therefore$ $y_1 - 2 = 0 \Rightarrow y_1 = 2$

Substituting $x = x_1$ and $y_1 = 2$ in (1), we get

$x_1^2 + 4 - 2x_1 - 8 + 1 = 0 \Rightarrow x_1^2 - 2x_1 - 3 = 0$

$\Rightarrow$ $(x_1 - 3)(x_1 + 1) = 0 \Rightarrow x_1 = 3, -1$

Hence, the required points are (3, 2) and (–1, 2).

II. Given function is $f(x) = \tan^{-1}(\sin x + \cos x)$

Differentiating w.r.t. x, we get

$f'(x)=\frac{1}{1+(\sin x+\cos x)^2}\frac{d}{dx}(\sin x+\cos x)$

$=\frac{\cos x-\sin x}{1+(\sin x+\cos x)^2}$

We know that for all $x \in \left(0, \frac{\pi}{4}\right)$, $\cos x > \sin x$

$\Rightarrow \quad \cos x - \sin x > 0$ for all $x \in \left(0, \frac{\pi}{4}\right)$

$\Rightarrow \quad f'(x) > 0$ for all $x \in \left(0, \frac{\pi}{4}\right)$.

Hence, f is strictly increasing on $\left(0, \frac{\pi}{4}\right)$.

III. $\sin x + \sqrt{3}\cos x = 2\left\{\frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x\right\}, x \in R$

$= 2\left\{\sin x \cos\frac{\pi}{3} + \cos x \sin\frac{\pi}{3}\right\} = 2\left\{\sin\left(x + \frac{\pi}{3}\right)\right\}, x \in R$

Since maximum value of $\sin\left(x + \frac{\pi}{3}\right)$ is 1, therefore, maximum value of $\sin x + \sqrt{3}\cos x$ is 2 and it occurs when $\sin\left(x + \frac{\pi}{3}\right) = 1$,

i.e., when $x + \frac{\pi}{3} = n\pi + (-1)^n \frac{\pi}{2}, n \in I$,

i.e., $x = n\pi - \frac{\pi}{3} + (-1)^n \frac{\pi}{2}, n \in I$.

If we take $n = 1$, we find that $x = \frac{\pi}{6}$.

$\therefore$ Given function assumes maximum value at $\frac{\pi}{6}$.

14. (d): (P) We have, $\frac{dy}{dx} = 2^{y-x} \Rightarrow \frac{dy}{2^y} = \frac{dx}{2^x}$

Integrating both sides, we get $\frac{-2^{-y}}{\log 2} = \frac{-2^{-x}}{\log 2} + C$

$\Rightarrow \quad -2^{-y} + 2^{-x} = C\log 2 = k(\text{say}) \quad \Rightarrow \quad 2^{-x} - 2^{-y} = k$

(Q) We have, $\frac{dy}{dx} = 1 + x + y^2 + xy^2$

$\Rightarrow \frac{dy}{dx} = (1 + y^2)(1 + x) \Rightarrow \frac{dy}{1 + y^2} = (1 + x)dx$

Integrating both sides, we get $\tan^{-1} y = x + \frac{x^2}{2} + C$ ...(i)

When, $y = 0, x = 0 \Rightarrow \tan^{-1}(0) = 0 + 0 + C \quad \therefore \quad C = 0$

Now, from (i), $\tan^{-1} y = x + \frac{x^2}{2} \Rightarrow y = \tan\left(x + \frac{x^2}{2}\right)$

(R) We have, $x^2 \frac{dy}{dx} = x^2 + xy + y^2$

$\Rightarrow \frac{dy}{dx} = 1 + \frac{y}{x} + \left(\frac{y^2}{x^2}\right)$ ...(i)

Put $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

From (i), we get

$v + x\frac{dv}{dx} = 1 + v + v^2 \Rightarrow x\frac{dv}{dx} = 1 + v^2 \Rightarrow \frac{dv}{1 + v^2} = \frac{dx}{x}$

Integrating both sides, we get

$\tan^{-1} v = \log|x| + C \Rightarrow \tan^{-1}\left(\frac{y}{x}\right) = \log|x| + C$

(S) We have, $2(y + 3) - xy\frac{dy}{dx} = 0$

$\Rightarrow 2(y + 3) = xy\frac{dy}{dx} \quad \Rightarrow 2\frac{dx}{x} = \left(\frac{y}{y + 3}\right)dy$

$\Rightarrow 2 \cdot \frac{dx}{x} = \left(1 - \frac{3}{y + 3}\right)dy$

Integrating both sides, we get

$2\log x = y - 3\log(y + 3) + C$ ...(i)

When $x = 1, y = -2$

From (i), $2\log 1 = -2 - 3\log(-2 + 3) + C$

$\Rightarrow \quad 2 \cdot 0 = -2 - 3 \cdot 0 + C \quad \Rightarrow \quad C = 2$

On substituting the value of C in (i), we get

$2\log x = y - 3\log(y + 3) + 2$

$\Rightarrow \quad \log x^2 + \log(y + 3)^3 = (y + 2)$

$\Rightarrow \quad x^2(y + 3)^3 = e^{y + 2}$

15. (b) **16. (c)**

17. (d): We have, $\cot(\sin^{-1} x) = \cos(\tan^{-1}\sqrt{3})$

$\Rightarrow \cot(\sin^{-1} x) = \cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$

$\Rightarrow \sin^{-1} x = \cot^{-1}\left(\frac{1}{2}\right)$

$\Rightarrow x = \sin\left[\cot^{-1}\left(\frac{1}{2}\right)\right] = \sin\left[\sin^{-1}\left(\frac{2}{\sqrt{5}}\right)\right]$

$\therefore \; x = \frac{2}{\sqrt{5}} \quad \left[\because \cot^{-1}\left(\frac{1}{2}\right) = \theta \Rightarrow \cot\theta = \frac{1}{2} \therefore \sin\theta = \frac{2}{\sqrt{5}}\right]$

18. (d)

19. (a): $f(x) = \begin{cases} \frac{x^3 + x^2 - 16x + 20}{(x - 2)^2} & \text{if } x \neq 2 \\ b & \text{if } x = 2 \end{cases}$

$\therefore \quad \lim_{x \to 2} f(x) = \lim_{x \to 2} \frac{x^3 + x^2 - 16x + 20}{(x - 2)^2}$

$= \lim_{x \to 2} \frac{(x - 2)(x^2 + 3x - 10)}{(x - 2)^2} = \lim_{x \to 2} \frac{(x - 2)(x + 5)(x - 2)}{(x - 2)^2}$

$= \lim_{x \to 2} (x + 5) = 2 + 5 = 7$

$\because \quad f(x)$ is continuous for all x.

$\therefore \quad f(2) = \lim_{x \to 2} f(x) \quad \Rightarrow \quad b = 7$

20. (a) : (P) Here, $f(x) = \dfrac{x}{(1+x^2)} \Rightarrow f'(x) = \dfrac{(1-x^2)}{(1+x^2)^2}$

Given, $f(x)$ is decreasing function.

$\Rightarrow f'(x) < 0 \Rightarrow \dfrac{(1-x^2)}{(1+x^2)^2} < 0$

$\Rightarrow 1 - x^2 < 0 \Rightarrow x^2 - 1 > 0$

$\therefore x \in (-\infty, -1) \cup (1, \infty)$

(Q) Here, $f(x) = \tan^{-1} x - x$

$\Rightarrow f'(x) = \dfrac{1}{1+x^2} - 1 = -\dfrac{x^2}{1+x^2} < 0$

($\because f(x)$ is decreasing function)

$\therefore f(x)$ decreases for all $x \in (-\infty, \infty)$.

(R) Here, $f(x) = x - e^x + \tan\left(\dfrac{2\pi}{7}\right) \Rightarrow f'(x) = 1 - e^x > 0$

($\because f(x)$ is increasing function)

$\Rightarrow e^x < 1 \Rightarrow e^x < e^0 \Rightarrow x < 0$

$\Rightarrow x \in (-\infty, 0)$

21. (d) **22. (c)**

23. (b): The given function $f : R \to R$ is defined by $f(x) = (x - 1)(x - 2)(x - 3)$

Since, $1, 2, 3 \in R$ and $f(1) = 0, f(2) = 0, f(3) = 0$

Now, $f(1) = f(2)$ but $1 \neq 2$

$\therefore f$ is not a one-one mapping.

Again if $y = (x - 1)(x - 2)(x - 3)$, then

$x^3 - 6x^2 + 11x - 6 - y = 0$

and any third degree equation must have at least one real root. Hence, it is an onto mapping.

24. (b)

25. (b): $|\vec{a}-\vec{b}|^2 + |\vec{b}-\vec{c}|^2 + |\vec{c}-\vec{a}|^2$

$= 2\left[|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2\right] - 2\Sigma(\vec{a}\cdot\vec{b}) = 2(1 + 1 + 1) - 2\Sigma(\vec{a}\cdot\vec{b})$

$= 6 - 2\Sigma(\vec{a}\cdot\vec{b})$ ...(i)

But $(\vec{a}+\vec{b}+\vec{c})^2 \geq 0 \Rightarrow (1 + 1 + 1) + 2\Sigma(\vec{a}\cdot\vec{b}) \geq 0$

$\therefore 3 \geq -2\Sigma(\vec{a}\cdot\vec{b})$ ...(ii)

From (i) and (ii), we get

$|\vec{a}-\vec{b}|^2 + |\vec{b}-\vec{c}|^2 + |\vec{c}-\vec{a}|^2 \leq 6 + 3 = 9$

26. (c) : In Statement-I, the given cartesian equation is $\dfrac{x-5}{3} = \dfrac{y+4}{7} = \dfrac{z-6}{2}$.

$\Rightarrow \vec{a} = 5\hat{i} - 4\hat{j} + 6\hat{k}$ and $\vec{b} = 3\hat{i} + 7\hat{j} + 2\hat{k}$.

The vector equation of the line is given by

$\vec{r} = \vec{a} + \lambda\vec{b}, \lambda \in R.$

$\Rightarrow \vec{r} = 5\hat{i} - 4\hat{j} + 6\hat{k} + \lambda(3\hat{i} + 7\hat{j} + 2\hat{k})$

Thus Statement-I is true.

In Statement-II it is given that the line passes through the point (–2, 4, –5) and is parallel to

$\dfrac{x+3}{3} = \dfrac{y-4}{5} = \dfrac{z+8}{6}$

Clearly, the direction ratios of line are (3, 5, 6).

Now, the equation of the line (in cartesian form) is

$\dfrac{x-(-2)}{3} = \dfrac{y-4}{5} = \dfrac{z-(-5)}{6} \Rightarrow \dfrac{x+2}{3} = \dfrac{y-4}{5} = \dfrac{z+5}{6}$

Hence, Statement-II is false.

27. (b): When $x \geq 6$ and $y \geq 2$, then

$2x + y \geq 2 \times 6 + 2$, *i.e.*, $2x + y \geq 14$

Hence, $x \geq 0$, $y \geq 0$ and $2x + y \geq 10$ are automatically satisfied by every point of the region

$\{(x, y) : x \geq 6\} \cap \{(x, y) : y \geq 2\}$

28. (b): We know that, $\Sigma p_i = 1$

$\therefore a + 4a + 3a + 7a + 8a + 10a + 6a + 9a = 1$

$\Rightarrow 48a = 1 \Rightarrow a = \dfrac{1}{48}$

Now, $P(X < 3) = P(0) + P(1) + P(2)$

$= a + 4a + 3a = 8a = 8\cdot\dfrac{1}{48} = \dfrac{1}{6}$

$P(X \geq 4) = P(4) + P(5) + P(6) + P(7)$

$= 8a + 10a + 6a + 9a = 33a = \dfrac{33}{48}$

and $P(0 < X < 5) = P(1) + P(2) + P(3) + P(4)$

$= 4a + 3a + 7a + 8a = 22a = \dfrac{22}{48} = \dfrac{11}{24}$

29. (b): $f[g(x)] = f\left[\dfrac{3x+x^3}{1+3x^2}\right] = \log\left\{\dfrac{1+\dfrac{3x+x^3}{1+3x^2}}{1-\dfrac{3x+x^3}{1+3x^2}}\right\}$

$\Rightarrow f[g(x)] = \log\left(\dfrac{1+x}{1-x}\right)^3 = 3\log\left(\dfrac{1+x}{1-x}\right) = 3f(x)$

30. (a)

31. (c) : Since the given vectors are coplanar.

$\therefore \begin{vmatrix} 1 & -1 & 1 \\ 3 & 1 & 2 \\ 1 & \lambda & -3 \end{vmatrix} = 0$

$\Rightarrow 1(-3 - 2\lambda) + 1(-9 - 2) + 1(3\lambda - 1) = 0$

$\Rightarrow -3 - 2\lambda - 11 + 3\lambda - 1 = 0 \Rightarrow \lambda - 15 = 0 \Rightarrow \lambda = 15$

32. (b): (P) The equation of the plane passing through the points (2, 1, 0), (3, –2, –2) and (3, 1, 7) is given by

$$\begin{vmatrix} x-2 & y-1 & z-0 \\ 3-2 & -2-1 & -2-0 \\ 3-2 & 1-1 & 7-0 \end{vmatrix}=0 \Rightarrow \begin{vmatrix} x-2 & y-1 & z \\ 1 & -3 & -2 \\ 1 & 0 & 7 \end{vmatrix}=0$$

$\Rightarrow$ $(x-2)(-21+0)-(y-1)(7+2)+z(3)=0$

$\Rightarrow$ $-21x-9y+3z=-51 \Rightarrow 7x+3y-z=17$

(Q) Given, $A \equiv (2, 3, 4)$ and $B \equiv (4, 5, 8)$

$\therefore$ Mid-point of AB is $\left(\frac{2+4}{2}, \frac{3+5}{2}, \frac{4+8}{2}\right)$ *i.e.*, (3, 4, 6)

Also, $\vec{n}=(4-2)\hat{i}+(5-3)\hat{j}+(8-4)\hat{k}=2\hat{i}+2\hat{j}+4\hat{k}$

Hence, the required equation of the plane is

$(\vec{r}-\vec{a})\cdot\vec{n}=0$, where $\vec{a}=3\hat{i}+4\hat{j}+6\hat{k}$

$\Rightarrow$ $[(x-3)\hat{i}+(y-4)\hat{j}+(z-6)\hat{k}]\cdot(2\hat{i}+2\hat{j}+4\hat{k})=0$

$\Rightarrow$ $2x-6+2y-8+4z-24=0$

$\Rightarrow$ $2x+2y+4z=38 \Rightarrow x+y+2z=19$

(R) The equation of the plane passing through $(2, 1, -1)$ is

$a(x-2)+b(y-1)+c(z+1)=0$ ...(i)

Since, it passes through $(-1, 3, 4)$

$\therefore$ $a(-1-2)+b(3-1)+c(4+1)=0$

$\Rightarrow$ $-3a+2b+5c=0$ ...(ii)

Since, the plane (i) is perpendicular to the plane $x-2y+4z=10$

$\therefore$ $a-2b+4c=0$ ...(iii)

On solving (ii) and (iii), we get

$\frac{a}{18}=\frac{b}{17}=\frac{c}{4}=\lambda \Rightarrow a=18\lambda, b=17\lambda, c=4\lambda$

Hence, required equation of plane is

$18\lambda(x-2)+17\lambda(y-1)+4\lambda(z+1)=0$

$\Rightarrow$ $18x-36+17y-17+4z+4=0$

$\Rightarrow$ $18x+17y+4z=49$

(S) Since, the line drawn from the point $(-2, -1, -3)$ meets a plane at right angle at the point $(1, -3, 3)$. So, the plane passes through the point $(1, -3, 3)$ and normal to plane $\vec{n}$ is

$(-2-1)\hat{i}+(-1+3)\hat{j}+(-3-3)\hat{k}$

$\Rightarrow$ $\vec{n}=-3\hat{i}+2\hat{j}-6\hat{k}$ and $\vec{a}=\hat{i}-3\hat{j}+3\hat{k}$

Now, the required equation of plane is

$(\vec{r}-\vec{a})\cdot\vec{n}=0$

$\Rightarrow$ $[(x\hat{i}+y\hat{j}+z\hat{k})-(\hat{i}-3\hat{j}+3\hat{k})]\cdot(-3\hat{i}+2\hat{j}-6\hat{k})=0$

$\Rightarrow$ $[(x-1)\hat{i}+(y+3)\hat{j}+(z-3)\hat{k}]\cdot(-3\hat{i}+2\hat{j}-6\hat{k})=0$

$\Rightarrow$ $-3x+3+2y+6-6z+18=0$

$\Rightarrow$ $-3x+2y-6z=-27 \therefore 3x-2y+6z-27=0$

33. (a) : Since $Y=\{y \in N : y=4x+3 \text{ for some } x \in N\}$

$\therefore$ $Y=\{7, 11, ..., \infty\}$

Let $y=f(x)=4x+3 \Rightarrow x=\frac{y-3}{4}$

Inverse of $f(x)$ is $g(y)=\frac{y-3}{4}$

Hence, Statement-I is true.

Statement-II is also true.

34. (a) : Given $(e^y+1)\cos x\,dx+e^y\sin x\,dy=0$

$\Rightarrow$ $\frac{e^y}{1+e^y}dy=-\frac{\cos x}{\sin x}dx \Rightarrow \left(1-\frac{1}{1+e^y}\right)dy=-\cot x\,dx$

$\Rightarrow$ $\left(1+\frac{(-e^{-y})}{1+e^{-y}}\right)dy=-\cot x\,dx$

On integrating, we get $y+\log\left(\frac{1+e^y}{e^y}\right)=\log\left(\frac{c}{\sin x}\right)$

$\Rightarrow$ $y=\log\left(\frac{c}{\sin x}\right)+\log\left(\frac{e^y}{1+e^y}\right)$

$\Rightarrow$ $y=\log\left(\frac{ce^y}{\sin x(1+e^y)}\right)$

$\Rightarrow$ $e^y=\frac{ce^y}{\sin x(1+e^y)} \Rightarrow c=\sin x(1+e^y)$

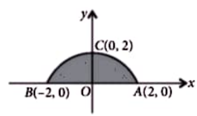
35. (b): We have, $y=\sqrt{4-x^2}$

On squaring both sides, we get

$y^2=4-x^2 \Rightarrow x^2+y^2=4$

Let us sketch the figure of the curve $x^2+y^2=4$ which represents a circle.

But $(y \geq 0)$.



$\therefore$ Required area = area of shaded region

$$=\int_{-2}^{2}\sqrt{4-x^2}\,dx=\left[\frac{x}{2}\sqrt{4-x^2}+2\sin^{-1}\left(\frac{x}{2}\right)\right]_{-2}^{2}$$

$$=\left[\left\{\frac{2}{2}\sqrt{4-4}+2\sin^{-1}\left(\frac{2}{2}\right)\right\}-\left\{\frac{-2}{2}\sqrt{4-(-2)^2}+2\sin^{-1}\left(\frac{-2}{2}\right)\right\}\right]$$

$$=\left[1\times0+2\times\frac{\pi}{2}+1\times0+2\times\frac{\pi}{2}\right]=2\pi \text{ sq. units}$$

36. (a) : $\int(3x+4)^3dx=\frac{(3x+4)^4}{4\cdot3}+c=\frac{(3x+4)^4}{12}+c$

37. (b): Let $I=\int\frac{(x+1)^2}{x(x^2+1)}dx=\int\frac{x^2+1+2x}{x(x^2+1)}dx$

$=\int\left(\frac{1}{x}+\frac{2}{x^2+1}\right)dx=\log|x|+2\tan^{-1}x+c$

38. (d) : $\int \sin^2 x\,dx = \int \frac{1-\cos 2x}{2}\,dx$

$= \frac{1}{2}\left(x - \frac{\sin 2x}{2}\right) + c = \frac{x}{2} - \frac{\sin 2x}{4} + c$

39. (c) : $\int \tan^2 x\,dx = \int (\sec^2 x - 1)\,dx = \tan x - x + c$

40. (a) : Let $I = \int \frac{dx}{\sin^2 x \cos^2 x} = \int \frac{4}{4\sin^2 x \cos^2 x}\,dx$

$= 4\int \text{cosec}^2 2x\,dx = -2\cot 2x + c$

41. (a) : Since, D.R.'s are proportional to D.C.'s, therefore L_1 will be perpendicular to L_2 iff $l_1 l_2 + m_1 m_2 + n_1 n_2 = 0$

42. (c) : Since, D.R.'s are proportional to D.C.'s, therefore L_1 will be parallel to L_2, iff $\frac{l_1}{l_2} = \frac{m_1}{m_2} = \frac{n_1}{n_2}$

43. (c) : Equation of line joining B and C is

$\frac{x-1}{4} = \frac{y-4}{0} = \frac{z-6}{-2}$

Let coordinates of foot of perpendicular be $D(x, y, z)$.

$\therefore$ D.R.'s of AD are $< x-1, y-2, z-1 >$.

Now, $4(x-1) + 0(y-2) - 2(z-1) = 0 \Rightarrow 4x - 2z = 2$

Also, (x, y, z) will satisfy equation of line BC.

Here, (3, 4, 5) satisfy both the conditions.

$\therefore$ Required coordinates are (3, 4, 5).

44. (b): Let a, b, c be the direction ratios of the required line. Since it is perpendicular to the lines whose direction ratios are (1, –2, –2) and (0, 2, 1) respectively.

$\therefore \quad a - 2b - 2c = 0$ ...(i)

$0 \cdot a + 2b + c = 0$ ...(ii)

On solving (i) and (ii) by cross-multiplication, we get

$\frac{a}{-2+4} = \frac{b}{0-1} = \frac{c}{2} \Rightarrow \frac{a}{2} = \frac{b}{-1} = \frac{c}{2}$

Thus, the direction ratios of the required line are $< 2, -1, 2 >$.

45. (b) : D.R.'s of given lines are $< 3, -2, 0 >$ and $< 1, \frac{3}{2}, 2 >$

Now, as $3\cdot 1 + (-2)\cdot\left(\frac{3}{2}\right) + 0\cdot 2 = 3 - 3 + 0 = 0$

$\therefore$ Given lines are perpendicular to each other.

SECTION B2 (APPLIED MATHEMATICS)

16. (a) : Quantitity of milk in mixture A = 7/12 part, quantity of milk in mixture B = 17/24 part and quantity of milk in new mixture = 5/8 part

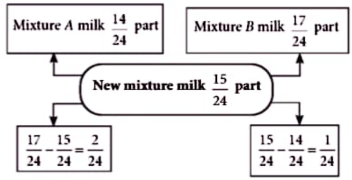
$\therefore$ L.C.M. of 12, 24 and 8 = 24

$\therefore$ Quantity of milk in mixture $A = \frac{7\times 2}{12\times 2} = \frac{14}{24}$ part

Quantity of milk in mixture $B = \frac{17}{24}$ part

Quantity of milk in new mixture $= \frac{5\times 3}{8\times 3} = \frac{15}{24}$ part

So,



$\therefore \frac{\text{Quantity of mixture } A}{\text{Quantity of mixture } B} = \frac{2/24}{1/24} = \frac{2}{1}$

Hence, the required ratio is 2 : 1.

17. (a) : Given $f(x) = 2x^3 - 9x^2 + 12x - 5$ ...(i)

Differentiating (i) w.r.t. x, we get

$f'(x) = 2\cdot 3x^2 - 9\cdot 2x + 12 = 6(x^2 - 3x + 2)$

Now, $f'(x) = 0 \Rightarrow 6(x^2 - 3x + 2) = 0 \Rightarrow (x-1)(x-2) = 0$

$\Rightarrow \quad x = 1, 2$

Also 1, 2 both are in [0, 3], therefore, 1 and 2 both are turning points.

Further, $f(1) = 2\cdot 1^3 - 9\cdot 1^2 + 12\cdot 1 - 5 = 2 - 9 + 12 - 5 = 0$, $f(2) = 2\cdot 2^3 - 9\cdot 2^2 + 12\cdot 2 - 5 = 16 - 36 + 24 - 5 = -1$, $f(0) = -5$ and $f(3) = 2\cdot 3^3 - 9\cdot 3^2 + 12\cdot 3 - 5 = 54 - 81 + 36 - 5 = 4$

Therefore, the absolute maximum value = 4

18. (a) : Since, $\begin{bmatrix} x+y & 2x+z \\ x-y & 2z+w \end{bmatrix} = \begin{bmatrix} 4 & 7 \\ 0 & 10 \end{bmatrix}$

$\Rightarrow \quad x + y = 4$ ...(i), $x - y = 0$ ...(ii), $2x + z = 7$ ...(iii)

and $2z + w = 10$ ...(iv)

On solving these equations, we get

$x = 2, y = 2, z = 3$ and $w = 4$

19. (b): Given, P = ₹1000000, $i = \frac{9}{12\times 100} = 0.0075$ and $n = 12 \times 10 = 120$

So, EMI $= \frac{P\times i\times(1+i)^n}{(1+i)^n - 1}$

$= \frac{1000000\times 0.0075\times(1.0075)^{120}}{(1.0075)^{120} - 1}$

$= \frac{7500\times 2.4514}{2.4514-1} = \frac{7500\times 2.4514}{1.4514} = 12667.42$

Hence, EMI = ₹ 12667.42

20. (d): Given that, $P(X = 0) = p$, $P(X = 1) = p$

Let $P(X = 2) = x \quad \because \Sigma p_i = 1 \Rightarrow p + p + x = 1$

$\Rightarrow \quad x = 1 - 2p$ ... (i)

Also, $E(X) = \Sigma x_i p_i = 0 \times p + 1 \times p + 2 \times x$

$= p + 2x = p + 2(1 - 2p) = p + 2 - 4p = 2 - 3p$

Now, $E(X^2) = \Sigma x_i^2 p_i = 0 + 1 \times p + 4 \times x$

$= p + 4x = p + 4(1 - 2p) = 4 - 7p$

$\because \quad E(X^2) = E(X)$ [Given]

$\Rightarrow \quad 4 - 7p = 2 - 3p \Rightarrow 2 = 4p \Rightarrow p = 1/2$

21. (a) : To find the last digit of 12^{12}, we find 12^{12} (mod 10).

Since $12 \equiv 2 \pmod{10} \Rightarrow 12^4 \equiv 2^4 \pmod{10}$

$\Rightarrow \quad 12^4 \equiv 16 \pmod{10} \Rightarrow 12^4 \equiv 6 \pmod{10}$

$\Rightarrow \quad (12^4)^3 \equiv 6^3$ (mod 10)2

$\Rightarrow \quad 12^{12} \equiv 216 \pmod{10} \Rightarrow 12^{12} \equiv 6 \pmod{10}$.

Hence, the last digit of 12^{12} is 6.

22. (a) : We have $\frac{3x-4}{2} \geq \frac{x+1}{4} - 1$

or $\frac{3x-4}{2} \geq \frac{x-3}{4}$ or $2(3x - 4) \geq (x - 3)$

or $6x - 8 \geq x - 3$ or $5x \geq 5$ or $x \geq 1$

Thus all real numbers which are greater than or equal to 1 is the solution set of the given inequality.

$\therefore \quad x \in [1, \infty)$

23. (d): Average cost (AC) = $\frac{\text{Total cost } (C(x))}{x}$

$= \frac{\frac{x^3}{4} + 9x^2 - 7}{x} = \frac{x^2}{4} + 9x - \frac{7}{x}$

24. (c) : Fisher's ideal index number satisfied both time reversal and factor reversal test since, why Fisher's price index is known as the ideal index number and it consider both the current and bare year quantities.

Time reversal test satisfied $P_{01} \times P_{10} = 1$

So, statement I is true and statement II is false.

25. (c) : Let the annual coupon rate be r %.

Given, F = ₹ 1000, then C = ₹ $1000 \times \frac{r}{200}$ = ₹ $5r$

d = 8% p.a. or 4% per half year $\Rightarrow i = \frac{4}{400} = 0.04$

N = 5 years = 10 half years and P.V. = ₹ 800

Using formula, $\frac{C[1-(1+i)^{-N}]}{i} + F(1+i)^{-N}$

$\therefore \quad 800 = \frac{5r[1-(1+0.04)^{-10}]}{0.04} \times 1000(1+0.04)^{-10}$

$\Rightarrow \quad 800 = \frac{5r[1-0.6761]}{0.04} + 1000 \times 0.6761$

$\Rightarrow \quad \frac{5r \times 0.3239}{0.04} = 800 - 676.10 = 123.90$

$\Rightarrow \quad r = \frac{123.90 \times 0.04}{0.3239 \times 5} \Rightarrow r = 3.06$

26. (b): Given, x = 4 km/h, y = 2 km/h

and t = 10 min = $\frac{10}{60}$h

We know that : $d = \frac{t(x^2 - y^2)}{2y}$

$\Rightarrow \quad d = \frac{10}{60}\frac{(4^2 - 2^2)}{2 \times 2} = \frac{16-4}{6 \times 4} = \frac{12}{24} = \frac{1}{2} = 0.5$

Hence, the distance between two points is 0.5 km.

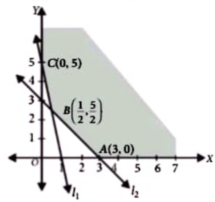
27. (d): We have, minimize $Z = 7x + y$

Subject to $5x + y \geq 5$, $x + y \geq 3$, $x, y \geq 0$

Let $l_1 : 5x + y = 5$, $l_2 : x + y = 3$, $l_3 : x = 0$ and $l_4 : y = 0$

For B : Solving l_1 and l_2,

we get $B\left(\frac{1}{2}, \frac{5}{2}\right)$



Shaded portion is the feasible region, where $A(3, 0)$, $B\left(\frac{1}{2}, \frac{5}{2}\right)$, $C(0, 5)$.

Now, minimize $Z = 7x + y$

Z at $A(3, 0)$

$= 7(3) + 0 = 21$

Z at $B\left(\frac{1}{2}, \frac{5}{2}\right) = 7\left(\frac{1}{2}\right) + \frac{5}{2} = 6$

Z at $C(0, 5) = 7(0) + 5 = 5$

As the feasible region is unbounded, so we draw the half plane $7x + y < 5$ and observe that there is no common point with the feasible region.

Thus, Z is minimized at $C(0, 5)$ and its minimum value is 5.

28. (d): Given, $f(x) = 1 - x^3 - x^5$

Differentiating w.r.t. x, we get $f'(x) = -3x^2 - 5x^4$

$\Rightarrow \quad f'(x) = -(3x^2 + 5x^4)$

$\Rightarrow \quad f'(x) < 0$ for all values of x.

29. (a) : Dividend on 1 share of ₹ 50 = 12% of ₹ 50 = ₹ 6

Let the man buy one share for ₹ x.

His profit on one share 15% of ₹ $x = \frac{15}{100}x$

Since the dividend paid by the company on share = ₹ 6

$\therefore \quad \frac{15}{100}x = 6 \Rightarrow x = 40.$

$\therefore \quad$ The man buys each share at ₹ 40.

30. (a) : Here, $\frac{d}{dx}(\cos^{-1}(1-x))$

$= \frac{-1}{\sqrt{1-(1-x)^2}} \times (-1) = \frac{1}{\sqrt{2x - x^2}}$

$\Rightarrow \quad \frac{d^2}{dx^2}(\cos^{-1}(1-x)) = \frac{d}{dx}(2x - x^2)^{-1/2}$

$= -\frac{1}{2}\frac{(2-2x)}{(2x-x^2)^{3/2}} = \frac{x-1}{(2x-x^2)^{3/2}}$

31. (d): First four years moving average

$= \frac{55+62+54+69}{4} = 60$

First 3 years moving average $= \frac{55+62+54}{3} = 57$

$\therefore$ Required difference = 60 – 57 = 3

32. (a) : Let the random variable (number of successes) be X. Clearly, on tossing a die twice, the number of successes can be 0, 1 or 2.

$\therefore$ X can take 0, 1 and 2.

$P(X = 0) = P$ (no success in both throws)

$= P$ (getting a number ≤ 4) $\cdot$ P (getting a number ≤ 4)

$= \frac{4}{6} \cdot \frac{4}{6} = \frac{4}{9}$

$P(X = 1) = P$(number of success = 1)

$= P$ (getting a number ≤ 4) $\cdot$ P(getting a number > 4) + P(getting a number > 4) $\cdot$ P(getting a number ≤ 4)

$= \frac{4}{6} \cdot \frac{2}{6} + \frac{2}{6} \cdot \frac{4}{6} = \frac{4}{9}$

$P(X = 2) = P$(number of success = 2)

$= P$(getting a number > 4). P(getting a number > 4)

$= \frac{2}{6} \cdot \frac{2}{6} = \frac{1}{9}$

Thus, the required probability distribution of X is

X	0	1	2
$P(X)$	$\frac{4}{9}$	$\frac{4}{9}$	$\frac{1}{9}$

33. (a) : $(AB)' = B'A'$ is true. This result is a standard result called "reversal law of transposes."

34. (a) **35. (c)**

36. (a) : Part of the tank filled by first pipe in 1 hour $= \frac{1}{3}$

Part of the tank emptied by second pipe in 1 hour $= \frac{1}{4}$

Part of the tank filled by both the pipes in 1 hour

$= \frac{1}{3} - \frac{1}{4} = \frac{1}{12}$

$\therefore$ The time taken to fill the tank = 12 hours.

37. (b): Marginal cost (MC)

$= \frac{d}{dx}(C(x)) = \frac{d}{dx}\left(\frac{x^3}{3} + 3x^2 - 7x + 16\right)$

$= \frac{1}{3} \cdot 3x^2 + 3 \cdot 2x - 7 \cdot 1 = x^2 + 6x - 7$

Average cost (AC) $= \frac{C(x)}{x} = \frac{1}{3}x^2 + 3x - 7 + \frac{16}{x}$

38. (c) : Since the man earns ₹ 30 per quarter, his annual income from 20 debentures = ₹ 120.

Now, assume that par value of debenture is ₹ x.

$\therefore$ Income from 20 debentures giving 8% annual return on investment

$= ₹\, 20 \times 8 \times \frac{x}{100} = ₹\frac{8x}{5}$

By given condition, $\frac{8x}{5} = 120 \Rightarrow x = \frac{5 \times 120}{8} = 75$

Hence, the par value of a debenture is ₹ 75.

39. (c) : (P) We have, $A = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 0+1 & 0+0 \\ 0+0 & 1+0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

(Q) $P(x)\, P(y) = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}\begin{bmatrix} \cos y & \sin y \\ -\sin y & \cos y \end{bmatrix}$

$$= \begin{bmatrix} \cos x \cos y - \sin x \sin y & \cos x \sin y + \sin x \cos y \\ -\sin x \cos y - \sin y \cos x & -\sin x \sin y + \cos x \cos y \end{bmatrix}$$

$$= \begin{bmatrix} \cos(x+y) & \sin(x+y) \\ -\sin(x+y) & \cos(x+y) \end{bmatrix}$$

(R) For skew-symmetric matrix $A = -A'$ and $a_{ii} = 0$

$$\therefore \begin{bmatrix} d & a & 3 \\ 2 & b & -1 \\ c & 1 & 0 \end{bmatrix} = -\begin{bmatrix} d & 2 & c \\ a & b & 1 \\ 3 & -1 & 0 \end{bmatrix}$$

On comparing, we get

$a = -2, c = -3$

Also, $d = b = 0$ ($\because$ Diagonal elements are zero)

40. (b)

41. (d): Statement I is false, as the samples taken from the same population will differ, but will share the same characteristics of the population.

42. (d)

43. (a): To find the last two digits of the product 2345 × 6789, we find 2345 × 6789 (mod 100).

Since 2345 ≡ 45 (mod 100) and 6789 ≡ 89 (mod 100).

So, 2345 × 6789 ≡ 45 × 89 (mol 100)

≡ 4005 (mod 100) ≡ 05 (mod 100).

Hence, the last two digits of product 2345 × 6789 is 05.

44. (d): Given, $y^2 = x + \sin x$ ...(i)

$\Rightarrow 2y\frac{dy}{dx} = 1 + \cos x$

Since, tangent is parallel to x-axis

$\therefore \frac{dy}{dx} = 0 \Rightarrow \cos x = -1 \Rightarrow \sin x = 0$

$\therefore$ From (i), $y^2 = x$

45. (a)

UCD
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- IIT Roorkee - Indian Institute of Technology Roorkee
- IILM University Greater Noida
- UPES Dehradun
- VIT Bhopal University

Quiz CLUB

Unlock Your Knowledge!

1. If 'ω' is a complex cube root of unity, then find $\omega^{\left(\frac{1}{3}+\frac{2}{9}+\frac{4}{27}+\ldots\infty\right)}+\omega^{\left(\frac{1}{2}+\frac{3}{8}+\frac{9}{32}+\ldots\infty\right)}$.
2. In how many ways 6 letters be posted in 5 different letter boxes?
3. Find the sixth term in the sequence is $3, 1, \frac{1}{3}, \ldots$.
4. Find the distance between the lines $5x - 12y + 65 = 0$ and $5x - 12y - 39 = 0$.
5. Find the eccentricity of the ellipse $\frac{x^2}{36}+\frac{y^2}{16}=1$.
6. Find the distance between the x-axis and the point (3, 12, 5).
7. Evaluate : $\lim\limits_{x \to 0} \frac{e^{5x} - e^{4x}}{x}$
8. If $f(x + y) = f(x) f(y)$ for all x and y and $f(5) = 2$, $f'(0) = 3$, then find the value of $f'(5)$.
9. Find the variance of the numbers 2, 4, 6, 8.
10. Find the probability that a non leap year selected at random will have 53 Sundays.
11. If $n(A) = 8$, then find the total number of bijections defined on A.
12. If A is a square matrix of order 3×3, then find the value of $|KA|$.
13. If $f(x) = \begin{cases} 3x-8, & \text{if } x \le 5 \\ 2k, & \text{if } x > 5 \end{cases}$ is continuous, then find k.
14. If $f(x) = \sqrt{2x} + \frac{4}{\sqrt{2x}}$, then find $f'(2)$.
15. The volume of a sphere is increasing at the rate of 1200 c.cm/sec. Find the rate of increase in its surface area when the radius is 10 cm.
16. Evaluate : $\int \frac{2^x}{\sqrt{1-4^x}}dx$
17. Find the area of the region bounded by the curve $y = \cos x$, $x = 0$ and $x = \pi$.
18. Find the order of the differential equation whose general solution is given by $y = c_1e^x + c_2e^x + (c_3 + c_4)e^{3x+5} + c_5e^{2x}$, where c_1, c_2, c_3, c_4 and c_5 are arbitrary constants.
19. If the set A contains 5 elements, then find the number of elements in the power set $P(A)$.
20. Find the value of sin 765°.

Readers can send their responses at editor@mtg.in or post us with complete address by 10th of every month. Winners' names and answers will be published in next issue.

SOLUTIONS TO FEBRUARY 2024 QUIZ CLUB

1. $(-5, \infty) - \{-3, -1\}$		2. 10
3. 4536	4. $24\sqrt{2}$	5. 63
6. $y = -3x + 7$	7. $\frac{2}{3}$	8. $\left(1, 4, \frac{1}{3}\right)$
9. $\frac{b+1}{2a}$	10. 1	11. $\sqrt{\frac{n^2-1}{3}}$
12. 1/3	13. $-\frac{7}{18}$	14. $\frac{3}{10}$
15. 231	16. $K^3\|A\|$	17. −1
18. $\left(0, \frac{\pi}{2}\right)$	19. 3	20. 21

Competency Based Questions

for CBSE BOARD EXAM 2023-24

As per the circular issued by CBSE for Assessment and Evaluation practices for the session 2023-24, the Internal/ year end Board Examination questions paper will be having minimum 40% Competency Based Questions. These can be in the form of Multiple Choice Questions, Assertion & Reason based, Integrated or any other types of questions. In this article, we have specially created Some Competency Based Questions for student for maximum practice.

MCQs Multiple Choice Questions

1. Let R be the relation "is congruent to" on the set of all triangles in a plane is

(a) Reflexive only

(b) Symmetric only

(c) Symmetric and reflexive only

(d) Equivalence relation

2. The function $f : R \to R$ defined by $f(x) = 3 - 4x$ is

(a) onto (b) not onto

(c) not one-one (d) none of these

3. $\tan^{-1} 1 + \cos^{-1}\left(\frac{-1}{2}\right) + \sin^{-1}\left(\frac{-1}{2}\right)$ is equal to

(a) $\frac{2\pi}{3}$ (b) $\frac{3\pi}{4}$

(c) $\frac{\pi}{2}$ (d) 6π

4. The range of $\sin^{-1}x + \cos^{-1}x + \tan^{-1}x$ is

(a) $[0, \pi]$ (b) $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$

(c) $[0, \pi)$ (d) $\left[0, \frac{\pi}{2}\right]$

5. If $\begin{bmatrix} a+b & 2 \\ 5 & ab \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$, then find the values of a and b respectively.

(a) 2, 4

(b) 4, 2

(c) Both (a) and (b)

(d) None of these

6. The matrix $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ is a

(a) unit matrix

(b) diagonal matrix

(c) symmetric matrix

(d) skew-symmetric matrix

7. Find x, if $\begin{bmatrix} 1 & 2 & x \\ 1 & 1 & 1 \\ 2 & 1 & -1 \end{bmatrix}$ is singular.

(a) 1 (b) 2

(c) 3 (d) 4

8. If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, then find A^{-1}.

(a) $\frac{1}{5}\begin{bmatrix}-3 & 2 & 2\\ 2 & -3 & 2\\ 2 & 2 & -3\end{bmatrix}$ (b) $\frac{1}{5}\begin{bmatrix}3 & 2 & 2\\ 2 & 3 & 2\\ 2 & 2 & 3\end{bmatrix}$

(c) $\frac{1}{5}\begin{bmatrix}3 & 2 & 2\\ 2 & -3 & 2\\ 2 & 2 & 3\end{bmatrix}$ (d) $\frac{1}{5}\begin{bmatrix}3 & 2 & 2\\ 2 & 3 & 2\\ 2 & 2 & -3\end{bmatrix}$

9. $\lim_{x\to 2}\left(\frac{x^2-3x+2}{x^2+x-6}\right)=$

(a) $\frac{1}{5}$ (b) $\frac{2}{5}$

(c) $\frac{3}{5}$ (d) 1

10. If $y=\sqrt{x^2+\sqrt{x^2+\sqrt{x^2+\ldots\text{to}\infty}}}$, then $\frac{dy}{dx}$ is equal to

(a) $\frac{2x}{2y-1}$ (b) $\frac{-2x}{2y-1}$

(c) $\frac{-x^2}{x(2y-1)}$ (d) none of these

11. Maximum slope of the curve $y=-x^3+3x^2+9x-27$ is

(a) 0 (b) 12

(c) 16 (d) 32

12. The function $(x-\sin x)$ decreases for

(a) all x (b) $x<\frac{\pi}{2}$

(c) $0<x<\frac{\pi}{4}$ (d) no value of x

13. $\int\frac{\sec^2 x}{\tan x+2}dx$ equals

(a) $\log|\tan x-2|+c$ (b) $\log|\tan x+2|+c$

(c) $\log|\cot x+2|+c$ (d) $\log|\cot x-2|+c$

14. The area of the region bounded by the curve $y=x+1$ and the lines $x=2$, $x=3$ is

(a) $\frac{7}{2}$ sq. units (b) $\frac{9}{2}$ sq. units

(c) $\frac{11}{2}$ sq. units (d) $\frac{13}{2}$ sq. units

15. The number of arbitrary constants in the particular solution of a differential equation of second order is

(a) 0 (b) 1

(c) 2 (d) 3

16. The general solution of $e^x\cos y\,dx-e^x\sin y\,dy=0$ is

(a) $e^x\cos y=k$ (b) $e^x\sin y=k$

(c) $e^x=k\cos y$ (d) $e^x=k\sin y$

17. If the direction cosines of a vector of magnitude 3 are $\frac{2}{3},\frac{-a}{3},\frac{2}{3}, a>0$, then the vector is

(a) $2\hat{i}+\hat{j}+2\hat{k}$ (b) $2\hat{i}-\hat{j}+2\hat{k}$

(c) $\hat{i}-2\hat{j}+2\hat{k}$ (d) $\hat{i}+2\hat{j}+2\hat{k}$

18. What is the angle between the two lines whose direction ratios are $(\sqrt{3}-1,-\sqrt{3}-1, 4)$ and $(-\sqrt{3}-1,\sqrt{3}-1, 4)$?

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

19. Optimization of the objective function is a process of

(a) Maximising the objective function

(b) Maximising or minimising the objective function

(c) Minimising the objective function

(d) none of these

20. A card is picked at random from a pack of 52 playing cards. Given that the picked card is a queen, the probability of this card to be a card of spade is

(a) $\frac{1}{3}$ (b) $\frac{4}{13}$

(c) $\frac{1}{4}$ (d) $\frac{1}{2}$

A & R Assertion & Reason Based Questions

In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

1. Assertion (A) : If R is a relation defined on the set of natural numbers N such that $R = \{ (x, y) : x, y \in N \text{ and } 2x + y = 24 \}$, then R is an equivalence relation.
Reason (R) : A relation is said to be an equivalence relation if it is reflexive, symmetric and transitive.

2. Assertion (A) : $f : N \rightarrow Y$ be a function defined as $f(x) = 4x + 3$, where $Y = \{y \in N : y = 4x + 3 \text{ for some } x \in N\}$ is invertible.
Reason (R) : If a function is one-one and onto, then it is a invertible function.

3. Assertion (A) : If $2(\sin^{-1}x)^2 - 5(\sin^{-1}x) + 2 = 0$, then x has 2 solutions.
Reason (R) : $\sin^{-1}(\sin x) = x$, if $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$

4. Let A be a 2 × 2 matrix.
Assertion (A) : adj(adj A) = A.
Reason (R) : $|\text{adj } A| = |A|$.

5. Let A and B be two events associated with an experiment such that $P(A \cap B) = P(A)P(B)$.
Assertion (A) : $P(A|B) = P(A)$ and $P(B|A) = P(B)$.
Reason (R) : $P(A \cup B) = P(A) + P(B)$.

6. Consider the function $f(x) = \begin{cases} x^2, & x \ge 1 \\ x+1, & x < 1 \end{cases}$
Assertion (A) : f is not derivable at $x = 1$ as
$\lim_{x \to 1^-} f(x) \ne \lim_{x \to 1^+} f(x)$.
Reason (R) : If a function f is derivable at a point 'a', then it is continuous at 'a'.

7. Assertion (A) : Both $\sin x$ and $\cos x$ are decreasing functions in $\left(\frac{\pi}{2}, \pi\right)$.
Reason (R) : If $f'(x) < 0$, then $f(x)$ is a decreasing function.

8. Assertion (A) : If $|\vec{a}| = 2, |\vec{b}| = 3, |2\vec{a} - \vec{b}| = 5$, then $|2\vec{a} + \vec{b}| = 5$.
Reason (R) : If vectors $\vec{P} \cdot \vec{Q} = |\vec{P}||\vec{Q}|$, then $\theta = 90°$.

9. Assertion (A) : The value of
$\int_{-3}^{3} (ax^5 + bx^3 + cx + k)dx$,
where a, b, c, k are constants, depends on only k.
Reason (R) : $\int_{-a}^{a} f(x)dx = 0$, if $f(-x) = -f(x)$ i.e., f is an odd function.

10. Assertion (A) : The area of the region bounded by the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ is $\frac{3}{2}(\pi - 2)$ sq. units.
Reason (R) : Area of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is πab sq. units.

11. Assertion (A) : The elimination of four arbitrary constants in $y = (c_1 + c_2 + c_3 e^{c_4})x$ results into a differential equation of the first order $x\frac{dy}{dx} = y$.
Reason (R) : Elimination of n arbitrary constants requires in general, a differential equation of the n^{th} order.

12. Assertion (A) : If the cartesian equation of a line is $\frac{x-5}{3} = \frac{y+4}{7} = \frac{z-6}{2}$, then its vector form is $\vec{r} = 5\hat{i} - 4\hat{j} + 6\hat{k} + \lambda(3\hat{i} + 7\hat{j} + 2\hat{k})$.
Reason (R) : The cartesian equation of the line which passes through the point (–2, 4, –5) and parallel to the line given by $\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}$ is $\frac{x+3}{-2} = \frac{y-4}{4} = \frac{z+8}{-5}$.

13. Assertion (A) : The length of projection of the vector $3\hat{i}-\hat{j}-2\hat{k}$ on the vector $\hat{i}+2\hat{j}-3\hat{k}$ is $\frac{7}{\sqrt{14}}$.

Reason (R) : The projection of a vector $\vec{a}$ on another vector $\vec{b}$ is $\frac{(\vec{a}\cdot\vec{b})}{|\vec{b}|}$.

14. Let E_1 and E_2 be any two events associated with an experiment.

Assertion (A) : $P(E_1) + P(E_2) \leq 1$.

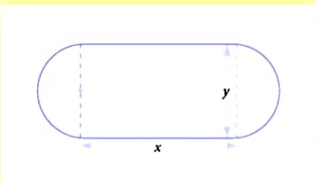
Reason (R) : $P(E_1) + P(E_2) = P(E_1 \cup E_2) + P(E_1 \cap E_2)$.

15. Assertion (A) : Let E and F be events associated with the sample space S of an experiment. Then, we have $P(S|F) = P(F|F) = 1$.

Reason (R) : If A and B are any two events associated with the sample space S and F is an event associated with S such that $P(F) \neq 0$, then $P((A \cup B)|F) = P(A|F) + P(B|F) - P((A \cap B)|F)$.

Case Based Questions

1. A man wants to make a park which consists of a rectangular region with semicircular ends having a perimeter of 100 m as shown below :



Design of Floor

Park

Based the above information, answer the following questions.

(i) If x and y represents the length and breadth of the rectangular region, then find the relation between the variables.

(ii) Find the area of the rectangular region A expressed as a function of x.

(iii) Find the maximum value of area A.

OR

The man is interested in maximizing the area of the whole park including the semicircular ends. For this to happen find the value of x.

2. Student of Grade 12, decided to plant trees along straight lines, parallel to each other to one side of the playground ensuring that they had enough play area. Let us assume that they planted one of the row of the trees along the line $y = 3x - 4$. Let L be the set of all lines which are parallel to the ground and R be a relation on L.

Based on the above information, answer the following questions.

(i) Let relation R be defined by $R = \{(L_1, L_2) : L_1$ is parallel to L_2, where $L_1, L_2 \in L\}$, then check whether R is equivalence or not.

(ii) Is the function $f: R \to R$ defined by $f(x) = 3x - 4$ is bijective?

(iii) Let $f: R \to R$ be defined by $f(x) = 3x - 4$, then find the range of $f(x)$.

OR

Draw the graph of the line $y = 3x - 4$. Also, find the sum of x and y intercepts.

3. A manufacturer produces three products Bolts, Nuts, and Screws which he sells in two markets. Annual sales are indicated below.

Market	Products (in numbers)		
	Nuts	Bolts	Screws
A	10000	2000	18000
B	6000	20000	8000

The unit sale price of Nuts, Bolts and Screws are ₹ 2.50, ₹ 1.50 and ₹ 1.00 respectively and the unit cost of the above three commodities are ₹ 2.00, ₹ 1.00 and ₹ 0.50 respectively.

Based on the above information, answer the following questions.

(i) Find the total revenue of market A.

(ii) Find the profit in market A and B respectively.

4. In answering a question in a multiple choice test for class XII, a student either knows the answer or guesses. Let 1/5 be the probability that he knows the answer and 4/5 be the probability that he guesses. Assume that a student who guesses the answer will be correct with probability 1/3. Let E_1, E_2 and A be the events that the student knows the answer, guesses the answer and answers correctly, respectively.

Based on the above information, answer the following questions.

(i) Find $\sum_{k=1}^{k=2} P(A|E_k)\, P(E_k)$.

(ii) What is the probability that the student knows the answer given that he answered it correctly?

5. $P(x) = -3x^2 + 100x + 27500$ is the total profit function of a company, where x is the production of the company.

Based on the above information, answer the following questions.

(i) What will be the maximum profit?

(ii) Check in which interval the profit is strictly increasing.

6. A veterinary doctor was examining a sick cat brought by a pet lover. When it was brought to the hospital, it was already dead. The pet lover wanted to find its time of death. He took the temperature of the cat at 10.30 p.m. which was 95.6°F. He took the temperature again after one hour; the temperature was lower than the first observation. Which was 91.4°F. The normal temperature of the cat is taken as 98.6°F when it was alive. The doctor estimated the time of death using *Newton law of cooling* which is governed by the differential equation : $\frac{dT}{dt} \propto (T-70)$, where 70°F is the room temperature and T is the temperature of the object at time t.

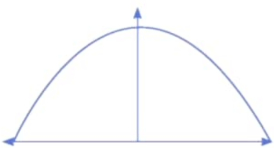
Substituting the two different observations of T and t made, in the solution of the differential equation $\frac{dT}{dt} = k(T-70)$ where, k is a constant of proportion, time of death is calculated.

Based on the above information, answer the following questions.

(i) Find the solution of the differential equation $\frac{dT}{dt} = k(T - 70)$.

(ii) If $t = 0$, when T is 72, then find the value of constant of integration c.

7. The bridge made on the river was 50 feet wide. The arch on the bridge is in a parabolic form. The highest point of the bridge is 5 feet above the road at the middle of the bridge as shown in the figure.



Based on the above information, answer the following questions.

(i) Find the equation of the parabola designed on the bridge.

(ii) Find the value of the integral $\int_{-25}^{25} y\,dx$.

(iii) Find the area formed by the curve $x^2 = 125y$, x-axis, y-axis and $y = 5$.

OR

Find the focus of the parabola. Also, find the equation of directrix.

SOLUTIONS

Multiple Choice Questions

1. (d): Let S denote the set of all triangles in a plane and R be the relation on S defined by $(\Delta_1, \Delta_2) \in R$ $\Rightarrow$ Triangle $\Delta_1 \cong \Delta_2$

(i) Let any triangle $\Delta \in S$, we have $\Delta \cong \Delta \Rightarrow (\Delta, \Delta) \in R\ \forall\ \Delta \in S \Rightarrow R$ is reflexive on S.

(ii) Let $\Delta_1, \Delta_2 \in S$ such that $(\Delta_1, \Delta_2) \in R$, then $\Delta_1 \cong \Delta_2 \Rightarrow \Delta_2 \cong \Delta_1 \Rightarrow (\Delta_2, \Delta_1) \in R \Rightarrow R$ is symmetric

(iii) Again, let $\Delta_1, \Delta_2, \Delta_3 \in S$ such that $(\Delta_1, \Delta_2) \in R$ and $(\Delta_2, \Delta_3) \in R$ $\therefore$ $\Delta_1 \cong \Delta_2 \cong \Delta_3$ $\therefore$ $(\Delta_1, \Delta_3) \in R$

$\Rightarrow$ R is transitive

$\therefore$ R is an equivalence relation.

2. (a): Let y be any real number such that $f(x) = y$

$\therefore$ $y = 3 - 4x$

$$\Rightarrow 4x = 3 - y \Rightarrow x = \frac{3-y}{4}$$

So, for any real number y, there exists $\frac{3-y}{4} \in R$ such that $f\left(\frac{3-y}{4}\right) = 3 - 4\left(\frac{3-y}{4}\right) = 3 - 3 + y = y$

Hence, f is onto.

3. (b): Let $\tan^{-1}(1) = \theta \Rightarrow \tan\theta = 1 = \tan\frac{\pi}{4}$

$$\Rightarrow \theta = \frac{\pi}{4} \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$$

$\therefore$ Principal value of $\tan^{-1}(1)$ is $\frac{\pi}{4}$.

Now, let

$$\cos^{-1}\left(\frac{-1}{2}\right) = \phi \Rightarrow \cos\phi = \frac{-1}{2} = -\cos\frac{\pi}{3}$$

$$= \cos\left(\pi - \frac{\pi}{3}\right) = \cos\frac{2\pi}{3} \Rightarrow \phi = \frac{2\pi}{3} \in [0, \pi]$$

$\therefore$ Principal value of $\cos^{-1}\left(\frac{-1}{2}\right)$ is $\frac{2\pi}{3}$.

Similarly, principal value of $\sin^{-1}\left(\frac{-1}{2}\right)$ is $\left(\frac{-\pi}{6}\right)$.

$\therefore$ The value of

$$\tan^{-1}(1) + \cos^{-1}\left(\frac{-1}{2}\right) + \sin^{-1}\left(\frac{-1}{2}\right)$$

$$= \frac{\pi}{4} + \frac{2\pi}{3} - \frac{\pi}{6} = \frac{3\pi}{4}$$

4. (b): Let $f(x) = \sin^{-1}x + \cos^{-1}x + \tan^{-1}x$

Then, Dom $(f) = [-1, 1] \cap [-1, 1] \cap R = [-1, 1]$

Clearly, $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$

and $\frac{-\pi}{4} \le \tan^{-1}x \le \frac{\pi}{4} \Rightarrow \frac{\pi}{4} \le \frac{\pi}{2} + \tan^{-1}x \le \frac{3\pi}{4}$

Thus, range is $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$.

5. (c): Since, $\begin{bmatrix} a+b & 2 \\ 5 & ab \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$

$\Rightarrow$ $a + b = 6$ and $ab = 8$

$\Rightarrow$ $a + \frac{8}{a} = 6$ $\qquad (\because ab = 8 \Rightarrow b = 8/a)$

$\Rightarrow$ $a^2 - 6a + 8 = 0$

$\Rightarrow$ $(a - 2)(a - 4) = 0 \Rightarrow a = 2, 4$

Hence, $a = 2, b = 4$ or $a = 4, b = 2$

6. (c): $A' = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = A$

Hence, A is a symmetric matrix.

7. (d): For singular matrix, $|A| = 0$

$$\therefore \begin{vmatrix} 1 & 2 & x \\ 1 & 1 & 1 \\ 2 & 1 & -1 \end{vmatrix} = 0$$

$\Rightarrow$ $1(-1 - 1) - 2(-1 - 2) + x(1 - 2) = 0$

$\Rightarrow$ $-2 + 6 - x = 0 \Rightarrow x = 4$

8. (a): $|A| = 1(1 - 4) - 2(2 - 4) + 2(4 - 2)$ (Expanding by R_1)

$= -3 + 4 + 4 = 5 \neq 0$

$\therefore$ A^{-1} exists.

$$\text{So, adj}(A) = \begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix}^T = \begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix}$$

$$\text{Now, } A^{-1} = \frac{1}{|A|}\cdot\text{adj}(A) = \frac{1}{5}\begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix}$$

9. **(a) :** We have, $\lim_{x\to 2}\left(\frac{x^2-3x+2}{x^2+x-6}\right)$

$= \lim_{x\to 2}\frac{(x-2)(x-1)}{(x+3)(x-2)} = \lim_{x\to 2}\left(\frac{x-1}{x+3}\right) = \frac{2-1}{2+3} = \frac{1}{5}$

10. **(a) :** Given $y=\sqrt{x^2+\sqrt{x^2+\sqrt{x^2+\ldots \text{ to } \infty}}}$

$\Rightarrow \quad y=\sqrt{x^2+y} \Rightarrow y^2-y=x^2$ ...(i)

Differentiating (i) w.r.t. x, we get

$2y\frac{dy}{dx}-\frac{dy}{dx}=2x \Rightarrow \frac{dy}{dx}=\frac{2x}{2y-1}$

11. **(b) :** We have given, $y=-x^3+3x^2+9x-27$

$\therefore$ Slope of the curve is $\frac{dy}{dx}=-3x^2+6x+9$

Now, $\frac{d^2y}{dx^2}=-6x+6=-6(x-1)$

For maximum slope, put $\frac{d^2y}{dx^2}=0$

$\Rightarrow -6(x-1)=0 \Rightarrow x=1$

Also, $\frac{d^3y}{dx^3}=-6<0$

Hence, the maximum slope of given curve is at $x=1$.

$\therefore \left(\frac{dy}{dx}\right)_{(x=1)}=-3\cdot 1^2+6\cdot 1+9=12$

12. **(d) :** Let $f(x)=x-\sin x$

Differentiating w.r.t x, we get, $f'(x)=1-\cos x$

For function to be decreasing $\Rightarrow 1-\cos x<0 \Rightarrow \cos x>1$, which is not possible because maximum value of $\cos x$ is 1.

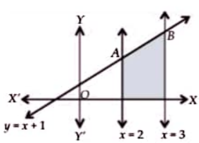
Therefore, no value of x for which $f(x)$ is decreasing.

13. **(b) :** Let $I=\int\frac{\sec^2 x}{\tan x+2}dx$

Put $\tan x+2=t \Rightarrow \sec^2 x\,dx=dt$

$\therefore I=\int\frac{dt}{t}=\log|t|+c = \log|\tan x+2|+c$

14. **(a) :** We have, $y=x+1$ and lines $x=2$, $x=3$. Points of intersection are $A(2, 3)$ and $B(3, 4)$.



$\therefore$ Required area of the shaded region $=\int_2^3 (x+1)dx$

$=\left[\frac{x^2}{2}+x\right]_2^3=\left[\frac{9}{2}+3-\frac{4}{2}-2\right]=\frac{7}{2}$ sq. units

15. **(a) :** In the particular solution of a differential equation of any order, there is no arbitrary constant because in the particular solution of any differential equation, we remove all the arbitrary constant by substituting some particular values.

16. **(a) :** We have, $e^x\cos y dx - e^x \sin y dy=0$

$\Rightarrow \quad e^x \cos y dx = e^x \sin y dy \Rightarrow dx=\tan y\, dy$

Integrating both sides, we get $x=\log\sec y+\log k$

$\Rightarrow \quad x=\log(k\sec y) \Rightarrow e^x=k\sec y \Rightarrow e^x\cos y=k$

17. **(b) :** Required vector $=3(l\hat{i}+m\hat{j}+n\hat{k})$,

where $l^2+m^2+n^2=1$

$\Rightarrow \frac{4}{9}+\frac{a^2}{9}+\frac{4}{9}=1 \Rightarrow \frac{a^2}{9}+\frac{8}{9}=1 \Rightarrow a^2=1 \Rightarrow a=1$

$(\because a>0)$

$\therefore$ Required vector $=3\left(\frac{2}{3}\hat{i}-\frac{1}{3}\hat{j}+\frac{2}{3}\hat{k}\right)=2\hat{i}-\hat{j}+2\hat{k}$

18. **(c) :** Here, $a_1=\sqrt{3}-1,\ b_1=-\sqrt{3}-1,\ c_1=4$

$a_2=-\sqrt{3}-1,\ b_2=\sqrt{3}-1,\ c_2=4$

$\therefore \quad \cos\theta=$

$$\frac{(\sqrt{3}-1)(-\sqrt{3}-1)+(-\sqrt{3}-1)(\sqrt{3}-1)+(4)(4)}{\sqrt{(\sqrt{3}-1)^2+(-\sqrt{3}-1)^2+16}\sqrt{(-\sqrt{3}-1)^2+(\sqrt{3}-1)^2+16}}$$

$\Rightarrow \cos\theta=\frac{-(3-1)-(3-1)+16}{\sqrt{24}\sqrt{24}}=\frac{12}{24}=\frac{1}{2} \Rightarrow \theta=\frac{\pi}{3}$

19. **(b)**

20. **(c) :** Let A be the event that the card is a spade and B be the event that the picked card is a queen.

We have a total of 13 spades and 4 queen cards.

Also only one queen is from spade.

$\therefore \quad P(A)=\frac{13}{52},\ P(B)=\frac{4}{52},\ P(A\cap B)=\frac{1}{52}$

$\therefore \quad P(A|B)=\frac{P(A\cap B)}{P(B)}=\frac{\frac{1}{52}}{\frac{4}{52}}=\frac{1}{4}$

Assertion & Reason Based Questions

1. **(d):** Here, A is false but R is true.

2. **(a) :** Since $Y=\{y\in N: y=4x+3 \text{ for some } x\in N\}$

$\therefore \quad Y=\{7, 11, \ldots \infty\}$

Let $f(x)=4x+3 \Rightarrow x=\frac{y-3}{4} \Rightarrow f(x)=y$

$\Rightarrow f$ is one-one and onto.

$\Rightarrow f^{-1}(x)$ exist

Inverse of $f(x)$ is $g(y) = \frac{y-3}{4}$

Hence, Assertion is true.

Reason is also true and is the correct explanation of Assertion.

3. (d): $2(\sin^{-1}x)^2 - 5(\sin^{-1}x) + 2 = 0$

$$\Rightarrow \sin^{-1}x = \frac{5 \pm \sqrt{25-16}}{4} \Rightarrow \sin^{-1}x = \frac{1}{2}, \sin^{-1}x = 2$$

$\Rightarrow x = \sin\left(\frac{1}{2}\right)$ is only solution

$\left[\because \frac{-\pi}{2} \le \sin^{-1}x \le \frac{\pi}{2}, \sin^{-1}x = 2 \text{ is not possible}\right]$

$\therefore$ Assertion is false but Reason is true.

4. (b): Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, adj $A = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$\Rightarrow$ adj (adj A) $= \begin{bmatrix} a & b \\ c & d \end{bmatrix} = A$

adj $A = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \Rightarrow |\text{adj } A| = ad - bc = |A|$

5. (c): Since, $P(A \cap B) = P(A)P(B)$, therefore, A and B are independent events.

$$\therefore \quad P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)P(B)}{P(B)} = P(A)$$

Similarly, $P(B|A) = P(B)$.

Thus, Assertion is true.

However, Reason is not true as $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

6. (a): Reason is a standard result.

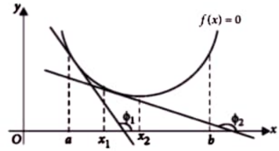
Also, $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (x+1) = 2$

and $\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} x^2 = 1$

$\Rightarrow \lim_{x \to 1^-} f(x) \ne \lim_{x \to 1^+} f(x) \Rightarrow f$ is not continuous at $x = 1$

$\Rightarrow f$ is not derivable at $x = 1$

7. (c): We know that, $\sin x$ and $\cos x$ decrease in $\frac{\pi}{2} < x < \pi$, so Assertion is correct.



Reason is incorrect which is clear from the graph that $f(x)$ is differentiable in (a, b).

Also, $a < x_1 < x_2 < b$

But $f'(x_1) = \tan\phi_1 < \tan\phi_2 = f'(x_2)$

$\Rightarrow$ Derivative is increasing.

8. (c): Given, $|2\vec{a} - \vec{b}| = 5$

$\Rightarrow |2\vec{a} - \vec{b}|^2 = 5^2 \Rightarrow 4|\vec{a}|^2 + |\vec{b}|^2 - 4\vec{a}\cdot\vec{b} = 25$

$\Rightarrow 16 + 9 - 4\vec{a}\cdot\vec{b} = 25 \Rightarrow \vec{a}\cdot\vec{b} = 0$

Now, $|2\vec{a} + \vec{b}| = \sqrt{|2\vec{a} + \vec{b}|^2} = \sqrt{4|\vec{a}|^2 + |\vec{b}|^2 + 4\vec{a}\cdot\vec{b}}$

$= \sqrt{16 + 9 + 0} = \sqrt{25} = 5$

$\therefore$ Assertion is true.

Given, $\vec{P}\cdot\vec{Q} = |\vec{P}||\vec{Q}|$ $\qquad \because \vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

$\Rightarrow |\vec{P}||\vec{Q}|\cos\theta = |\vec{P}||\vec{Q}| \Rightarrow \cos\theta = 1 \Rightarrow \theta = 0°$

$\therefore$ Reason is false.

9. (a): Clearly, Reason is true.

Let $I = \int_{-3}^{3} (ax^5 + bx^3 + cx + k)dx$

$= a\int_{-3}^{3} x^5 dx + b\int_{-3}^{3} x^3 dx + c\int_{-3}^{3} x dx + k\int_{-3}^{3} 1 dx$

Since, x^5, x^3, x are odd function.

$\therefore \quad I = 0 + 0 + 0 + k[x]_{-3}^{3} = 6k,$

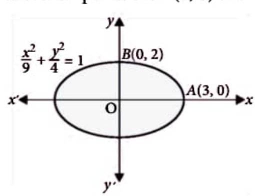
which is dependent only on k.

So, Assertion is true.

Thus, both Assertion and Reason are true and Reason is the correct explanation of Assertion.

10. (d): Given curve is $\frac{x^2}{9} + \frac{y^2}{4} = 1$, which represents an ellipse with centre at (0, 0).

Clearly, intersection points are $A(3, 0)$ and $B(0, 2)$.



Required area = $\pi \times 3 \times 2 = 6\pi$ sq. units

Assertion is false but Reason is true.

11. (a) : Let $c_1 + c_2 + c_3e^{c_4} = A$ (constant)

Then, $y = Ax$ ...(i)

$\Rightarrow \quad \frac{dy}{dx} = A$

$\Rightarrow \quad \frac{dy}{dx} = \frac{y}{x}$ [Using(i)]

$\Rightarrow \quad x\frac{dy}{dx} = y$

Both Assertion and Reason are true and Reason is the correct explanation of Assertion.

12. (c) : Given cartesian equation is

$\frac{x-5}{3} = \frac{y+4}{7} = \frac{z-6}{2}$.

$\Rightarrow \vec{a} = 5\hat{i} - 4\hat{j} + 6\hat{k}$ and $\vec{b} = 3\hat{i} + 7\hat{j} + 2\hat{k}$.

The vector equation of the line is given by $\vec{r} = \vec{a} + \lambda\vec{b}, \lambda \in R$.

$\Rightarrow \quad \vec{r} = 5\hat{i} - 4\hat{j} + 6\hat{k} + \lambda(3\hat{i} + 7\hat{j} + 2\hat{k})$

Thus Assertion is true.

In Reason, it is given that the line passes through the point (–2, 4, –5) and parallel to the line is

$$\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}$$

Clearly, the direction ratios of line are (3, 5, 6).

Now, the equation of the line (in cartesian form) is

$$\frac{x-(-2)}{3} = \frac{y-4}{5} = \frac{z-(-5)}{6} \Rightarrow \frac{x+2}{3} = \frac{y-4}{5} = \frac{z+5}{6}$$

Hence, Reason is false.

13. (a) : Required length = $\left|\frac{(3\hat{i} - \hat{j} - 2\hat{k}) \cdot (\hat{i} + 2\hat{j} - 3\hat{k})}{\sqrt{1^2 + 2^2 + (-3)^2}}\right|$

$$= \left|\frac{3-2+6}{\sqrt{1+4+9}}\right| = \frac{7}{\sqrt{14}}$$

Also, vector projection of $\vec{a}$ on $\vec{b} = (\vec{a} \cdot \hat{b}) = \left(\frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}\right)$

Hence, both A and R are true and R is the correct explanation of A.

14. (d): Reason is true.

Now, we know $P(E_1) + P(E_2) > 1$

For example, when a dice is rolled once and

E_1 : 'a number < 5' shows up, E_2 : 'a number > 1' shows up

then, $P(E_1) = \frac{4}{6} = \frac{2}{3}$ and also $P(E_2) = \frac{5}{6}$.

Here, $P(E_1) + P(E_2) > 1$.

So Assertion is false.

15. (b): We know that,

$$P(S|F) = \frac{P(S \cap F)}{P(F)} = \frac{P(F)}{P(F)} = 1$$

Also, $P(F|F) = \frac{P(F \cap F)}{P(F)} = \frac{P(F)}{P(F)} = 1$

Thus, $P(S | F) = P(F | F) = 1$

Now, we have,

$$P((A \cup B)|F) = \frac{P[(A \cup B) \cap F]}{P(F)}$$

$$= \frac{P[(A \cap F) \cup (B \cap F)]}{P(F)}$$

(by distributive law of intersection over union)

$$= \frac{P(A \cap F) + P(B \cap F) - P(A \cap B \cap F)}{P(F)}$$

$$= \frac{P(A \cap F)}{P(F)} + \frac{P(B \cap F)}{P(F)} - \frac{P((A \cap B) \cap F)}{P(F)}$$

$= P(A | F) + P(B | F) - P((A \cap B) | F)$

Case Based Questions

1. (i) According to given information,

perimeter of the park $= 2x + 2\left(\pi \cdot \frac{y}{2}\right) = 100$

$\Rightarrow \quad 2x + \pi y = 100$ ....(i)

(ii) Area of rectangular region of the park,

$A = xy$

$= x\left(\frac{100-2x}{\pi}\right) = \frac{2}{\pi}(50x - x^2)$ [$\because$ Using (i)]

(iii) We have, $A = \frac{2}{\pi}(50x - x^2)$

$\Rightarrow \frac{dA}{dx} = \frac{2}{\pi}(50 - 2x)$ and $\frac{d^2A}{dx^2} = \frac{2}{\pi}(0-2) = -\frac{4}{\pi}$

Now, $\frac{dA}{dx} = 0 \Rightarrow \frac{2}{\pi}(50 - 2x) = 0 \Rightarrow x = 25$

When $x = 25, \frac{d^2A}{dx^2} = -\frac{4}{\pi} < 0$

$\Rightarrow \quad A$ is maximum when $x = 25$.

Maximum value of $A = \frac{2}{\pi}(50 \times 25 - (25)^2) = \frac{1250}{\pi}m^2$

OR

Let Z be the area of the whole park, then

$$Z = xy + 2\cdot\frac{1}{2}\pi\left(\frac{y}{2}\right)^2 = xy + \frac{\pi}{4}\cdot y^2$$

$$= x\cdot\frac{100-2x}{\pi} + \frac{\pi}{4}\cdot\left(\frac{100-2x}{\pi}\right)^2$$

$$= \frac{2}{\pi}(50x - x^2) + \frac{(50-x)^2}{\pi}$$

$$\Rightarrow \frac{dZ}{dx} = \frac{2}{\pi}(50-2x) + \frac{2}{\pi}(50-x)(-1) = -\frac{2x}{\pi}$$

and $\frac{d^2Z}{dx^2} = -\frac{2}{\pi}$

$$\frac{dZ}{dx} = 0 \Rightarrow -\frac{2x}{\pi} = 0 \Rightarrow x = 0$$

When $x = 0$, $\frac{d^2Z}{dx^2} = -\frac{2}{\pi} < 0$

$\Rightarrow$ Z is maximum when $x = 0$.

2. (i) $R = \{(L_1, L_2) : L_1$ is parallel to L_2, where $L_1, L_2 \in L$

(a) **Reflexive :** Let $L_1 \in L$, Then, a line is always parallel to itself. So, R reflexive.

(b) **Symmetric :** As $L_1, L_2 \in L$

Let $(L_1, L_2) \in R \Rightarrow L_1$ is parallel to L_2.

$\Rightarrow L_2$ is also parallel to L_1, *i.e.*, $L_2 || L_1$

Thus, R is symmetric.

(c) **Transitive :** As L_1, L_2 and $L_3 \in L$

Let $(L_1, L_2) \in R$ and $(L_2, L_3) \in R$ which means L_1 is parallel to L_2 and L_2 is parallel to L_3

$\Rightarrow$ $L_1 || L_3$

Thus, R is transitive.

Hence, R is an equivalence Relation.

(ii) $f(x) = 3x - 4$

Let $f(x_1) = f(x_2)$

$\Rightarrow$ $3x_1 - 4 = 3x_2 - 4 \Rightarrow x_1 = x_2$

So, $f(x)$ is one-one.

Now, let $y = f(x) = 3x - 4$

$$\Rightarrow x = \frac{y+4}{3} \in R\ \forall\ y \in R$$

Therefore, $f(x)$ is onto. Thus, it is bijective function.

(iii) As $f(x)$ is bijective function. So, its co-domain is equal to its range. Thus, Range of $f(x) = R$

OR

We have, $y = 3x - 4$.

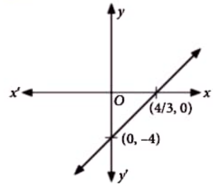
When $x = 0$,

then $y = 3 \times 0 - 4 = -4$.

When $y = 0$,

then $0 = 3x - 4$

$\Rightarrow$ $x = 4/3$



Sum of x and y-intercepts

$$= -4 + \frac{4}{3} = \frac{-12+4}{3} = \frac{-8}{3}.$$

3. Let,

$$X = \begin{bmatrix} 10000 & 2000 & 18000 \\ 6000 & 20000 & 8000 \end{bmatrix}, Y = \begin{bmatrix} 2.50 \\ 1.50 \\ 1.00 \end{bmatrix}\ Z = \begin{bmatrix} 2.00 \\ 1.00 \\ 0.50 \end{bmatrix}$$

$$XY = \begin{bmatrix} 10000 \times 2.50 + 2000 \times 1.50 + 18000 \times 1.00 \\ 6000 \times 2.50 + 20000 \times 1.50 + 8000 \times 1.00 \end{bmatrix}$$

$$= \begin{bmatrix} 46000 \\ 53000 \end{bmatrix}$$

(i) Total revenue of market A = ₹ 46000

(ii) $$XZ = \begin{bmatrix} 10000 \times 2.00 + 2000 \times 1.00 + 18000 \times 0.50 \\ 6000 \times 2.00 + 20000 \times 1.00 + 8000 \times 0.50 \end{bmatrix}$$

$$= \begin{bmatrix} 31000 \\ 36000 \end{bmatrix}$$

$\therefore$ Profit in market A = Revenue – Total cost of market A

= ₹ 46000 – ₹ 31000 = ₹ 15000

Profit in market B = ₹ 53000 – ₹ 36000

₹ 17000 = Revenue – Total cost of market B

4. (i) We have,

$$P(E_1) = \frac{1}{5}, P(E_2) = \frac{4}{5}, P(A/E_1) = 1, P(A/E_2) = \frac{1}{3}$$

Now, $\sum_{k=1}^{k=2} P(A/E_k)\,P(E_k)$

$$= P(A/E_1)P(E_1) + P(A/E_2)P(E_2)$$

$$= \frac{1}{5} \times 1 + \frac{4}{5} \times \frac{1}{3} = \frac{3+4}{15} = \frac{7}{15}$$

(ii) $$P(E_1/A) = \frac{P(A/E_1) \times P(E_2)}{P(A/E_1) \times P(E_1) + P(E_2) \times P(A/E_2)}$$

$$= \frac{\frac{1}{5}\times 1}{\frac{1}{5}+\frac{4}{15}} = \frac{1}{5}\times\frac{15}{7} = \frac{3}{7}$$

5. (i) $P(x) = -3x^2 + 100x + 27500$

$P'(x) = -3(2x) + 100 = -6x + 100$

$P''(x) = -6 < 0$, so profit is maximum.

For critical points $P'(x) = 0$

$\Rightarrow \quad 6x = 100 \Rightarrow x = \frac{100}{6} = 16.66$

$\therefore$ Profit is maximum at 16.66 and maximum profit is

$P(16.66) = -3(16.66)^2 + 100\,(16.66) + 27500 = 28333.33$

(ii) For strictly increasing function, $P'(x) > 0$

$\Rightarrow \quad -6x + 100 > 0$

$\Rightarrow \quad -6x > -100$

$\Rightarrow \quad x < \frac{100}{6} = 16.66$

So, $x \in (-\infty, 16.66)$ for strictly increasing function.

6. (i) $\frac{dT}{dt} = k(T-70)$

Using variable separable method,

$$\frac{dT}{T-70} = k\cdot dt$$

Integrating both sides, we get

$$\int\frac{dT}{T-70} = \int k\cdot dt \Rightarrow \log|T-70| = kt + c \qquad \text{...(i)}$$

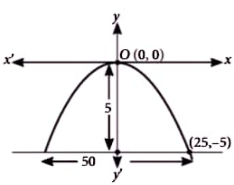
which is the required solution. Here, c is constant of integration.

(ii) $t = 0$, when $T = 72$

$\log|72 - 70| = k \times 0 + c$ [Using (i)]

$\Rightarrow \quad \log 2 = c$

7. (i) We know that



Equation of parabola along y-axis is

$x^2 = -4ay$ ...(i)

Equation (i) passes through $(25, -5)$

$\therefore \quad 625 = 20a \Rightarrow a = \frac{625}{20} = \frac{125}{4}$

$\therefore \quad x^2 = -4\times\frac{125}{4}\times y = -125y$

(ii) $\int_{-25}^{25} -\frac{x^2}{125}dx = \frac{-1}{125}\left[\frac{x^3}{3}\right]_{-25}^{25}$

$= \frac{-1}{375}\left[(25)^3 - (-25)^3\right] = \frac{-1}{375}\times 31250 = -\frac{250}{3}$

(iii) The area formed by the curve $x^2 = 125y$, x-axis, y-axis, $y = 5$ is given by

$$\int_0^5 \sqrt{125y}\,dy = 5\sqrt{5}\int_0^5\sqrt{y}\,dy = 5\sqrt{5}\times\frac{2}{3}\left[y^{3/2}\right]_0^5$$

$= \frac{10\sqrt{5}}{3}\times 5\sqrt{5} = \frac{250}{3}$ sq. feet.

OR

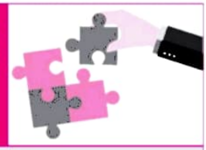
We have, $x^2 = -125y$

$a = 125/4$

$\therefore$ Focus is $(-a, 0)$, *i.e.*, $\left(\frac{-125}{4}, 0\right)$.

Also, equation of directrix is $y = \frac{125}{4}$.

PUZZLE CORNER



MATHDOKU

Introducing MATHDOKU, a mixture of ken-ken, sudoku and Mathematics. In this puzzle 6 × 6 grid is given, your objective is to fill the digits 1-6 so that each appear exactly once in each row and each column.

Notice that most boxes are part of a cluster. In the upper-left corner of each multibox cluster is a value that is combined using a specified operation on its numbers. For example, if that value is 3 for a two-box cluster and operation is multiply, you know that only 1 and 3 can go in there. But it is your job to determine which number goes where! A few cluster may have just one box and that is the number that fills that box.

1−	20×		3÷	3÷	
	3÷			24×	5+
2−		12+			
1−	8+			11+	
		2−	3×		9+
9+			2×		

Readers can send their responses at editor@mtg.in or post us with complete address. Winners' name with their valuable feedback will be published in next issue.

QUANTITATIVE APTITUDE

For Various Competitive Exams

1. The value of $\frac{3}{4}+\frac{5}{36}+\frac{7}{144}+....+\frac{17}{5184}+\frac{19}{8100}$ is

(a) 0.99 (b) 0.09
(c) 0.9 (d) None of these

2. 16 children are to be divided into groups A and B of 10 and 6 children respectively. The average % of marks scored by children of group A is 75 and the average % of marks scored by both groups is 76. What is the average % of marks of the group B?

(a) $77\frac{1}{3}$ (b) $77\frac{2}{3}$ (c) $78\frac{1}{3}$ (d) $78\frac{2}{3}$

3. The present age of Ravi's father is 4 times Ravi's age. Five year back, Ravi's father was seven times as old as Ravi was at that time. The present age of Ravi's father is

(a) 40 years (b) 35 years
(c) 84 years (d) 70 years

4. In an examination, a candidate needs 40% marks to pass. All questions carry equal marks. The candidate just passed by getting 10 answers correct of the total number of questions. How many questions are there in total?

(a) 25 (b) 30 (c) 40 (d) 45

5. A merchant earn a profit of 20% by selling a basket containing 80 apples, which cost ₹ 240, but he gave $\frac{1}{4}$ of it to his friend at cost price and sells the remaining apple. In order to earn the same profit, at what price he sells the each apple?

(a) ₹ 3.00 (b) ₹ 3.60 (c) ₹ 3.80 (d) ₹ 4.80

6. A company reduces the number of employees in the ratio 16 : 15 and increases their wages in the ratio 12 : 17. The over all wages bill is

(a) decreased in the ratio 64 : 85
(b) increased in the ratio 64 : 85
(c) increased in the ratio 15 : 17
(d) decreased in the ratio 17 : 15

7. Sonu invested 10% more than the investment of Mona and Mona invested 10% less than the investment of Raghu. If the total investment of all the three persons is ₹ 5780, what is the investment of Raghu?

(a) ₹ 1980 (b) ₹ 2000 (c) ₹ 1800 (d) ₹ 3800

8. Every day a baby drinks 500 mL milk. If the cost of 1 litre milk is ₹ 46. Find the cost of milk for 45 days.

(a) ₹ 935 (b) ₹ 1500
(c) ₹ 1035 (d) None of these

9. Four men can do a work by working 2 hours daily, the same work can be done by 3 girls working 6 hours daily and 4 boys can do the same work by working 2 hours daily. Find the number of days in which 1 man, 1 girl and 1 boy can do the work.

(a) $3\frac{3}{11}$ days (b) $3\frac{2}{11}$ days
(c) $3\frac{1}{11}$ days (d) $3\frac{4}{11}$ days

10. If 3 men and 4 boys earn ₹ 264 in 8 days and 2 men and 3 boys earn ₹ 184 for the same time period. Find the time in which 6 men and 7 boys earn ₹ 378.

(a) 6 days (b) 5 days
(c) 4 days (d) 3 days

11. Three pipes A, B and C can fill a tank in 30 mins, 20 mins and 40 mins respectively. When tank is empty, all the three pipes are opened. If A, B and C discharges chemical solutions P, Q, R respectively then the part of solution Q in the liquid in the tank after 4 mins is

(a) $\frac{8}{13}$ (b) $\frac{9}{13}$ (c) $\frac{7}{13}$ (d) $\frac{6}{13}$

12. A car travels along the four sides of square at the speed of v, $2v$, $3v$ and $4v$ respectively. If u is average speed of the car when it travels around the square, then which one of the following is correct?

(a) $u = 2.25\ v$ (b) $u = 3v$
(c) $v < u < 2v$ (d) $3v < u < 4v$

13. A drum contains a mixture of two liquids A and B in the ratio 5 : 3. When 6 litres of mixture are drawn off and the drum is filled with B, the ratio of A and B becomes 15 : 17. How many litres of liquid B was contained in the drum initially?

(a) 9 litres (b) 15 litres
(c) 12 litres (d) 8 litres

14. What is the least number of complete year in which a sum of money at 20% compound interest will be more than doubled?

(a) 7 (b) 6 (c) 5 (d) 4

15. A square $ABCD$ is inscribed in a circle of unit radius, semicircles are described on each side of square as diameters, the area of the region bounded by the four semicircles & the circle is

(a) 4 sq. units (b) 3 sq. units
(c) 2.5 sq. units (d) 2 sq. units

Directions for questions 16 to 20: Answer the questions on the basis of the information given below:

Study the table given below and answer the questions.

Number of T.V. sold

	2011	2012	2013	2014	2015
LG	30000	38000	36000	42000	40000
Samsung	17000	28000	33000	32000	27000
Sony	12500	20000	35000	40000	50000
National Panasonic	30000	25000	22000	20000	15000
Toshiba	15725	18625	13275	14375	16000

16. The average annual sale of which brand is the highest?

(a) LG (b) Sony
(c) National Panasonic (d) Samsung

17. Which of the following statements is/are true?

(a) LG is showing an increase in sales every year.
(b) Samsung has recorded a fall in sales thrice during the given 5 years.
(c) The % increase in the number of units sold from 2011 to 2015 is 300% for Sony.
(d) The average annual sale of Samsung is more than that of Sony.

18. The installed capacity of each company is 75000 units and all the units produced by each company are sold. Then, the least and highest values for annual capacity utilization for any company are respectively

(a) 17.5% and 72.5% (b) 14.28% and 75.5%
(c) 16.67% and 66.67% (d) 5.24% and 95.44%

19. If for the year 2016 there is 25% increase in the total sale of TV's and there is 10% decrease in the sale of Toshiba TV's in comparison from 2016, then what % of the total sale in 2016 is for Toshiba TV's ?

(a) 17% (b) 8.2% (c) 9.3% (d) 7.8%

20. The brand which showed a decrease of 50% during the given 5 years period registered maximum % decrease during the period

(a) 2014 – 2015 (b) 2013 – 2014
(c) 2012 – 2013 (d) 2011 – 2012

SOLUTIONS

1. (a) : We have, $\frac{3}{4}+\frac{5}{36}+\frac{7}{144}+......+\frac{17}{5184}+\frac{19}{8100}$

$$=\frac{3}{1^2\cdot 2^2}+\frac{5}{2^2\cdot 3^2}+\frac{7}{3^2\cdot 4^2}+.....+\frac{17}{8^2\cdot 9^2}+\frac{19}{9^2\cdot 10^2}$$

$$=\left(1-\frac{1}{2^2}\right)+\left(\frac{1}{2^2}-\frac{1}{3^2}\right)+\left(\frac{1}{3^2}-\frac{1}{4^2}\right)+..........$$
$$+\left(\frac{1}{8^2}-\frac{1}{9^2}\right)+\left(\frac{1}{9^2}-\frac{1}{10^2}\right)$$

$$=1-\frac{1}{10^2}=\frac{99}{100}=0.99$$

2. (b) : Let n_1, n_2, $\bar{x}_1$, $\bar{x}_2$ and $\bar{x}$ are the number of children in group A, number of children in group B, average of marks scored by group A, average of marks scored by group B and average of marks scored by both the groups respectively.

Using $\bar{x}=\frac{n_1\bar{x}_1+n_2\bar{x}_2}{n_1+n_2}$

where $n_1 = 10$, $n_2 = 6$, $\bar{x}_1 = 75$, $\bar{x}_2 = ?$ and $\bar{x} = 76$

$$\therefore\quad 76=\frac{10\times 75+6\times\bar{x}_2}{10+6}\Rightarrow\bar{x}_2=\frac{233}{3}=77\frac{2}{3}$$

3. (a) : Let father's present age be x years.

Ravi's present age be y years.

According to question,

$x = 4y$ ...(i)

and $x - 5 = 7(y - 5)$

$\Rightarrow 7y - x = 30$ ...(ii)

From (i), we get

$7y - 4y = 30$

$\Rightarrow 3y = 30 \Rightarrow y = 10$

From (i), we get

$x = 4 \times 10 = 40$

Present age of Ravi's father = 40 years.

4. (a) : Let the total number of questions in exam be x.
Given each question carry equal marks, say 1 mark.
The total marks of examination = $1 \times x = x$
The candidate will pass the exam by attempting 10 questions which means if he/she gets 10 marks, he/she will pass the exam.
According to problem, 40% are passing marks

$\therefore \quad \frac{40 \times x}{100} = 10 \Rightarrow x = \frac{100}{4} = 25$

$\therefore$ Total questions = 25

5. (c) : $\because$ C.P. of 80 apples = ₹ 240
S.P. of the basket (at a profit of 20%)

$= ₹\,\frac{240 \times 120}{100} = ₹\,288$

$\therefore$ Profit = ₹ 48

Cost of $\frac{3}{4}$ apples = ₹ $\frac{3}{4} \times 240$ = ₹ 180

Now, S.P. of 60 apples = C.P. of 60 apples + Profit
= ₹ (180 + 48) = ₹ 228

$\therefore$ S.P. of 1 apple $= \frac{228}{60} = ₹\,3.80$

6. (b) : Ratio of numbers of employees = 16 : 15
and ratio of their wages = 12 : 17
Let total employees in the starting = $16k$
and number of employees at present = $15k$
and their wages in the starting and at present are 12λ and 17λ respectively.
$\therefore$ Wages bill in starting = $16 \times 12k\lambda$
and wages bill at present = $15 \times 17k\lambda$
$\therefore$ Required ratio = $16 \times 12k\lambda : 15 \times 17k\lambda$
or 64 : 85
$\therefore$ Over all wages bills is increased in the ratio 64 : 85.

7. (b) : Let the share of Raghu be ₹ x.
$\therefore$ Mona's share of investment is 10% less than Raghu

$= \frac{(100-10) \times x}{100} = \frac{90x}{100}$

and Sonu's share of investment 10% more than the share of Mona $= \left(\frac{100+10}{100}\right) \times \frac{90}{100}x = \frac{99x}{100}$

$\therefore$ Sonu's share : Mona's share : Raghu's share

$= \frac{99x}{100} : \frac{90x}{100} : x$

or 99 : 90 : 100
Total investment = ₹ 5780

$\therefore$ Investment of Raghu's $= \frac{5780 \times 100}{99+90+100}$

$= \frac{5780 \times 100}{289} = ₹\,2000$

8. (c) : Total quantity of milk drink for 45 days

$= \frac{45 \times 500}{1000} = \frac{45}{2}$ litres

Now, cost of 1 litre milk = ₹46

Thus, cost of $\frac{45}{2}$ litres milk = $\frac{46 \times 45}{2}$ = ₹ 1035

9. (a) : Four men can do a work by working 2 hours daily
$\therefore$ 1 man can do the work = $4 \times 2 = 8$ hours per day or 8 days
Similarly 1 girl can do it = $3 \times 6 = 18$ days
and 1 boy can do it = $4 \times 2 = 8$ days

Now, one day work of a man $= \frac{1}{8}$,

One day work of a boy $= \frac{1}{8}$

and 1 day work of a girl $= \frac{1}{18}$

Therefore, one day work of a man, a girl and a boy is given by
(1 man + 1 girl + 1 boy)'s 1 day work

$= \frac{1}{8} + \frac{1}{18} + \frac{1}{8} = \frac{9+4+9}{72} = \frac{22}{72} = \frac{11}{36}$

$\therefore$ 1 man, 1 girl and 1 boy together finish the whole work in $\frac{36}{11}$ hours working per day

i.e., $\frac{36}{11}$ days *i.e.*, $3\frac{3}{11}$ days

10. (a) : Earnings of (3 men + 4 boys) in 8 days = ₹ 264
$\therefore$ Earnings of (3 men + 4 boys) in 1 day = ₹ 33 ...(i)
Again, earnings of (2 men + 3 boys) in 1 day = ₹ 23 ...(ii)
Now (i) × 2 and (ii) × 3 gives
Earnings of (6 men + 8 boys) in 1 day = ₹ 66 ...(iii)
Earnings of (6 men + 9 boys) in 1 day = ₹ 69 ...(iv)
On subtracting (iii) from (iv), we get
Earnings of 1 boy in 1 day = ₹ 69 – ₹ 66 = ₹ 3
$\therefore$ Earnings of 3 men + ₹(4 × 3) in 1 day = ₹ 33
$\Rightarrow$ Earnings of 3 men in 1 day = ₹ (33 – 12) = ₹ 21

$\therefore$ Earnings in 1 day for 1 man $= ₹\,\frac{21}{3} = ₹\,7$

Hence, earnings of (6 men + 7 boys) in 1 day

$= ₹(6 \times 7 + 7 \times 3) = ₹\,63$

$\therefore$ 6 men + 7 boys earn ₹ 378 in $\frac{378}{63} = 6$ days.

11. (d) : The part of tank filled by pipes A, B and C in 1 min are $\frac{1}{30}, \frac{1}{20}, \frac{1}{40}$ respectively.

Now, work done by pipes A, B and C in 1 min when working together

$$= \frac{1}{30} + \frac{1}{20} + \frac{1}{40} = \frac{4+6+3}{120} = \frac{13}{120}$$

$\therefore$ Work done by A, B and C in 4 mins = quantity of solution P, Q and R from A, B and C respectively in 4 mins $= 4 \times \frac{13}{120} = \frac{13}{30}$

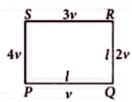
Now, quantity of solution Q in liquid in 4 mins

$$= \frac{1 \times 4}{20} = \frac{4}{20}$$

$\therefore$ Part of solution $Q = \frac{4/20}{13/30} = \frac{1}{5} \times \frac{30}{13} = \frac{6}{13}$

12. (c) : Let length of the side of square be l.

$\therefore$ Time to cover the distance (length of side PQ) $= \frac{l}{v}$



Time to cover the distance (length of side QR) $= \frac{l}{2v}$

and the time to cover the distances (side RS and SP) are respectively $\frac{l}{3v}$ and $\frac{l}{4v}$.

$\therefore$ Average speed $= \frac{\text{Total distance}}{\text{Total time}}$

$$= \frac{4 \times l}{l\left(\frac{1}{v} + \frac{1}{2v} + \frac{1}{3v} + \frac{1}{4v}\right)}$$

$$= \frac{4v}{1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}} = \left(\frac{4 \times 12 \times v}{12+6+4+3}\right) \text{ i.e., } u = \frac{48v}{25} = 1.92\,v$$

$\Rightarrow v < u < 2v$

13. (a) : The ratio of mixture of two liquids A and B in the drum is 5 : 3.

Let the drum initially contains $5x$ and $3x$ litres of liquids A and B respectively.

Now, 6 litres of liquid (mixture) is drawn off

$\therefore$ Quantity of A in the mixture left

$$= 5x - \frac{5}{8} \times 6 = \left(5x - \frac{15}{4}\right) \text{litres}$$

Quantity of B in the mixture left

$$= 3x - \frac{3}{8} \times 6 = \left(3x - \frac{9}{4}\right) \text{litres}$$

Now, according to problem, we have

$$\frac{5x - \frac{15}{4}}{\left(3x - \frac{9}{4}\right) + 6} = \frac{15}{17} \Rightarrow \frac{20x - 15}{12x + 15} = \frac{15}{17}$$

$\Rightarrow (20x - 15) \times 17 = 15(12x + 15)$

$\Rightarrow (340 - 180)x = 15 \times 15 + 15 \times 17 = 15(32)$

$\Rightarrow 160x = 15 \times 32 \quad \Rightarrow x = \frac{15 \times 32}{160} = 3$

$\therefore$ Quantity of liquid B in the drum initially $= 3x$ $= 3 \times 3 = 9$ litres

14. (d) : Let the sum of money = ₹ P

$\therefore$ Amount = ₹ $2P$

Now, $A = P\left(1 + \frac{r}{100}\right)^n \Rightarrow 2P = P\left(1 + \frac{20}{100}\right)^n$

$\Rightarrow \left(\frac{6}{5}\right)^n = 2$

Putting $n = 1$ $\quad \therefore \left(\frac{6}{5}\right) = 1.2 < 2 \therefore n \neq 1$

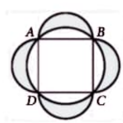
Putting $n = 2$ $\quad \therefore \frac{36}{25} = 1.44 < 2 \therefore n \neq 2$

Putting $n = 3$ $\quad \therefore \frac{216}{125} < 2 \therefore n \neq 3$

Putting $n = 4$ $\quad \therefore \frac{1296}{625} > 2 \therefore n \geq 4$

$\therefore$ Least value of $n = 4$

15. (d) : Radius of inscribed circle (r) = 1 unit



$\therefore$ Area of circle $= \pi r^2$ $= \pi$ sq. units

Let side of square $ABCD = x$ unit

$\therefore x^2 + x^2 = (2)^2$

$\Rightarrow x^2 = 2 \Rightarrow x = \sqrt{2}$

$\therefore$ Area of square $= (\text{side})^2 = (\sqrt{2})^2 = 2$ sq.units

Now, radius of semi circles $= \frac{\text{side of square}}{2} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$

$\therefore$ Area of one semi-circle $= \frac{1}{2}\pi\left(\frac{1}{\sqrt{2}}\right)^2 = \frac{\pi}{4}$ sq. units

Area of 4 semi-circles $= 4 \times \frac{\pi}{4} = \pi$ sq. units

Required area (shaded area) = Area of 4 semi circles of radius $\sqrt{2}$ + area of square – area of larger circle
$= \pi + 2 - \pi = 2$ sq. units

16. (a) : Average sale of LG $= \frac{\text{Total sale in all years}}{\text{Number of years}}$

$$= \frac{30000 + 38000 + 36000 + 42000 + 40000}{5}$$

$$= \frac{186000}{5} = 37200$$

Average sale of Samsung

$$= \frac{(17 + 28 + 33 + 32 + 27)}{5} \times 1000 = \frac{137 \times 1000}{5} = 27400$$

Average sale of Sony

$$= \frac{12500 + 20000 + 35000 + 40000 + 50000}{5}$$

$$= \frac{157500}{5} = 31500$$

Average sale of National Panasonic

$$= \frac{30000 + 25000 + 22000 + 20000 + 15000}{5}$$

$$= \frac{112000}{5} = 22400$$

Average sale of Toshiba

$$= \frac{15725 + 18625 + 13275 + 14375 + 16000}{5}$$

$$= \frac{78000}{5} = 15600$$

Thus, the company LG sold the highest number of TV's.

17. (c) : (a) The sale of LG decreases during 3rd and 5th years.

(b) Samsung has recorded a fall in sale twice but not thrice.

(c) Sale of Sony in 2011 is 12500 and in 2015 is 50000

$\therefore$ Sale % increased by

$$= \frac{50000 - 12500}{12500} \times 100 = 300\%$$

(d) Average sale of Sony is 31500 and that of Samsung is 27400

$\therefore$ Average annual sale of Samsung is less than that of Sony.

18. (c) :

	Least annual capacity utilization	**Highest annual capacity utilization**
LG	$\frac{30000 \times 100}{75000} = 40\%$	$\frac{42000 \times 100}{75000} = 56\%$
Samsung	$\frac{17000 \times 100}{75000} = 22.67\%$	$\frac{33000 \times 100}{75000} = 44\%$
Sony	$\frac{12500 \times 100}{75000} = 16.67\%$	$\frac{50000 \times 100}{75000} = 66.67\%$
National Panasonic	$\frac{15000 \times 100}{75000} = 20\%$	$\frac{30000 \times 100}{75000} = 40\%$
Toshiba	$\frac{13275 \times 100}{75000} = 17.7\%$	$\frac{18625 \times 100}{75000} = 24.83\%$

$\therefore$ Least annual capacity utilization = 16.67 % and highest annual utilization = 66.67%

19. (d) : Total sale of TV's in 2015 = 40000 + 27000 + 50000 + 15000 + 16000 = 148000

Now, for 2016 sale is increased by 25%.

$\therefore$ Sale in 2016

$$= \frac{\text{Sale of TV's in 2015} \times (100 + 25)}{100}$$

$$= \frac{148000 \times 125}{100} = 185000$$

Sale of Toshiba TV in 2015 = 16000, which is decreased by 10% in 2016.

$\therefore$ Sale of Toshiba TV in 2016

$$= \frac{16000 \times (100 - 10)}{100} = 14400$$

$\therefore$ Required % for Toshiba T.V

$$= \frac{\text{Sale of Toshiba in 2016}}{\text{Total Sale in 2016}} \times 100 = \frac{14400}{185000} \times 100$$

$= 7.78\% \approx 7.8\%$

20. (a) : In 2011 to 2015

National Panasonic shows a decrease of sale by 50%

i.e., $\frac{30000 - 15000}{30000} \times 100 = 50\%$

and % decrease in 2012 $= \frac{30000 - 25000}{30000} \times 100$

$= 16.67\%$

% decrease in 2013 $= \frac{25000 - 22000}{25000} \times 100 = 12\%$

% decrease in 2014 $= \frac{22000 - 20000}{22000} \times 100 = 9.09\%$

% decrease in 2015 $= \frac{20000 - 15000}{20000} \times 100 = 25\%$

$\therefore$ Maximum % decrease in sale of National Panasonic is shown during the year 2014–2015. ◆◆

Brain Teaser

LOGICAL REASONING

For Various Competitive Exams

Direction Q.(1 and 2) : Identify what will come in place of '?'.

1. Z,S,W,O,T,K,Q,G,?
(a) N,C (b) N,D (c) O,C (d) O,D

2. AD, EH, IL, ?, QT
(a) LM (b) MN (c) MP (d) OM

Direction Q.(3 and 4) : The two words on the left of (: :) are related in the some way as the words on the right of (: :). Identify what will come in place of '?'

3. Igloo : Ice : : Marquee : ?
(a) Canvas (b) Silk (c) Buckram (d) Sateen

4. Dilatory : Expeditious : : Direct : ?
(a) Tortuous (b) Circumlocutory
(c) Straight (d) None of these

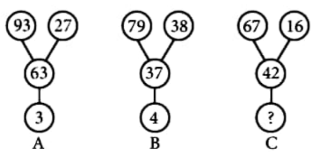
Direction Q.(5 and 6) : Three of the following four are alike in a certain way and so form a group. Which is the one that does not belong to that group?

5. (a) Kwashiorkor (b) Cretinism
(c) Marasmus (d) Goitre

6. (a) Cataract (b) Hypermetropia
(c) Trachoma (d) Eczema

Direction Q.(7 and 8) : In each of the following questions, find the number which replaces the sign of '?'.

7.



(a) 5 (b) 6 (c) 8 (d) 9

8.

7	4	5
8	7	6
3	3	?
29	19	31

(a) 3 (b) 4 (c) 5 (d) 6

Directions Q.(9 and 10) : In each of the question below are given three statements followed by two conclusions numbered (I) and (II). You have to take the two given statements to be true even if they seem to be at variance from commonly known facts and then decide which of the given conclusions logically follows from the three given statements, disregarding commonly known facts. Give your answer as

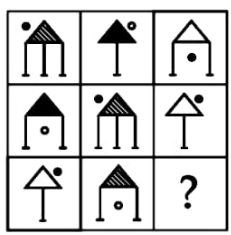
(a) if only conclusion (I) follows
(b) if only conclusion (II) follows
(c) if either (I) or (II) follows
(d) if both (I) and (II) follow

9. Statements:
Some trees are boxes.
All boxes are bricks.
All bricks are dogs.
Conclusions:
I. Some dogs are trees.
II. Some bricks are trees.

10. Statements:
All pots are rings.
All bangles are rings.
All rings are paints.
Conclusions:
I. Some paints are pots.
II. Some bangles are paints.

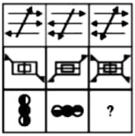
Direction Q.(11 and 12) : Select a figure from the options which will complete the given figure matrix.

11.



(a) (b) (c) (d)

12.



(a)  (b)  (c)  (d) 

Direction Q.(13 and 14) : In each of the following questions, there are three figures X, Y, Z showing a sequence of folding of a piece of paper Fig. (Z) shows the manner in which the folded paper has been cut. Select the figure from the options that resembles the unfolded form of Fig. (Z).

13.

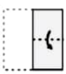
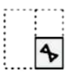
  

X Y Z

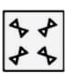
(a)  (b) 

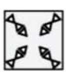
(c)  (d) 

14.

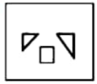
X Y Z

(a)  (b) 

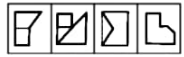
(c)  (d) 

Direction Q.(15 and 16) : In each of the following questions, find out which of the figures (a), (b), (c) and (d) can be formed from the pieces given in (X).

15.

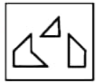


(X)

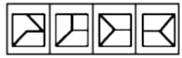


(a) (b) (c) (d)

16.



(X)



(a) (b) (c) (d)

Direction Q.(17 to 20): Study the following arrangement carefully and answer the questions given below:

B K 5 # M A 3 R % J 2 D E N @ 7 W 8 © 9 P T I V F 6 1 H Q ★ Y 4 $ L Z

17. Which of the following is the eighth to the right of the eleventh from the right end of the above arrangement?

(a) W (b) © (c) 5 (d) $

18. How many such consonants are there in the above arrangement, each of which is immediately preceded by a symbol but not immediately followed by a letter?

(a) None (b) One (c) Two (d) Three

19. If all the numbers from the above arrangement are removed, which of the following will be the twelfth from the left end?

(a) @ (b) P (c) I (d) N

20. How many such symbols are there in the above arrangement, each of which is immediately followed by a number but not immediately preceded by a number?

(a) None (b) One (c) Two (d) Three

SOLUTIONS

1. (a) : The given sequence consists of two series:

I. Z,W, T, Q, ? in which each letter is moved three steps backward to obtain the next term.

II. S, O, K, G in which each letter is moved four steps backward to obtain the next term.

2. (c) : The first and second letters of each term are moved four steps forward to obtain the corresponding letters of the next term.

3. (a) : First is made up of the second.

4. (b) : The words in each pair are opposites of each other.

5. (b) : All except Cretinism are deficiency diseases, while Cretinism is a hormonal disease.

6. (d) : All except Eczema are eye infections, while eczema is a skin infection.

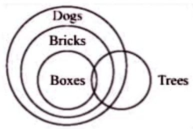
7. (d) : In fig. (A), $93 - (27 + 63) = 3$

In fig. (B), $79 - (38 + 37) = 4$

$\therefore$ In fig. (C), missing number $= 67 - (16 + 42) = 9$

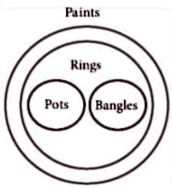
8. (c) : In the first column, 29 – 8 = 7 × 3 = 21
In the second column, 19 – 7 = 4 × 3 = 12
Let the missing number in the third column be x.
Then, $31 - 6 = 5 \times x \Rightarrow 5x = 25 \Rightarrow x = 5$

9. (d) : All boxes are bricks will mean that boxes are a subset of bricks and all bricks are dogs will mean that bricks are a subset of dogs. Now, some trees are boxes will means that trees will intersect with boxes and we get the following diagram:



As both dogs and bricks are intersecting with trees, thus both the conclusions (I) and (II) are valid.
Hence, both the conclusions follow.

10. (d) : All pots are rings and all bangles are rings will mean both pots and bangles are subset of rings. Also all rings are paints will mean that rings are a subset of paints. We get the following diagram:



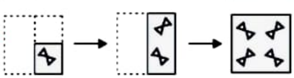
Pots are a subset of paints, thus conclusion (I) is valid. Also, bangles is a subset of paints, thus conclusion (II) is valid as well.
Hence, both the conclusions follow.

11. (c) : Each row must have a black, a white and a striped triangle; 1, 2 and 3 lines below the triangles and 2 black and 1 white circles.

12. (c)

13. (b) :

14. (b) :



15. (b) **16. (a)**

17. (d) : 8th to the right of the 11th from the right end means third from the right end, i.e., $.

18. (c) :

Symbol	Consonant	~~Letter~~

Such combinations are:

% J 2 ; ★ Y 4

19. (a) :

B	K	#	M	A	R	%	J	D	E	N	@	W
©	P	T	I	V	F	H	Q	★	Y	$	L	Z

20. (b) :

~~Number~~	Symbol	Number

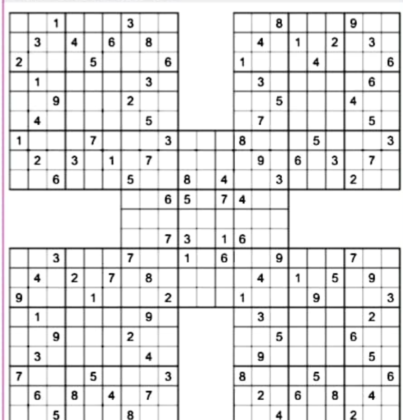
There is only one such combination:

N @ 7

◈◈

SAMURAI SUDOKU

Samurai Sudoku puzzle consists of five overlapping sudoku grids. The standard sudoku rules apply to each 9 × 9 grid. Place digits from 1 to 9 in each empty cell. Every row, every column and every 3 × 3 box should contain one of each digit.
The puzzle has a unique answer.



Readers can send their responses at editor@mtg.in or post us with complete address. Winners' name with their valuable feedback will be published in next issue.

YOU ASK WE ANSWER

Do you have a question that you just can't get answered?
Use the vast expertise of our MTG team to get to the bottom of the question. From the serious to the silly, the controversial to the trivial, the team will tackle the questions, easy and tough. The best questions and their solutions will be printed in this column each month.

1. Evaluate the following indefinite integral:

$$\int \frac{dx}{(x+1)^5\sqrt{x^2+2x}}$$ *(Priyanka Malhotra, Gujarat)*

Ans. Let $I = \int \frac{dx}{(x+1)^5\sqrt{x^2+2x}} = \int \frac{dx}{(x+1)^5\sqrt{(x+1)^2-1}}$

$$= \int \frac{dx}{(x+1)^6\sqrt{1-\frac{1}{(x+1)^2}}} = \int \frac{-t^4 dt}{\sqrt{1-t^2}} \quad \left[\text{Putting } \frac{1}{x+1} = t\right]$$

Here, t^4 is a polynomial of degree 4.

$$\therefore \quad I = \int \frac{-t^4 dt}{\sqrt{1-t^2}} = (At^3 + Bt^2 + Ct + D)\sqrt{1-t^2}$$

$$+\lambda \int \frac{dt}{\sqrt{1-t^2}} \quad \ldots\text{(i)}$$

Differentiating both sides and multiplying by $\sqrt{1-t^2}$, we get

$-t^4 = (3At^2 + 2Bt + C)(1 - t^2) - t(At^3 + Bt^2 + Ct + D) + \lambda$

Comparing the coefficients, we have

$-4A = -1$, $3B = 0$, $3A - 2C = 0$, $2B - D = 0$ and $C + \lambda = 0$

$\Rightarrow A = 1/4$, $B = 0$, $C = 3/8$, $D = 0$ and $\lambda = -3/8$

From (i) $I = \left(\frac{1}{4}t^3 + \frac{3}{8}t\right)\sqrt{1-t^2} - \frac{3}{8}\int \frac{dt}{\sqrt{1-t^2}}$

$$= \frac{1}{8}(2t^3 + 3t)\sqrt{1-t^2} - \frac{3}{8}\sin^{-1} t + C, \text{ where } t = \frac{1}{x+1}.$$

2. If $f(x)$ is a polynomial of degree <3, then prove that

$$\begin{vmatrix} 1 & a & \frac{f(a)}{x-a} \\ 1 & b & \frac{f(b)}{x-b} \\ 1 & c & \frac{f(c)}{x-c} \end{vmatrix} \div \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = \frac{f(x)}{(x-a)(x-b)(x-c)}.$$

(Ankita, U.P.)

Ans. $\frac{f(x)}{(x-a)(x-b)(x-c)} = \frac{A}{(x-a)} + \frac{B}{(x-b)} + \frac{C}{(x-c)}$

[By partial fractions]

i.e., $f(x) = A(x-b)(x-c) + B(x-a)(x-c) + C(x-a)(x-b)$ ...(i)

Putting $x = a$ in equation (i), we have

$f(a) = A(a-b)(a-c) \Rightarrow A = \frac{f(a)}{(a-b)(a-c)}$

Similarly, putting $x = b, c$ in (i) respectively, we get

$B = \frac{f(b)}{(b-a)(b-c)}$ and $C = \frac{f(c)}{(c-a)(c-b)}$

Hence, we have $\frac{f(x)}{(x-a)(x-b)(x-c)}$

$$= \frac{(c-b)\frac{f(a)}{x-a} - (c-a)\frac{f(b)}{(x-b)} + (b-a)\frac{f(c)}{(x-c)}}{(a-b)(b-c)(c-a)}$$

$$= \begin{vmatrix} 1 & a & \frac{f(a)}{x-a} \\ 1 & b & \frac{f(b)}{x-b} \\ 1 & c & \frac{f(c)}{x-c} \end{vmatrix} \div \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$, which is the desired result.

COMIC CAPSULE

Challenging PROBLEMS on Integration

For JEE

NUMERICAL VALUE TYPE

1. Let $f(x)$ be differentiable function such that $f(x)=x^2+\int_0^x e^{-t}f(x-t)dt$, then $6f(1)$ = ______.

2. If the value of definite integral $\int_1^a x\cdot a^{-[\log_a x]}dx$, where $a>1$, and $[\cdot]$ denotes the greatest integer, is $\frac{e-1}{2}$, then the value of $5[a]$ is _____ .

3. If $I=\int_0^{\pi} x(\sin^2(\sin x)+\cos^2(\cos x))dx$, then $[I]$ = ______, where $[\cdot]$ denotes the greatest integer function.

4. The integral $\int_{\pi/4}^{5\pi/4}(|\cos t|\sin t+|\sin t|\cos t)\,dt$ has the value equal to _____.

5. Let f is a differentiable function such that $f'(x)=f(x)+\int_0^2 f(x)dx,\ f(0)=\frac{4-e^2}{3}$, then the value of $|[f(2)]|$, where $[\cdot]$ denotes the greatest integer function, is _____.

6. Let $I_n=\int_0^{\pi/2}(\sin x+\cos x)^n dx\ (n\geq 2)$. Then the value of $nI_n-2(n-1)I_{n-2}$ is _____.

7. If $\lim_{x\to 0}\int_0^x \frac{t^2dt}{(x-\sin x)\sqrt{a+t}}=1$, then the value of a is _______.

8. Given $\int_0^{\pi/2}\ln\sin x\,dx=\frac{\pi}{2}\ln\frac{1}{2}$ and $\int_0^{\pi/2}\left(\frac{x}{\sin x}\right)^2dx=\frac{k\cdot\pi}{2}\ln 2$, then k = ______.

9. The value of $\frac{29\int_0^1(1-x^4)^7dx}{4\int_0^1(1-x^4)^6dx}$ is equal to ______.

10. Let $F(x)$ be a non-negative continuous function defined on R such that $F(x)+F\left(x+\frac{1}{2}\right)=3$ and the value of $\int_0^{1500}F(x)dx$ is $\frac{9000}{\lambda}$. Then the numerical value of λ is _____.

11. Let $f(x)=\frac{4^x}{4^x+2}$, $I_1=\int_{f(1-a)}^{f(a)}xf(x(1-x))dx$ and $I_2=\int_{f(1-a)}^{f(a)}f(x(1-x))dx$, where $f(a)>f(1-a)$, then the value of $\frac{I_2}{I_1}$ is _____.

12. The value of $\int_{-1}^{1}\left[x\left[1+\sin\pi x\right]+1\right]dx$ is _______. ($[\cdot]$ denote the greatest integer function)

13. $\lim_{n\to\infty}\sum_{k=1}^{n}\frac{k^{1/a}\left\{n^{a-\frac{1}{a}}+k^{a-\frac{1}{a}}\right\}}{n^{a+1}}$ is equal to ______.

By : Tapas Kr. Yogi, Visakhapatnam Mob : 09533632105

14. If $f(0) = 1, f(2) = 3, f'(2) = 5$, then the value of the definite integral $\int_0^1 x f''(2x)dx$ is ______.

15. If $f(x)=\int_a^x \frac{1}{f(x)}dx$ and $\int_a^1 \frac{1}{f(x)}dx=\sqrt{2}$, then the value of $f(2)$ = ______.

16. If $\int_0^\pi x\sin^5 x\cos^6 x dx=\frac{k\pi}{693}$, then k = ______.

17. If $\int_0^{\pi/2} x^n \sin x\, dx=(3/4)(\pi^2-8)$, then the value of n is ______.

18. $I=\int_{-1}^{1}(1+x)^{1/2}(1-x)^{3/2}dx$, then the value of $\sec^2(I/2)$ is ______ .

19. The value of $\int_{-\pi/4}^{\pi/4}[(x^9-3x^5+7x^3-x+1)/\cos^2 x]dx$ is ______.

MATRIX-MATCH TYPE

20. List-I below gives values of integrals involving parameters A and B while List-II gives values of these parameters for which the results given are correct. Match the integrals in List-I with the values of parameters A, B in List-II so that the given result is correct.

	List-I		List-II
(A)	$\int\frac{e^{2x}-2e^x}{e^{2x}+1}dx$ $= A\ln(e^{2x}+1) + B\tan^{-1}(e^x) + C$	(p)	$A=-\frac{1}{2},\ B=-\frac{1}{4}$
(B)	$\int\sqrt{x+\sqrt{x^2+2}}\,dx = A\{x+\sqrt{x^2+2}\}^{3/2}+\frac{B}{\sqrt{x+\sqrt{x^2+2}}}+C$	(q)	$A=\frac{1}{3},\ B=-2$
(C)	$\int\frac{\cos 8x-\cos 7x}{1+2\cos 5x}dx$ $= A\sin 3x + B\sin 2x + C$	(r)	$A=\frac{1}{2},\ B=-2$
(D)	$\int\frac{\ln x}{x^3}dx=A\frac{\ln x}{x^2}+\frac{B}{x^2}+C$	(s)	$A=\frac{1}{3},\ B=-\frac{1}{2}$

21. Match the following :

	List-I		List-II
(A)	$\int\frac{\cos x}{\cos 3x}dx=$	(p)	$-\frac{1}{\tan x+2\sec x}+c$
(B)	$\int\frac{\cos^3 x}{(1+\sin^2 x)^2}dx=$	(q)	$-\frac{1}{2\sqrt{3}}\ln\left\|\frac{\sqrt{3}\tan x-1}{\sqrt{3}\tan x+1}\right\|+c$
(C)	$\int\frac{dx}{4\sin^2 x+\cos^2 x}=$	(r)	$\frac{1}{2}\tan^{-1}(2\tan x)+c$
(D)	$\int\frac{1+2\sin x}{(2+\sin x)^2}dx=$	(s)	$(\sin x + \operatorname{cosec} x)^{-1} + c$

22. For $0 < x < 1$, match the following :

	List-I		List-II
(A)	$\int\frac{dx}{(1-\sqrt{x})\sqrt{1-x}}=$	(p)	$2\left(\frac{\sqrt{x}+1}{\sqrt{1-x}}\right)+c$
(B)	$\int\frac{dx}{(1+\sqrt{x})\sqrt{1-x}}=$	(q)	$2\left(\frac{1-\sqrt{x}}{\sqrt{1-x}}+\sin^{-1}\sqrt{x}\right)+c$
(C)	$\int\frac{dx}{(1-\sqrt{x})\sqrt{x-x^2}}=$	(r)	$2\left(\frac{\sqrt{x}-1}{\sqrt{1-x}}\right)+c$
(D)	$\int\frac{dx}{(1+\sqrt{x})\sqrt{x-x^2}}=$	(s)	$2\left(\frac{1+\sqrt{x}}{\sqrt{1-x}}-\sin^{-1}\sqrt{x}\right)+c$

23. A function F is defined by $F(x)=\int_1^x \frac{e^t}{t}dt\ \forall\ x>0.$

Now express the functions in list-I in terms of F.

	List-I		List-II
(A)	$\int_1^x \frac{e^t}{t+2}dt$	(p)	$F(x)-\frac{e^x}{x}+e$
(B)	$\int_1^x \frac{e^{3t}}{t}dt$	(q)	$xe^{1/x}-e-F\left(\frac{1}{x}\right)$
(C)	$\int_1^x \frac{e^t}{t^2}dt$	(r)	$e^{-2}[F(x+2)-F(3)]$
(D)	$\int_1^x e^{\frac{1}{t}}dt$	(s)	$F(3x) - F(3)$

SOLUTIONS

1. **(8)** : $f(x)=x^2+\int_0^x e^{-t}f(x-t)dt=x^2+e^{-x}\int_0^x e^t f(t)dt$

$\Rightarrow f'(x)=2x-e^{-x}(e^x(f(x)-x^2))+e^{-x}\cdot e^x f(x)$

$\Rightarrow f'(x)=2x+x^2 \Rightarrow f(x)=\frac{x^3}{3}+x^2+k$

But $f(0)=0 \Rightarrow k=0. \therefore f(1)=\frac{4}{3}$

2. **(5)** : Let $I=\int_1^a x\cdot a^{-[\log_a x]}dx$

Let $\log_a x=t \Rightarrow a^t=x \Rightarrow dx=a^t\log_e a$

$\therefore I=\ln a\int_0^1 a^t\cdot a^{-[t]}\cdot a^t dt=\ln a\int_0^1 a^{2t}dt=\frac{a^2-1}{2}=\frac{e-1}{2}$

$\Rightarrow a=\sqrt{e}$

3. **(4)** : We have, $I=\int_0^\pi x(\sin^2(\sin x)+\cos^2(\cos x))dx$,

$I=\int_0^\pi(\pi-x)\left((\sin^2(\sin x))+\cos^2(\cos x)\right)dx$

$\therefore 2I=2\pi\int_0^{\pi/2}\left(\sin^2(\sin x)+\cos^2(\cos x)\right)dx$

$\Rightarrow I=\pi\int_0^{\pi/2}(\sin^2(\sin x))+\cos^2(\cos x)dx$

$=\pi\int_0^{\pi/2}(\sin^2(\cos x)+\cos^2(\sin x))dx$

$\Rightarrow 2I=\pi\int_0^{\pi/2}2dx \Rightarrow I=\frac{\pi^2}{2}$

4. **(0)** : Let $I=\int_{\pi/4}^{5\pi/4}(|\cos t|\sin t+|\sin t|\cos t)\,dt$

$=\int_{\pi/4}^{\pi/2}2\sin t\cos t\,dt$

$+\int_{\pi/2}^{\pi}\{(-\sin t\cos t)+(\sin t\cos t)\}dt+\int_\pi^{5\pi/4}-2\sin t\cos t\,dt$

$=\int_{\pi/4}^{\pi/2}\sin 2t\,dt-\int_\pi^{5\pi/4}\sin 2t\,dt=0$

5. **(5)** : Given, $f'(x)=f(x)+\int_0^2 f(x)\,dx$

$\Rightarrow f''(x)=f'(x) \Rightarrow \int\frac{f''(x)}{f'(x)}=\int dx$

$\Rightarrow \log(f'(x))=x+\lambda$

$\Rightarrow f'(x)=e^{x+\lambda}=Ae^x$ (Put $e^\lambda=A$)

$\Rightarrow f(x)=Ae^x+B \Rightarrow f(0)=A+B=\frac{4-e^2}{3}$ ...(i)

Also, $f'(x)=f(x)+\int_0^2 f(x)\,dx$

$\Rightarrow Ae^x=(Ae^x+B)+\int_0^2(Ae^x+b)\,dx$

$\Rightarrow Ae^x=Ae^x+\left(\frac{4-e^2}{3}-A\right)+(Ae^x+Bx)_0^2$

$\Rightarrow \left(\frac{4-e^2}{3}-A\right)+(Ae^2+2B)+A=0$

$\Rightarrow \frac{4-e^2}{3}+Ae^2+2\left(\frac{4-e^2}{3}-A\right)=0 \Rightarrow A=\frac{4-e^2}{2-e^2}$

From (i), we have $B=\frac{(4-e^2)(1+e^2)}{e^2-2}$

$\therefore f(x)=\left(\frac{4-e^2}{2-e^2}\right)x+\frac{(4-e^2)(1+e^2)}{e^2-2}$

$\therefore f(2)=\left(\frac{4-e^2}{2-e^2}\right)(2)+\frac{(4-e^2)(1+e^2)}{e^2-2}$

$=\left(\frac{4-e^2}{2-e^2}\right)(2-1-e^2)=\frac{(4-e^2)(1-e^2)}{2-e^2}=-4.017$

$\Rightarrow [f(2)]=-5 \quad \therefore \|[f(2)]\|=5$

6. **(2)** : $I_n=\int_0^{\pi/2}(\sin x+\cos x)^{n-1}(\sin x+\cos x)'dx$

$=2+(n-1)\int_0^{\pi/2}(\sin x+\cos x)^{n-2}(\cos x-\sin x)^2dx$

$=2+(n-1)\int_0^{\pi/2}(\sin x+\cos x)^{n-2}[2-(\sin x+\cos x)^2]\,dx$

$=2+2(n-1)I_{n-2}-(n-1)I_n \Rightarrow nI_n-2(n-1)I_{n-2}=2$

7. **(4)** : We have,

$\lim_{x\to 0}\frac{\int_0^x\frac{t^2}{\sqrt{a+t}}dt}{x-\sin x}=\lim_{x\to 0}\frac{x^2}{\sqrt{a+x}(1-\cos x)}$

$=\lim_{x\to 0}\frac{x^2}{\sqrt{a+x}\left(2\sin^2\frac{x}{2}\right)}=\frac{2}{\sqrt{a}}=1 \Rightarrow a=4$

8. (2) : $I=\int_0^{\pi/2} x^2 \operatorname{cosec}^2 x\,dx$

$$=\left[-x^2\cot x\right]_0^{\pi/2}+\int_0^{\pi/2} 2x\cot x\,dx$$

$$=0+\lim_{x\to 0^+}\frac{x^2}{\tan x}+2\left[x\ln|\sin x|\right]_0^{\pi/2}-2\int_0^{\pi/2}\ln|\sin x|\,dx$$

$$=0-2\lim_{x\to 0^+}x\ln|\sin x|-2\frac{\pi}{2}\ln\frac{1}{2}=\pi\ln 2$$

$$\left(\because \lim_{x\to 0^+}\frac{\ln\sin x}{1/x}=\lim_{x\to 0^+}\frac{\frac{1}{\sin x}.\cos x}{-\frac{1}{x^2}}=0\right)$$

9 (7) : Let $I=\int_0^1(1-x^4)^7dx$

$$=[(1-x^4)^7(x)]_0^1-\int_0^1\left(\frac{d}{dx}(1-x^4)^7\,dx\int dx\right)dx$$

$$=0-\int_0^1\left(7(1-x^4)^6(-4x^3)\cdot x\right)dx$$

$$=-28\int_0^1(1-x^4-1)(1-x^4)^6dx$$

$$=-28\int_0^1(1-x^4)^7dx+28\int_0^1(1-x^4)^6dx$$

$$=-28I+28\int_0^1(1-x^4)^6dx \Rightarrow 29I=28\int_0^1(1-x^4)^6dx$$

Now, $\dfrac{29\int_0^1(1-x^4)^7dx}{4\int_0^1(1-x^4)^6dx}=\dfrac{28\int_0^1(1-x^4)^6dx}{4\int_0^1(1-x^4)^6dx}=7$

10. (4) : We have, $F(x)+F\left(x+\frac{1}{2}\right)=3$ ...(i)

Replace x by $x+\frac{1}{2}$ in (i), we get

$F\left(x+\frac{1}{2}\right)+F(x+1)=3$ ...(ii)

$\therefore$ From (i) and (ii), we get $F(x)=F(x+1)$ ...(iii)
$\Rightarrow$ $F(x)$ is periodic function.

Now, consider $I=\int_0^{1500}F(x)dx=1500\int_0^1F(x)dx$

$$=1500\left|\int_0^{1/2}F(x)dx+\int_{1/2}^1F(x)dx\right|$$

(Using property of periodic function)

Put $x=y+\frac{1}{2}$ in 2^{nd} integral, we get

$$I=1500\left|\int_0^{1/2}F(x)dx+\int_0^{1/2}F\left(y+\frac{1}{2}\right)dy\right|$$

$$=1500\int_0^{1/2}\left(F(x)+F\left(x+\frac{1}{2}\right)\right)dx=1500\int_0^{1/2}3dx$$

(Using (i))

Hence, $I=1500(3)\left(\frac{1}{2}\right)=750\times 3=2250$

11. (2) : We have, $f(x)=\dfrac{4^x}{4^x+2}$

$\Rightarrow f(a)=\dfrac{4^a}{4^a+2}$ and $f(1-a)=\dfrac{4^{1-a}}{4^{1-a}+2}$

$\therefore\ f(a)+f(1-a)=1$

Now, $I_1=\int_{f(1-a)}^{f(a)}x\,f(x(1-x))dx$

$$=\int_{f(1-a)}^{f(a)}(1-x)\int(1-x)(2-1+x)dx$$

$$=\int_{f(1-a)}^{f(a)}(1-x)\int(1-x)(x)dx$$

$$=\int_{f(1-a)}^{f(a)}((1-x)\,(x))dx-\int_{f(1-a)}^{f(a)}xf\,((1-x)(x))dx$$

$\Rightarrow\ I_1=I_2-I_1\ \Rightarrow\ 2I_1=I_2\ \Rightarrow\ \dfrac{I_2}{I_1}=2$

12. (2) : Let $I=\int_{-1}^1\left[x[1+\sin\pi x]+1\right]dx$

$$=\int_{-1}^1\left[x[1+\sin\pi x]+1\right]dx+\int_1^0\left[x[1+\sin\pi x]+1\right]dx$$

Now, $-1<x<0\ \Rightarrow\ [1+\sin\pi x]=0$
and $0<x<1\ \Rightarrow\ [1+\sin\pi x]=1$

$$\therefore\ I=\int_{-1}^0 1\cdot dx+\int_0^1[x+1]\,dx=\int_{-1}^0 1\cdot dx+\int_0^1[x]\,dx+\int_0^1 1\cdot dx$$

$$=[0-(-1)]+0+(1-0)=2$$

13. (1) : $\lim_{n\to\infty}\sum_{k=1}^{n}\frac{k^{1/a}\left\{n^{a-\frac{1}{a}}+k^{a-\frac{1}{a}}\right\}}{n^{a+1}}$

$$=\lim_{n\to\infty}\sum_{k=1}^{n}\frac{1}{n}\cdot\left\{\left(\frac{k}{n}\right)^{1/a}+\left(\frac{k}{n}\right)^{a}\right\}=\int_0^1(x^{1/a}+x^a)dx$$

$$=\left\{\frac{x^{(1/a)+1}}{\frac{1}{a}+1}+\frac{x^{a+1}}{a+1}\right\}_0^1=\frac{a}{a+1}+\frac{1}{a+1}=1$$

14. (2) : $\int \underset{\text{I}}{x}\,\underset{\text{II}}{f''(2x)}\,dx=x\frac{f'(2x)}{2}-\int\frac{f'(2x)}{2}dx$

$$=x\frac{f'(2x)}{2}-\frac{f(2x)}{4}$$

$$\int_0^1 x\,f''(2x)\,dx=\left[\frac{xf'(2x)}{2}-\frac{f(2x)}{4}\right]_0^1$$

$$=\left(\frac{f'(2)}{2}-\frac{f(2)}{4}\right)-\left(0-\frac{f(0)}{4}\right)=\left(\frac{5}{2}-\frac{3}{4}\right)-\left(0-\frac{1}{4}\right)=2$$

15. (2) : Since, $f(x)=\int_a^x\frac{1}{f(x)}dx$

Differentiating both sides w.r.t. x, then

$$f'(x)=\frac{1}{f(x)}\Rightarrow 2f(x)f'(x)=2$$

Integrating both sides, then $(f(x))^2=2x+c$

$\therefore\ f(x)=\sqrt{(2x+c)}$. But $\int_a^1\frac{1}{f(x)}dx=\sqrt{2}$

And $f(1)=\int_a^1\frac{1}{f(x)}dx=\sqrt{2}\ \Rightarrow\ \sqrt{(2+c)}=\sqrt{2}$

$\therefore\ c=0$. Then, $f(x)=\sqrt{2x}$

$\therefore\ f(2)=2$

16. (8) : Let $I=\int_0^{\pi}x\sin^5x\cdot\cos^6x\,dx$

$$=\int_0^{\pi}(\pi-x)\cdot\sin^5(\pi-x)\cdot\cos^6(\pi-x)dx$$

$$=\int_0^{\pi}(\pi-x)\sin^5x\cdot\cos^6x\,dx$$

$$=\int_0^{\pi}\pi\sin^5x\cdot\cos^6x\,dx-\int_0^{\pi}x\sin^5x\cos^6x\,dx$$

$$\Rightarrow 2I=\pi\cdot 2\int_0^{\pi/2}\sin^5x\cdot\cos^6x\,dx\Rightarrow I=\pi\left[\frac{4}{11}\cdot\frac{2}{9}\cdot\frac{1}{7}\right]=\frac{8\pi}{693}$$

$\therefore\ k=8$

17. (3) : Let $I_n=\int_0^{\pi/2}x^n\sin x\,dx$

Integrating by parts choosing $\sin x$ as the second function, we get

$$I_n=[x^n(-\cos x)]_0^{\pi/2}-\int_0^{\pi/2}nx^{n-1}(-\cos x)dx$$

$$=0+n\int_0^{\pi/2}x^{n-1}\cos x\,dx$$

Again integrating by parts, we get

$$I_n=n[x^{n-1}\sin x]_0^{\pi/2}-n(n-1)\int_0^{\pi/2}x^{n-2}\sin x\,dx$$

$$\Rightarrow I_n=n\left(\frac{\pi}{2}\right)^{n-1}-n(n-1)I_{n-2}$$

R.H.S. contains π^2. So putting $n=3$, we get

$$I_3=3\left(\frac{\pi}{2}\right)^2-3\times 2I_1=\frac{3\pi^2}{4}-6\int_0^{\pi/2}x\sin x\,dx$$

$= \frac{3\pi^2}{4} - 6[x(-\cos x) + \sin x]_0^{\pi/2}$

$= \frac{3\pi^2}{4} - 6(1) = \frac{3}{4}(\pi^2 - 8)$, which is true. Hence, $n = 3$

18. (2) : Using property $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$

$$I = \int_{-1}^{1} (1-x)^{1/2}(1+x)^{3/2}\,dx$$

$$\Rightarrow\ 2I = \int_{-1}^{1} (1+x)^{1/2}(1-x)^{1/2}[(1-x)+(1+x)]dx$$

$$\Rightarrow\ 2I = 2\int_{-1}^{1} \sqrt{1-x^2}\,dx\ \Rightarrow\ I = 2\int_{0}^{1} \sqrt{1-x^2}\,dx$$

Put $x = \sin\theta \Rightarrow dx = \cos\theta\, d\theta \Rightarrow\ I = 2\int_0^{\pi/2} \cos^2\theta\, d\theta = \frac{\pi}{2}$

19. (2) : $f(x) = \frac{x^9 - 3x^5 + 7x^3 - x}{\cos^2 x} + \sec^2 x$

$= \sec^2 x(x^9 - 3x^5 + 7x^3 - x) + \sec^2 x$

$$\Rightarrow \int_{-\pi/4}^{\pi/4} f(x)dx = \int_{-\pi/4}^{\pi/4} \sec^2 x\,dx = 2\int_0^{\pi/4} \sec^2 x\,dx$$

$= 2[\tan x]_0^{\pi/4} = 2$

20. A-r; B-q; C-s; D-p

21. A-q; B-s; C-r; D-p

(A) $\int \frac{\cos x}{\cos 3x}dx = \int \frac{dx}{4\cos^2 x - 3}$

(B) $\int \frac{(1-\sin^2 x)\cos x}{(1+\sin^2 x)^2}dx$

(C) $\int \frac{\sec^2 x dx}{1 + 4\tan^2 x}$

(D) $\int \left[\frac{(2\sin x + 1 + 3) - 3}{(2+\sin x)^2}\right]dx$

22. A-r; B-q; C-p; D-r

Put $\sqrt{x} = \sin\theta,\ x = \sin^2\theta$

$\sqrt{1-x} = \cos\theta,\ dx = \sin 2\theta\, d\theta$ etc.

23. A-r; B-s; C-p; D-q

(A) put $t + 2 = z$ (B) put $3t = z$

(C) Integrate by part (D) put $1/t = z$

❖❖

GK CORNER

Enhance Your General Knowledge with Current Updates!

SCHEMES AND MISSIONS

1. Amit Shah, Union Home Minister and Minister of Cooperation, inaugurated a computerization scheme for the offices of the Registrar of Cooperative Societies (RCSs) and Agriculture and Rural Development Banks (ARDBs) in New Delhi. The scheme aims to modernise the cooperative systems, fulfilling the vision of "Sahkaar Se Samriddhi" under PM Modi's leadership.

2. The Union Cabinet has recently extended the scheme to distribute subsidised sugar to Antyodya Anna Yojana (AAY) families for two more years till March 31, 2026. These initiatives are expected to significantly influence India's economy and facilitates access of sugar to the poorest of the poor.

3. The University Grants Commission (UGC) has issued 'Guidelines to Provide Equitable Opportunity for the Socio-Economically Disadvantaged Groups (SEDGs) in the Higher Education Institutions (HEIs) and it also recommended that all universities and HEIs in India follow these guidelines to improve educational opportunities for students from disadvantaged backgrounds.

4. The Government of Uttar Pradesh launched the Annual Mass Drug Administration (MDA) Campaign from February 5 to 15. The campaign was conducted in 17 districts of the state. The MDA campaign involved door-to-door visits by health workers to administer filariasis prevention medication to the entire population, excluding children under two years of age, pregnant women, and those facing with serious illnesses.

5. The All India Council for Technical Education (AICTE) has launched its "Support to Students for Participating in Competitions Abroad (SSPCA)" scheme for encouraging Indian students to participate in global competitions and it provides financial assistance of up to Rs 2 lakh per student on a reimbursement basis.

6. PM Narendra Modi has launched "Pradhan Mantri Suryodaya Yojana" which aimed at making electricity bills of poor and middle-class citizens 'zero' while helping them make substantial gains from the rooftops panels, installed on their terrace.

7. The Tamil Nadu Government has introduced guidelines for the "Ungalai Thedi, Ungal Ooril" (Come Look For You, In Your Village) outreach program. It is an initiative spearheaded by Chief Minister M.K. Stalin, aimed at strengthening public services and scheme implementation across the state.

8. Prithvi Vigyan Scheme of the Ministry of Earth Sciences has been approved by Government of India recently. Prithvi scheme comprehensively addresses the five components of Earth System Sciences: atmosphere, hydrosphere, geosphere, cryosphere, and biosphere. This holistic approach aims to enhance understanding and deliver reliable services for the country.

9. The 10th phase of Sagar Parikrama, has been inaugurated by India's Fisheries Minister, Parshottam Rupala, which focused on the welfare of fishing communities and coastal development.

10 The cabinet of Chhattisgarh has decided to start the "Shri Ramlala Darshan (Ayodhya Dham) Scheme" in the state. Under this scheme, people will get the chance to visit Ayodhya and have darshan of Shri Ramlala. Every year around 20,000 beneficiaries will be taken on a pilgrimage for Shri Ramlala darshan under this scheme.

11 The government of India has been extended the Production Linked Incentive (PLI) program for the automobile sector by one year through March 2028.

12 The Uttar Pradesh Tourism Department's Paying Guest Scheme aims to introduce the rich culinary heritage of Awadh to saints and visitors from India and abroad, providing them with a unique gastronomic experience.

Test Yourself!

1. Which of the following is not the feature of Shri Ramlala Darshan (Ayodhya Dham) Scheme which has started in Chhattisgarh?
   (a) A Shri Ramlala Darshan Committee has been formed in every district
   (b) Chhattisgarh residents between 18-75 years of age are eligible for this scheme
   (c) Every year, around 20,000 beneficiaries will be taken on a pilgrimage
   (d) Disabled individuals are not eligible for this scheme

2. Which Indian Southern state's government has introduced "Ungalai Thedi, Ungal Ooril" program?
   (a) Kerala (b) Telangana
   (c) Andhra Pradesh (d) Tamil Nadu

3. Which of the following members were not excluded in the Annual Mass Drug Administration (MDA) Campaign, which was held in Uttar Pradesh?
   (a) Pregnant women
   (b) People having serious illness
   (c) Population of above 60 years of age
   (d) Children under 2 years of age

4. Which phase of Sagar Parikrama, has been inaugurated by India's Fisheries Minister, Parshottam Rupala, recently?
   (a) 8^{th} (b) 9^{th}
   (c) 10^{th} (d) 11^{th}

5. Which scheme aimed at making electricity bills of poor and middle-class citizens 'zero'?
   (a) Pradhan Mantri Suryodaya Yojana
   (b) Pradhan Mantri Solar Yojana
   (c) Pradhan Mantri Suryast Yojana
   (d) Pradhan Mantri Suraj Yojana

6. What is the main motive of extending Subsidised Sugar Scheme?
   (a) To boost India's economy
   (b) To increase contribution of poor people in various government's activities
   (c) To facilitate access of sugar to the poorest of poor
   (d) Both (a) and (c)

7. How much financial assistance is given to each student under SSPCA scheme by AICTE?
   (a) Up to Rs. 1 lakh (b) Up to Rs. 2 lakh
   (c) Up to Rs. 2.5 lakh (d) Up to Rs. 3 lakh

8. Which of the following components does not address by PRITHVI scheme?
   (a) Atmosphere (b) Hydrosphere
   (c) Geosphere (d) None of these

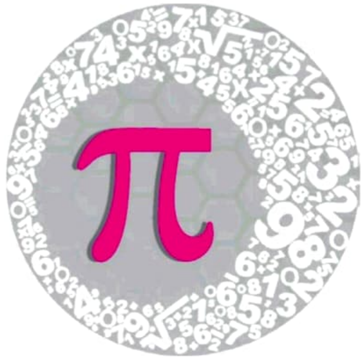
9. What is the vision of Scheme which was recently launched to modernise the cooperative system?
   (a) Sahkaar Se Unnati (b) Sahkaar Se Samriddhi
   (c) Saath Se Samriddhi (d) Saath Se Unnati

10. Which organisation has issued 'Guidelines to Provide Equitable Opportunity for the Socio-Economically Disadvantaged Groups (SEDGs) in the Higher Education Institutions (HEIs) recently?
    (a) Quality Council of India (QCI)
    (b) National Institute of Educational Planning and Administration (NIEPA)
    (c) Tata Institute of Fundamental Research (TIFR)
    (d) University Grants Commission (UGC)

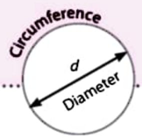
Answer Key

1. (d) 2. (d) 3. (c) 4. (c) 5. (a)
6. (d) 7. (b) 8. (d) 9. (b) 10. (d)

Happy Pi DAY!



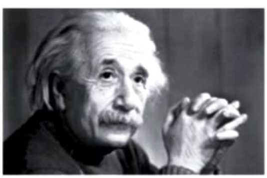
Welcome to **Pi** Day. It will be observed on 14th March (3/14). This annual celebration honors the mathematical constant π, representing the ratio of a circle's circumference to its diameter. Embrace the joy of mathematics, indulge in circular treat and join the global enthusiasm for the intriguing world of **Pi**.



π

The two famous physicists in their own changed the way we see the universe share this day, **14th March**.

This year, we will celebrate 145th birthday of great German-born theoretical physicist and mathematician Albert Einstein.



Albert Einstein's Birthday
March 14, 1879

Coincidently 14th March also marks the day when the legendary scientist Stephen Hawking passed away in 2018, after suffering from a rare slow progressing neurodegenerative disease for more than five decades.

Stephen Hawking's Death Anniversary
March 14, 2018

MONTHLY TEST DRIVE

XI

This specially designed column enables students to self analyse their extent of understanding of specified chapter. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Total Marks : 80

Series 9 : Statistics and Probability

Time Taken : 60 Min.

Only One Option Correct Type

1. If the ratio of mode and median of a distribution is 6 : 5, then the ratio of its mean and median is

(a) 8 : 9 (b) 9 : 10
(c) 9 : 7 (d) 8 : 11

2. The scores of a batsman in ten innings are: 38, 70, 48, 34, 42, 55, 63, 46, 54, 44. Find the mean deviation about the median.

(a) 7.8 (b) 8.6 (c) 6.4 (d) 3.4

3. The mean and variance of 7 observations are 8 and 16 respectively. If 5 of the observations are 2, 4, 10, 12 and 14, then find the remaining two observations.

(a) 2, 6 (b) 6, 8 (c) 8, 2 (d) 8, 4

4. If $P(B)=\frac{3}{4}, P(A\cap B\cap \bar{C})=\frac{1}{3}$ and $P(\bar{A}\cap B\cap \bar{C})=\frac{1}{3}$, then $P(B\cap C)=$

(a) $\frac{1}{12}$ (b) $\frac{1}{6}$ (c) $\frac{1}{15}$ (d) $\frac{1}{9}$

5. The letters of the word "QUESTION" are arranged in a row at random. The probability that there are exactly two letters between Q and S is

(a) $\frac{1}{14}$ (b) $\frac{5}{7}$ (c) $\frac{1}{7}$ (d) $\frac{5}{28}$

6. A person writes four letters and addresses on 4 envelopes. If the letters are placed in the envelopes at random, what is the probability that all letters are not placed in the right envelopes?

(a) $\frac{1}{24}$ (b) $\frac{11}{24}$ (c) $\frac{15}{24}$ (d) $\frac{23}{24}$

One or More Than One Option(s) Correct Type

7. If A, B are two events such that $P(A\cup B)\geq 3/4$and $\frac{1}{8}\leq P(A\cap B)\leq\frac{3}{8}$, then

(a) $P(A)+P(B)\leq\frac{11}{8}$ (b) $P(A)+P(B)\leq\frac{3}{8}$
(c) $P(A)+P(B)\geq\frac{7}{8}$ (d) None of these

8. If the probability for A to fail in an examination is 0.2 and that for B is 0.3, then the probability that either A or B fails, is

(a) 0.38 (b) 0.44 (c) 0.50 (d) 0.94

9. Two persons A and B have $n+1$ and n coins respectively, which they toss simultaneously. Then the probability that A will have more heads than B is

(a) 1/2 (b) > 1/2 (c) < 1/2 (d) > 1/3

10. A natural number is selected at random from the first 100 natural numbers. Let A, B and C denote the events of selection of even number, a multiple of 3 and a multiple of 5, respectively. Then

(a) $P(A\cap B)=\frac{4}{25}$ (b) $P(B\cap C)=\frac{3}{50}$
(c) $P(C\cap A)=\frac{1}{10}$ (d) $P(A\cup B\cup C)=\frac{37}{50}$

Recipe for Success

"To succeed in life and achieve results, you must understand and master three mighty forces – desire, belief, and expectation."

–Dr. A.P.J. Abdul Kalam

11. The variable x takes two values x_1 and x_2 with frequencies f_1 and f_2, respectively. If σ denotes the standard deviation of x, then

(a) $\sigma^2 = \dfrac{f_1x_1^2 + f_2x_2^2}{f_1 + f_2} - \left(\dfrac{f_1x_1 + f_2x_2}{f_1 + f_2}\right)^2$

(b) $\sigma^2 = \dfrac{f_1f_2}{(f_1 + f_2)^2}(x_1 - x_2)^2$

(c) $\sigma^2 = \dfrac{(x_1 - x_2)^2}{f_1 + f_2}$

(d) None of these

12. Coefficient of range of the data 5, 2, 3, 4, 6, 8, 10 is of the form $\dfrac{p}{q}$, then

(a) $p = 2$ (b) $q = 3$
(c) $p = 1$ (d) $q = 5$

13. Mean of the numbers 1, 2, 3, ..., n with respective weights $1^2 + 1, 2^2 + 2, 3^2 + 3, ..., n^2 + n$ is

(a) $\dfrac{3n(n+1)}{2(2n+1)}$ (b) $\dfrac{3n^2+7n+2}{2(2n+4)}$

(c) $\dfrac{3n+1}{4}$ (d) $\dfrac{3n+1}{2}$

Comprehension Type

Paragraph for Q. No. 14 and 15

Consider the series $x_1, x_2, ..., x_n$ whose mean is x and variance is σ^2.

14. If each observation is multiplied by 15, then new mean is

(a) $5\overline{x}$ (b) $15\overline{x}$
(c) $225\overline{x}$ (d) None of these

15. If 5 is added in each observation, then the new variance is

(a) σ^2 (b) $\sigma^2 + 5$
(c) $\sigma^2 - 5$ (d) None of these

Matrix Match Type

16. Match the following.

	Column-I		Column-II
(P)	A card is drawn from a pack of 100 cards numbered 1 to 100. Find the probability of drawing a number which is a perfect square.	(1)	$\frac{1}{6}$
(Q)	Find the probability that in a random arrangement of letters of the word 'UNIVERSITY', two 'I's do not come together.	(2)	$\frac{1}{10}$
(R)	If letters of the word 'PENCIL' are arranged in random order, what is the probability that N is always next to E?	(3)	$\frac{4}{5}$

	P	Q	R		P	Q	R
(a)	1	3	2	(b)	2	3	1
(c)	2	1	3	(d)	1	2	3

Numerical Answer Type

17. Six dice are thrown simultaneously. The probability that all dice show different faces is $n/324$. Then n is _____.

18. Five persons entered the lift cabin on the ground floor of an 8-floor building. Each of them independently and with equal probability can leave the cabin at any floor beginning with the first. If the favourable outcomes of all five persons leaving at different floors is 21λ, then λ = _____.

19. In the frequency distribution of the discrete data given below, the frequency k against value 0 is missing.

Variable (x)	0	1	2	3	4	5
Frequency (f)	k	20	40	40	20	4

If the mean is 2.5, then the missing frequency k is ____.

20. The mean of 5 observations is 5 and their variance is 9.2. If three of the observations are 1, 2 and 6, then the mean deviation from the mean of the data, is ____ .

◆◆

Keys are published in this issue. Search now! ☺

SELF CHECK

No. of questions attempted ......
No. of questions correct ......
Marks scored in percentage ......

Check your score! If your score is

> 90%	EXCELLENT WORK !	You are well prepared to take the challenge of final exam.
90-75%	GOOD WORK !	You can score good in the final exam.
74-60%	SATISFACTORY !	You need to score more next time.
< 60%	NOT SATISFACTORY!	Revise thoroughly and strengthen your concepts.

CBSE warm-up!

Exam on 9th March

CLASS-XII

Practice questions for CBSE Exams as per the latest pattern and rationalised syllabus by CBSE for the academic session 2023-24.

Practice Paper 2023-24

Time Allowed : 3 hours
Maximum Marks : 80

General Instructions

(a) This question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
(b) Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
(c) Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
(d) Section C has 6 Short Answer (SA)-type questions of 3 marks each.
(e) Section D has 4 Long Answer (LA)-type questions of 5 marks each.
(f) Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

SECTION A (MULTIPLE CHOICE QUESTIONS)

Each question carries 1 mark

1. If $A=[a_{ij}]=\begin{bmatrix}2 & -1\\ -3 & 4\\ 1 & 2\end{bmatrix}$ and $B=[b_{ij}]=\begin{bmatrix}2 & 3 & -5\\ 1 & 4 & 9\\ 0 & 7 & -2\end{bmatrix}$, then the value of $a_{11}\,b_{11} + a_{22}\,b_{22}$ is

(a) 8 (b) 20 (c) 16 (d) 24

2. If $\begin{bmatrix}x-y & 2y\\ 2y+z & x+y\end{bmatrix}=\begin{bmatrix}1 & 4\\ 9 & 5\end{bmatrix}$, then write the value of $x+y+z$.

(a) 10 (b) 9 (c) 8 (d) 7

3. If $y=\sec^{-1}\left(\frac{1}{1-2x^2}\right)$, then $\frac{dy}{dx}=$

(a) $\frac{1}{\sqrt{1-x^2}}$ (b) $\frac{2}{\sqrt{1-x^2}}$

(c) $\frac{1}{\sqrt{1+x^2}}$ (d) $\frac{2}{\sqrt{1+x^2}}$

4. If $\lambda(3\hat{i}+2\hat{j}-6\hat{k})$ is a unit vector, then the values of λ are

(a) $\pm\frac{1}{7}$ (b) ± 7

(c) $\pm\sqrt{43}$ (d) $\pm\frac{1}{\sqrt{43}}$

5. $\int(\sin 2x-4e^{3x})dx$ is equal to

(a) $\frac{1}{2}\cos 2x-\frac{4}{3}e^{3x}+c$

(b) $-\frac{1}{2}\cos 2x+\frac{4}{3}e^{3x}+c$

(c) $-\frac{1}{2}\cos 2x-\frac{4}{3}e^{3x}+c$

(d) None of these

6. The general solution of $\frac{dy}{dx}=2xe^{x^2-y}$ is

(a) $e^{x^2-y}=c$ (b) $e^{-y}+e^{x^2}=c$

(c) $e^{y}=e^{x^2}+c$ (d) $e^{x^2+y}=c$

7. If the direction cosines of a vector of magnitude 3 are $\frac{2}{3}, \frac{-a}{3}, \frac{2}{3}, a > 0$, then the vector is

(a) $2\hat{i}+\hat{j}+2\hat{k}$ (b) $2\hat{i}-\hat{j}+2\hat{k}$

(c) $\hat{i}-2\hat{j}+2\hat{k}$ (d) $\hat{i}+2\hat{j}+2\hat{k}$

8. The direction cosines of the vector $(2\hat{i}+2\hat{j}-\hat{k})$ are

(a) $<\frac{2}{3}, \frac{2}{3}, \frac{-1}{3}>$ (b) $<\frac{-2}{3}, \frac{2}{3}, \frac{1}{3}>$

(c) $<\frac{1}{3}, \frac{1}{3}, \frac{2}{3}>$ (d) $<\frac{-1}{3}, \frac{2}{3}, \frac{1}{3}>$

9. Which of the following sets is convex?

(a) $\{(x, y) : x^2 + y^2 \geq 1\}$

(b) $\{(x, y) : y^2 \geq x\}$

(c) $\{(x, y) : 3x^2 + 4y^2 \geq 5\}$

(d) $\{(x, y) : y \geq 2, y \leq 4\}$

10. If a matrix $A = [1\ 2\ 3]$, then the matrix AA' (where A' is the transpose of A) is

(a) 14 (b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}$ (d) [14]

11. Let $f(x) = \begin{cases} ax^2 + 1, & x > 1 \\ x + \frac{1}{2}, & x \leq 1 \end{cases}$.

Then $f(x)$ is derivable at $x = 1$, if $a =$

(a) 2 (b) 1 (c) 0 (d) $\frac{1}{2}$

12. The order and degree respectively of the differential equation $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^{1/4} + x^{1/5} = 0$ are

(a) 2 and 4 (b) 2 and 2

(c) 2 and 3 (d) 3 and 3

13. Find the magnitude of $(\hat{i}+3\hat{j}-2\hat{k}) \times (-\hat{i}+3\hat{k})$.

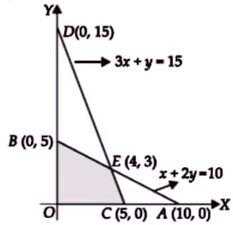
(a) 9 (b) $\sqrt{91}$ (c) $\sqrt{89}$ (d) $3\sqrt{10}$

14. If $f(x) = \begin{cases} mx + 1, & x \leq \frac{\pi}{2} \\ \sin x + n, & x > \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$, then

(a) $m = 1, n = 0$ (b) $m = \frac{n\pi}{2} + 1$

(c) $n = m\frac{\pi}{2}$ (d) $m = n = \frac{\pi}{2}$

15. Maximize $Z = 3x + 2y$, if the shaded region is the feasible region.



(a) Max. $Z = 10$ (b) Max. $Z = 15$

(c) Max. $Z = 18$ (d) None of these

16. Let A and B be independent events with $P(A) = 1/4$ and $P(A \cup B) = 2P(B) - P(A)$. Find $P(B)$.

(a) $\frac{1}{5}$ (b) $\frac{2}{5}$ (c) $\frac{3}{5}$ (d) $\frac{4}{5}$

17. If l, m and n are the direction cosines of line l, then equation of the line (l) passing through (x_1, y_1, z_1) is

(a) $\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$

(b) $\left(\frac{x-x_1}{l}\right)\left(\frac{y-y_1}{m}\right) = \left(\frac{z-z_1}{n}\right)$

(c) $\frac{x+x_1}{l} = \frac{y+y_1}{m} = \frac{z+z_1}{n}$

(d) None of these

18. If $A = \begin{bmatrix} 4 & k & k \\ 0 & k & k \\ 0 & 0 & k \end{bmatrix}$ and det $(A) = 256$, then $|k|$ equals

(a) 4 (b) 5 (c) 6 (d) 8

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

19. Consider the system of equations,
$x + y + z = 1, 2x + 2y + 2z = 2, 4x + 4y + 4z = 3.$

Assertion (A) : The above system has infinitely many solutions.

Reason (R) : For the above system, det $A = 0$ and (adj A) $B = O$, where

$$A=\begin{bmatrix}1 & 1 & 1\\ 2 & 2 & 2\\ 4 & 4 & 4\end{bmatrix} \text{ and } B=\begin{bmatrix}1\\ 2\\ 3\end{bmatrix}$$

20. **Assertion (A) :** Let $A = \{-1, 1, 2, 3\}$ and $B = \{1, 4, 9\}$, where $f : A \to B$ given by $f(x) = x^2$, then f is a many-one function.

Reason (R) : If $x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$, for every $x_1, x_2 \in$ domain, then f is one-one or else many-one.

SECTION B

This section comprises of very short answer type questions (VSA) of 2 marks each

21. Evaluate : $\int \frac{x^2}{\sqrt{1-x^2}}\,dx$

22. Let $A = R - \{3\}$, $B = R - \{1\}$ and $f : A \to B$ be defined by $f(x)=\frac{x-2}{x-3}$. Then, prove that f is surjective.

OR

Find the value of $\cos^{-1}\left(\frac{-1}{2}\right)+2\sin^{-1}\left(\frac{-1}{2}\right)$.

23. Find the interval in which $f(x) = \frac{\log x}{x}$ is strictly increasing.

24. Find the solution of differential equation $y^3 - \frac{dy}{dx} = x^2\frac{dy}{dx}$.

25. Evaluate : $\int x^2 (ax+b)^{-2}\,dx$

OR

Solve the differential equation $\frac{dy}{dx} + y\cot x = 2\cos x$.

SECTION C

This section comprises of short answer type questions (SA) of 3 marks each

26. Find the value of k, for which

$$f(x)=\begin{cases}\frac{\sqrt{1+kx}-\sqrt{1-kx}}{x}, & \text{if } -1\le x<0\\ \frac{2x+1}{x-1}, & \text{if } 0\le x<1\end{cases},$$

is continuous at $x = 0$.

27. Solve the initial value problem $2xy + y^2 - 2x^2\frac{dy}{dx} = 0,\ y(1) = 2$.

OR

Evaluate : $\int \frac{\sin 2x}{a^2\sin^2 x + b^2\cos^2 x}\,dx$

28. Find the value of $\int_{\pi/4}^{3\pi/4} \frac{x}{1+\sin x}\,dx$.

29. Find the solution of the differential equation $y^2\,dx + (xy + x^2)\,dy = 0$.

OR

Find the particular solution of $\ln\left(\frac{dy}{dx}\right) = 3x + 4y,\ y(0) = 0$.

30. Solve the following linear programming problem graphically.
Maximize $Z = 11x + 9y$
subject to constraints : $180x + 120y \le 1500$,
$x + y \le 10,\ x, y \ge 0$

OR

Find the number of points at which the objective function $Z = 3x + 2y$ can be maximized subject to $3x + 5y \le 15$, $5x + 2y \le 20$, $x \ge 0$, $y \ge 0$.

31. Three events A, B and C have probabilities $\frac{2}{5}, \frac{1}{3}$ and $\frac{1}{2}$ respectively. Given that $P(A \cap C) = \frac{1}{5}$ and $P(B \cap C) = \frac{1}{4}$, find the value of $P(C|B)$ and $P(\bar{A}\cap\bar{C})$.

SECTION D

This section comprises of long answer type questions (LA) of 5 marks each

32. Let $A = R - \{2\}$ and $B = R - \{1\}$. If $f : A \to B$ is a function defined by $f(x)=\frac{x-1}{x-2}$, then show that f is one-one and onto.

OR

Show that the relation R in the set of real numbers, defined as $R = \{(a, b) : a \le b^2\}$ is neither reflexive, nor symmetric, nor transitive.

33. Find the matrix A such that

$$\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix} A = \begin{bmatrix} -1 & -8 \\ 1 & -2 \\ 9 & 22 \end{bmatrix}.$$

34. Make a rough sketch of the region bounded by the curve $y = x^2$, line $x = 2$ and x-axis. Hence, find the area of the region using integration.

35. Using vectors, find the area of the ΔABC with vertices $A(1, 2, 3)$, $B(2, -1, 4)$ and $C(4, 5, -1)$.

OR

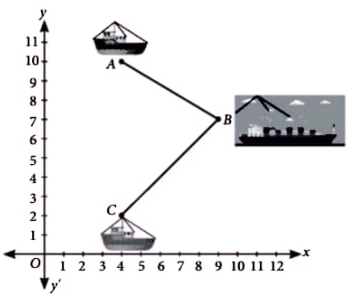
Find the direction cosines l, m, n of a line which are connected by the relations $l + m + n = 0$, $2mn + 2ml - nl = 0$.

SECTION E

This section comprises of 3 case-study/passage based questions of 4 marks each with sub parts. The first two case study questions have three sub parts (i), (ii), (iii) of marks 1, 1, 2 respectively. The third case study question has two sub parts of 2 marks each.

36. Read the following passage and answer the questions given below.

A barge is pulled into harbour by two tug boats as shown in the figure.



(i) Find the vector $\overrightarrow{AC}$.

(ii) If $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$, then find its unit vector.

(iii) If $\vec{a} = 4\hat{i} + 3\hat{j}$ and $\vec{b} = 3\hat{i} + 4\hat{j}$, then find $|\vec{a}| + |\vec{b}|$.

OR

If $\vec{a} = 2\hat{i} + 2\hat{j}$ and $\vec{b} = \hat{i} + \hat{j}$, then find $|\vec{a} - \vec{b}|$.

37. Read the following passage and answer the questions given below.

An open water tank of aluminium sheet of negligible thickness, with a square base and vertical sides, is to be constructed in a farm for irrigation. It should hold 32000 l of water, that comes out from a tube well.

(i) If the length, width and height of the open tank be x, x and y m respectively, then find the total surface area of tank.

(ii) Find the relation between x and y.

(iii) Find the length when the outer surface area of tank will be minimum.

OR

If cost of aluminium sheet is ₹ 360/m^2, then find the minimum cost for the construction of tank.

38. Read the following passage and answer the questions given below.

In a family, on the occasion of Diwali celebration father, mother, daughter and son line up at random for a family photograph.

(i) Find the probability that son is at one end, given that father and mother are in the middle.

(ii) Find the probability that mother is at left end, given that son and daughter are together.

SOLUTIONS

1. **(b) :** We have, $A = \begin{bmatrix} 2 & -1 \\ -3 & 4 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 & -5 \\ 1 & 4 & 9 \\ 0 & 7 & -2 \end{bmatrix}$

Here, $a_{11} = 2$, $a_{22} = 4$, $b_{11} = 2$, $b_{22} = 4$

$\therefore \quad a_{11}b_{11} + a_{22}b_{22} = 2(2) + 4(4) = 4 + 16 = 20$

2. (a) : By definition of equal matrices, we have

$x - y = 1$ ...(i) ; $2y = 4 \Rightarrow y = 2$ ...(ii)

$2y + z = 9$ ...(iii) ; $x + y = 5$ ...(iv)

Solving equations (i) and (iv), we get

$2x = 6 \Rightarrow x = 3$

From equations (ii) and (iii), we get

$2(2) + z = 9 \Rightarrow z = 5$

$\therefore \quad x + y + z = 3 + 2 + 5 = 10$

3. (b) : $y = \sec^{-1}\left(\frac{1}{1-2x^2}\right)$

Put $x = \sin\theta$

$\therefore \quad y = \sec^{-1}\left(\frac{1}{1-2\sin^2\theta}\right) = \sec^{-1}\left(\frac{1}{\cos 2\theta}\right)$

$= \sec^{-1}(\sec 2\theta) = 2\theta$ or $y = 2\sin^{-1}x$

Taking derivative w.r.t. x, we get $\frac{dy}{dx} = \frac{2}{\sqrt{1-x^2}}$.

4. (a) : $|\lambda \cdot (3\hat{i} + 2\hat{j} - 6\hat{k})| = 1$

$\Rightarrow \ |\lambda|\sqrt{9+4+36} = 1 \Rightarrow |\lambda|\sqrt{49} = 1$

$\Rightarrow \ 7|\lambda| = 1 \Rightarrow \lambda = \pm\frac{1}{7}$

5. (c) : Let $I = \int(\sin 2x - 4e^{3x})\,dx$

$= \int \sin 2x\,dx - 4\int e^{3x}\,dx = -\frac{\cos 2x}{2} - \frac{4}{3}e^{3x} + c$

6. (c) : We have, $\frac{dy}{dx} = 2x\,e^{x^2 - y}$

$\Rightarrow \frac{dy}{dx} = 2xe^{x^2} \cdot e^{-y} \ \Rightarrow \ e^y\,dy = 2x\,e^{x^2}\,dx$

Integrating both sides, we get $e^y = e^{x^2} + c$

7. (b) : We have, $l = \frac{2}{3}, m = -\frac{a}{3}$ and $n = \frac{2}{3}, a > 0$

We know that $l^2 + m^2 + n^2 = 1$

$\Rightarrow \ \frac{4}{9} + \frac{a^2}{9} + \frac{4}{9} = 1 \Rightarrow \frac{a^2}{9} + \frac{8}{9} = 1 \Rightarrow a^2 = 1$

$\Rightarrow a = \pm 1 \Rightarrow a = 1$ $\quad (\because a > 0)$

Magnitude of given vector = 3

$\therefore$ Required vector $= 3\left(\frac{2}{3}\hat{i} - \frac{1}{3}\hat{j} + \frac{2}{3}\hat{k}\right) = 2\hat{i} - \hat{j} + 2\hat{k}$

8. (a) : Direction cosines of $(2\hat{i} + 2\hat{j} - \hat{k})$ are

$\frac{2}{\sqrt{(2)^2+(2)^2+(-1)^2}}, \frac{2}{\sqrt{(2)^2+(2)^2+(-1)^2}}, \frac{-1}{\sqrt{(2)^2+(2)^2+(-1)^2}}$

$= \frac{2}{\sqrt{4+4+1}}, \frac{2}{\sqrt{4+4+1}}, \frac{-1}{\sqrt{4+4+1}}$, *i.e.*, $\frac{2}{3}, \frac{2}{3}, \frac{-1}{3}$

9. (d)

10. (d) : $A = [1 \ 2 \ 3]$ and $A' = \begin{bmatrix}1\\2\\3\end{bmatrix}$

So, $AA' = [1 \ 2 \ 3]\begin{bmatrix}1\\2\\3\end{bmatrix} = [1 + 4 + 9] = [14]$

11. (d) : Given $f(x)$ is derivable at $x = 1$.

$\therefore$ (R.H.D. at $x = 1$) = (L.H.D. at $x = 1$)

$\Rightarrow \ \lim_{x\to1^+}\frac{f(x)-f(1)}{x-1} = \lim_{x\to1^-}\frac{f(x)-f(1)}{x-1}$

$\Rightarrow \ \lim_{h\to0}\frac{f(1+h)-f(1)}{1+h-1} = \lim_{h\to0}\frac{f(1-h)-f(1)}{1-h-1}$

$\Rightarrow \ \lim_{h\to0}\left[\frac{a(1+h)^2+1-\frac{3}{2}}{h}\right] = \lim_{h\to0}\left[\frac{(1-h)+\frac{1}{2}-\frac{3}{2}}{-h}\right]$

$\Rightarrow \ \lim_{h\to0}\frac{a-\frac{1}{2}}{h} + 2a = 1 \Rightarrow \lim_{h\to0}\frac{a-\frac{1}{2}}{h} = 1-2a$

$\Rightarrow \ a - \frac{1}{2} = 0 \Rightarrow a = \frac{1}{2}$

12. (a) : We have, $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^{1/4} + x^{1/5} = 0$

$\Rightarrow \ \left(\frac{dy}{dx}\right)^{1/4} = -\left(x^{1/5} + \frac{d^2y}{dx^2}\right)$

On squaring both sides, we get

$\left(\frac{dy}{dx}\right)^{1/2} = \left(x^{1/5} + \frac{d^2y}{dx^2}\right)^2$

On squaring both sides again, we get

$\frac{dy}{dx} = \left(x^{1/5} + \frac{d^2y}{dx^2}\right)^4$

$\therefore$ Order = 2 and degree = 4

13. (b) : Let $\vec{a} = (\hat{i} + 3\hat{j} - 2\hat{k}) \times (-\hat{i} + 0\hat{j} + 3\hat{k})$

$= \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 3 & -2\\ -1 & 0 & 3\end{vmatrix} = (9-0)\hat{i} - (3-2)\hat{j} + (0+3)\hat{k} = 9\hat{i} - \hat{j} + 3\hat{k}$

$\therefore \ |\vec{a}| = \sqrt{9^2 + (-1)^2 + 3^2} = \sqrt{91}$

14. (c) : Since, $f(x)$ is continuous at $x=\frac{\pi}{2}$.

So, $\lim_{x\to\frac{\pi^-}{2}} f(x)=\lim_{x\to\frac{\pi^+}{2}} f(x) \Rightarrow m\frac{\pi}{2}+1=\sin\frac{\pi}{2}+n$

$\Rightarrow m\frac{\pi}{2}+1=1+n \Rightarrow \frac{m\pi}{2}=n$

15. (c) : Let us find the value of Z at corner points of the feasible region.
Clearly, $Z(O) = 0$; $Z(C) = 3 \times 5 = 15$;
$Z(E) = 3\times4+2\times3 = 18$ and $Z(B) = 2 \times 5 = 10$
Thus, maximum value of Z is 18 at point $E(4, 3)$.

16. (b) : We have, $P(A) = 1/4$
Now, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$= P(A) + P(B) - P(A)\,P(B)$ ($\because$ A, B are independent)
$\Rightarrow 1/4 + P(B) - (1/4)\,P(B) = 2P(B) - 1/4$ (Given)

$\frac{1}{4}+\frac{1}{4}=2P(B)-\frac{3}{4}P(B)$

$\Rightarrow \frac{1}{2}=\frac{5}{4}P(B) \Rightarrow P(B)=\frac{2}{5}$

17. (a) : If l, m, n are the direction cosines of the line, then the equation of the line which passes through (x_1, y_1, z_1) is

$\frac{x-x_1}{l}=\frac{y-y_1}{m}=\frac{z-z_1}{n}$

18. (d) : We have, $\det(A) = \begin{vmatrix}4 & k & k\\ 0 & k & k\\ 0 & 0 & k\end{vmatrix}$

Now, expanding along C_1, we get
$\Rightarrow \det(A) = 4(k^2)$
But we have, $\det(A) = 256$ (Given)
$\therefore$ On comparing, we get
$4k^2 = 256 \Rightarrow k^2 = 64 \Rightarrow k = \pm8$
$\therefore |k| = 8$

19. (d) : The given system of equations can be written in matrix form as

$$\begin{bmatrix}1 & 1 & 1\\ 2 & 2 & 2\\ 4 & 4 & 4\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}1\\ 2\\ 3\end{bmatrix}, \text{ i.e., } AX = B,$$

where $A=\begin{bmatrix}1 & 1 & 1\\ 2 & 2 & 2\\ 4 & 4 & 4\end{bmatrix}$, $X=\begin{bmatrix}x\\ y\\ z\end{bmatrix}$ and $B=\begin{bmatrix}1\\ 2\\ 3\end{bmatrix}$

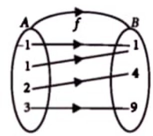
Here, $\det A=\begin{vmatrix}1 & 1 & 1\\ 2 & 2 & 2\\ 4 & 4 & 4\end{vmatrix}=(2\times4)\begin{vmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{vmatrix}$

$= 8 \times \{1(1-1) - 1(1-1) + 1(1-1)\} = 0$

Also, $(\text{adj } A)B=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}^T\begin{bmatrix}1\\ 2\\ 3\end{bmatrix}=O$

Thus, Statement-II is true.
However, the Statement-I is not true as the given system is inconsistent. Here, the third equation contradicts the first and second which are identical.
So, the given system has no solution.

20. (a) : Here $f(-1) = 1, f(1) = 1$, $f(2) = 4, f(3) = 9$
Two elements 1 and -1 have the same image $1 \in B$.
So, f is a many-one function.
Statement-I and II are true and Statement-II is the correct explanation of Statement-I.



21. Let $I=\int\frac{x^2}{\sqrt{1-x^2}}dx$

Put $x = \sin\theta$. Then, $dx = \cos\theta\, d\theta$

$\therefore I=\int\frac{\sin^2\theta}{\sqrt{1-\sin^2\theta}}\cos\theta\, d\theta$

$=\int\sin^2\theta\, d\theta=\frac{1}{2}\int(1-\cos2\theta)d\theta$ $\left[\because \sqrt{1-\sin^2\theta}=\cos\theta\right]$

$\Rightarrow I=\frac{1}{2}\left(\theta-\frac{1}{2}\sin2\theta\right)+C=\frac{1}{2}\theta-\frac{1}{2}\sin\theta\cos\theta+C$

$\Rightarrow I=\frac{1}{2}\sin^{-1}x-\frac{1}{2}x\sqrt{1-x^2}+C$

22. Let y be any arbitrary element in B, then $f(x) = y$

$\Rightarrow \frac{x-2}{x-3}=y \Rightarrow x=\frac{3y-2}{y-1}, (y\neq1)$

Clearly, $\forall\, y\in B$, $x=\frac{3y-2}{y-1}\in A$, thus for all $y\in B$, there exists $x\in A$ such that $f(x)=f\left(\frac{3y-2}{y-1}\right)=\frac{\frac{3y-2}{y-1}-2}{\frac{3y-2}{y-1}-3}=y$

$\Rightarrow f(x) = y$

Thus, every element in the co-domain B has its pre-image in A, so f is a surjective.

OR

Principal value of $\cos^{-1}\left(\frac{-1}{2}\right)$ is $\frac{2\pi}{3}$ and principal value of $\sin^{-1}\left(\frac{-1}{2}\right)$ is $\left(\frac{-\pi}{6}\right)$.

$\therefore$ Principal value of $\cos^{-1}\left(\frac{-1}{2}\right)+2\sin^{-1}\left(\frac{-1}{2}\right)$

$$=\frac{2\pi}{3}+\left(2\times\frac{-\pi}{6}\right)=\frac{2\pi}{3}-\frac{\pi}{3}=\frac{\pi}{3}$$

23. $f(x)=\frac{\log x}{x} \Rightarrow f'(x)=-\frac{\log x}{x^2}+\frac{1}{x^2}=\frac{1-\log x}{x^2}$

$f(x)$ is strictly increasing if $f'(x)>0$ *i.e.*, if $\frac{1-\log x}{x^2}>0$
i.e., if $1-\log x>0$ if $\log x<1$ *i.e.*, if $x<e$
Also, $f(x)$ is defined for $x>0$
$\therefore$ $f(x)$ is increasing on $(0, e)$.

24. We have, $y^3-\frac{dy}{dx}=x^2\frac{dy}{dx}\Rightarrow y^3=(1+x^2)\frac{dy}{dx}$

$$\Rightarrow \int\frac{dx}{1+x^2}=\int\frac{dy}{y^3}+c\Rightarrow \tan^{-1}x=\frac{-1}{2y^2}+c$$

25. Let $I=\int\frac{x^2}{(ax+b)^2}dx$

Put $ax+b=t \Rightarrow dx=\frac{1}{a}dt$

$$\therefore\ I=\frac{1}{a^3}\int\frac{(t-b)^2}{t^2}dt=\frac{1}{a^3}\int\left(1+\frac{b^2}{t^2}-\frac{2b}{t}\right)dt$$

$$=\frac{1}{a^3}\left(t-\frac{b^2}{t}-2b\log t\right)+C$$

$$=\frac{1}{a^3}\left(ax+b-\frac{b^2}{ax+b}-2b\log|ax+b|\right)+C$$

where C is constant

OR

The given D.E. is $\frac{dy}{dx}+y\cot x=2\cos x$ ...(i)

This is a linear differential equation of the form $\frac{dy}{dx}+Py=Q$, where $P=\cot x$; $Q=2\cos x$

Now, I.F. $=e^{\int P dx}=e^{\int\cot x dx}=e^{\log\sin x}=\sin x$

$\therefore$ The solution of equation (i) is given by

$$y\times \text{I.F.}=\int Q\times \text{I.F.}\,dx+c$$

$$\Rightarrow y\sin x=\int 2\cos x\cdot\sin x\,dx+c$$

$$=\int\sin 2x\,dx+c=-\frac{1}{2}\cos 2x+c$$

$$\Rightarrow y=-\frac{1}{2}\cos 2x\ \text{cosec}\,x+c\,\text{cosec}\,x$$

This is the required solution of the given differential equation.

26. $\because f(x)$ is continuous at $x=0$

$$\therefore \lim_{x\to 0^+}f(x)=f(0)=\lim_{x\to 0^-}f(x)$$

Now, $f(0)=\frac{2\times 0+1}{0-1}=-1$

$$\text{R.H.L.}=\lim_{x\to 0^+}f(x)=\lim_{h\to 0}f(0+h)=\lim_{h\to 0}\frac{2h+1}{h-1}=-1$$

$$\text{L.H.L.}=\lim_{x\to 0^-}f(x)=\lim_{h\to 0}f(0-h)=\lim_{h\to 0}\frac{\sqrt{1-kh}-\sqrt{1+kh}}{-h}$$

$$=\lim_{h\to 0}\frac{\sqrt{1-kh}-\sqrt{1+kh}}{-h}\times\frac{\sqrt{1-kh}+\sqrt{1+kh}}{\sqrt{1-kh}+\sqrt{1+kh}}$$

$$=\lim_{h\to 0}\frac{(1-kh)-(1+kh)}{-h[\sqrt{1-kh}+\sqrt{1+kh}]}$$

$$=\lim_{h\to 0}\frac{2k}{\sqrt{1-kh}+\sqrt{1+kh}}=\frac{2k}{2}=k$$

$\therefore$ R.H.L. = L.H.L. $\Rightarrow$ $k=-1$

27. We have $2xy+y^2-2x^2\frac{dy}{dx}=0 \Rightarrow \frac{dy}{dx}=\frac{2xy+y^2}{2x^2}$

Putting $y=vx \Rightarrow \frac{dy}{dx}=v+x\frac{dv}{dx}$

$$\therefore\ v+x\frac{dv}{dx}=\frac{2v+v^2}{2} \Rightarrow x\frac{dv}{dx}=\frac{2v+v^2}{2}-v$$

$$\frac{x\,dv}{dx}=\frac{2v+v^2-2v}{2}=\frac{v^2}{2}$$

$$\Rightarrow 2\int\frac{1}{v^2}dv=\int\frac{1}{x}dx$$

$$\Rightarrow -\frac{2}{v}=\log|x|+c \Rightarrow -\frac{2x}{y}=\log|x|+c \quad \text{...(i)}$$

It is given that $y(1)=2$, *i.e.*, $y=2$ when $x=1$
$\therefore$ Equation (i) becomes $-1=0+c\Rightarrow c=-1$
Putting $c=-1$ in equation (i), we get

$$-\frac{2x}{y}=\log|x|-1 \Rightarrow y=\frac{2x}{1-\log|x|}$$

Clearly, y is defined if $x\neq 0$, and $1-\log|x|\neq 0$
Now, $1-\log|x|=0\Rightarrow \log|x|=1\Rightarrow |x|=e\Rightarrow x=\pm e$

Hence, $y=\frac{2x}{1-\log|x|}$, where $x\neq 0,\pm e$ gives the solution of the given differential equation.

OR

Let $I=\int \frac{\sin 2x}{a^2\sin^2 x+b^2\cos^2 x}dx$

Let $a^2\sin^2 x+b^2\cos^2 x=t$

$\Rightarrow (a^2\cdot 2\sin x\cos x-b^2\cdot 2\cos x\sin x)\,dx=dt$

$\Rightarrow (a^2\sin 2x-b^2\sin 2x)\,dx=dt$

$\Rightarrow (a^2-b^2)\sin 2x\,dx=dt$

$\therefore\ I=\frac{1}{(a^2-b^2)}\int\frac{1}{t}dt=\frac{1}{(a^2-b^2)}\log|t|+C$

$\Rightarrow I=\frac{1}{(a^2-b^2)}\log\left|a^2\sin^2 x+b^2\cos^2 x\right|+C$

where C is constant

28. Let $I=\int_{\pi/4}^{3\pi/4}\frac{x}{1+\sin x}dx$ ...(i)

$\Rightarrow I=\int_{\pi/4}^{3\pi/4}\frac{\left(\frac{3\pi}{4}+\frac{\pi}{4}-x\right)}{1+\sin\left(\frac{3\pi}{4}+\frac{\pi}{4}-x\right)}dx$

$\left[\because \int_a^b f(x)\,dx=\int_a^b f(a+b-x)\,dx\right]$

$\Rightarrow I=\int_{\pi/4}^{3\pi/4}\frac{(\pi-x)dx}{1+\sin x}$ ...(ii)

Adding (i) and (ii), we get

$2I=\int_{\pi/4}^{3\pi/4}\frac{\pi\,dx}{1+\sin x}=\pi\int_{\pi/4}^{3\pi/4}\frac{(1-\sin x)\,dx}{\cos^2 x}$

$\Rightarrow 2I=\pi\int_{\pi/4}^{3\pi/4}(\sec^2 x-\sec x\tan x)\,dx$

$\Rightarrow 2I=\pi\left[[\tan x]_{\pi/4}^{3\pi/4}-[\sec x]_{\pi/4}^{3\pi/4}\right]$

$\Rightarrow 2I=\pi\left[\left(-\tan\frac{\pi}{4}-1\right)-\left(\sec\frac{3\pi}{4}-\sec\frac{\pi}{4}\right)\right]$

$2I=\pi\left[(-1-1)-\left(-\sec\frac{\pi}{4}-\sqrt{2}\right)\right]$

$2I=\pi\left[-2-\left(-\sqrt{2}-\sqrt{2}\right)\right]$

$2I=\pi\left[-2+2\sqrt{2}\right]\ \Rightarrow 2I=2\pi\left[\sqrt{2}-1\right]$

$\Rightarrow I=\pi\left[\sqrt{2}-1\right]$

29. We have, $y^2dx+(xy+x^2)dy=0\Rightarrow\frac{dy}{dx}=\frac{-y^2}{xy+x^2}$

which is homegeneous differential equation,

Put $y=vx\Rightarrow\frac{dy}{dx}=v+x\frac{dv}{dx}$

$\therefore\ v+x\frac{dv}{dx}=\frac{-v^2x^2}{vx^2+x^2}\Rightarrow x\frac{dv}{dx}=\frac{-v^2}{v+1}-v$

$\Rightarrow x\frac{dv}{dx}=-\left(\frac{2v^2+v}{v+1}\right)\Rightarrow\int\left(\frac{v+1}{v(2v+1)}\right)dv=-\int\frac{dx}{x}$

$\Rightarrow \int\left(\frac{1}{v}-\frac{1}{2v+1}\right)dv=-\log|x|+\log c$

$\Rightarrow \log|v|-\frac{1}{2}\log|2v+1|+\log|x|=\log c$

$2\log|v|-\log|2v+1|+2\log|x|=2\log c$

$\Rightarrow \log\left|\frac{v^2x^2}{2v+1}\right|=\log c^2\Rightarrow\frac{v^2x^2}{2v+1}=C$ [where $C=\log c^2$]

$\Rightarrow y^2=\left(\frac{2y}{x}+1\right)\Rightarrow xy^2=C(x+2y)$ is required solution.

OR

We have, $\ln\left(\frac{dy}{dx}\right)=3x+4y$

$\Rightarrow \frac{dy}{dx}=e^{3x}\cdot e^{4y}\ \Rightarrow e^{-4y}dy=e^{3x}dx$

On integration, we get $-\frac{1}{4}e^{-4y}=\frac{e^{3x}}{3}+C$

For particular solution, put $x=0$, $y=0$;

we have, $-\frac{1}{4}=\frac{1}{3}+C\ \Rightarrow C=-\frac{7}{12}$

$\therefore$ Solution is given by

$\frac{e^{-4y}}{4}+\frac{e^{3x}}{3}=\frac{7}{12}\ \Rightarrow 3e^{-4y}+4e^{3x}=7$

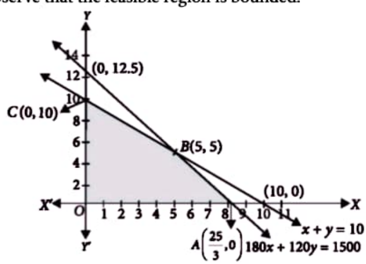
30. We have, maximize $Z=11x+9y$

Subject to the constraints,

$180x+120y\le 1500$ ...(i)

$x+y\le 10$ ...(ii) ; $x, y\ge 0$ ...(iii)

Now, plotting the graph of (i), (ii) and (iii), we get the required feasible region (shaded) as shown below. We observe that the feasible region is bounded.



We have corner points as, $A\left(\frac{25}{3},0\right)$, $B(5,5)$ and $C(0,10)$.

Corner points	Value of $Z = 11x + 9y$
$A\left(\frac{25}{3},0\right)$	$11\times\frac{25}{3}+9\times 0=\frac{275}{3}$
$B(5, 5)$	$11 \times 5 + 9 \times 5 = 100$ (Maximum)
C(0, 10)	$11 \times 0 + 9 \times 10 = 90$

Thus, Z is maximum at $x = 5$ and $y = 5$ and maximum value of Z is 100.

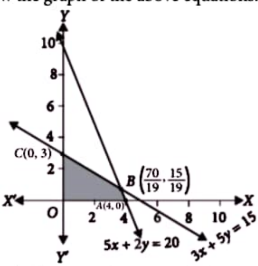
OR

Converting inequations into equations and drawing the corresponding lines.

$3x + 5y = 15$, $5x + 2y = 20$, *i.e.*, $\frac{x}{5}+\frac{y}{3}=1$, $\frac{x}{4}+\frac{y}{10}=1$

As $x \geq 0$, $y \geq 0$ solution lies in first quadrant.

Let us draw the graph of the above equations.



B is the point of intersection of the lines $3x + 5y = 15$ and $5x + 2y = 20$, *i.e.*, $B=\left(\frac{70}{19},\frac{15}{19}\right)$

We have corner points $O(0, 0)$, $A(4, 0)$, $B\left(\frac{70}{19},\frac{15}{19}\right)$ and $C(0, 3)$.

Now, $Z = 3x + 2y$ $\quad\therefore\ Z(O) = 3(0) + 2(0) = 0$

$Z(A) = 3(4) + 2(0) = 12$

$$Z(B)=3\left(\frac{70}{19}\right)+2\left(\frac{15}{19}\right)=12.63$$

$Z(C) = 3(0) + 2(3) = 6$

$\therefore$ Z has maximum value 12.63 at only one point *i.e.*, $B\left(\frac{70}{19},\frac{15}{19}\right)$

31. We have, $P(A)=\frac{2}{5}$, $P(B)=\frac{1}{3}$, $P(C)=\frac{1}{2}$

Also, $P(A\cap C)=\frac{1}{5}$ and $P(B\cap C)=\frac{1}{4}$

$$\therefore\quad P(C|B)=\frac{P(C\cap B)}{P(B)}=\frac{1/4}{1/3}=\frac{3}{4}$$

and, $P(\bar{A}\cap\bar{C})=P(\overline{A\cup C})=1-P(A\cup C)$

$=1-\{P(A)+P(C)-P(A\cap C)\}$

$$=1-\left(\frac{2}{5}+\frac{1}{2}-\frac{1}{5}\right)=1-\frac{7}{10}=\frac{3}{10}$$

32. Here, $f: A \to B$ is given by $f(x)=\frac{x-1}{x-2}$, where $A = R - \{2\}$ and $B = R - \{1\}$

Let $f(x_1) = f(x_2)$, where $x_1, x_2 \in A$ (*i.e.*, $x_1 \neq 2$, $x_2 \neq 2$)

$\Rightarrow \frac{x_1-1}{x_1-2}=\frac{x_2-1}{x_2-2}$

$\Rightarrow (x_1-1)(x_2-2)=(x_1-2)(x_2-1)$

$\Rightarrow x_1x_2-2x_1-x_2+2=x_1x_2-x_1-2x_2+2$

$\Rightarrow -2x_1-x_2=-x_1-2x_2 \Rightarrow x_1=x_2 \Rightarrow f$ is one-one

Let $y \in B = R - \{1\}$, *i.e.*, $y \in R$ and $y \neq 1$ such that $f(x) = y$

$\Rightarrow \frac{x-1}{x-2}=y \Rightarrow (x-2)y=x-1$

$\Rightarrow xy-2y=x-1 \Rightarrow xy-x=2y-1$

$\Rightarrow x(y-1)=2y-1 \Rightarrow x=\frac{2y-1}{y-1}$

$$\text{Now, } f(x)=f\left(\frac{2y-1}{y-1}\right)=\frac{\frac{2y-1}{y-1}-1}{\frac{2y-1}{y-1}-2}$$

$$=\frac{2y-1-y+1}{2y-1-2y+2}=\frac{y}{1}=y$$

$\therefore$ $f(x) = y$, when $x=\frac{2y-1}{y-1}\in A$ (as $y \neq 1$)

Hence, f is onto.

Thus, f is one–one and onto.

OR

Given relation is $R = \{(a, b): a \leq b^2\}$

Reflexive : Let $a \in R$ $\quad\therefore\ aRa \Rightarrow a \leq a^2$

but if $a < 1$, then $a \nless a^2$

For example, $a=\frac{1}{2}\Rightarrow a^2=\frac{1}{4}$ so, $\frac{1}{2}\nless\frac{1}{4}$

Hence, R is not reflexive.

Symmetric : $aRb \Rightarrow a \leq b^2$ if $bRa \Rightarrow b \leq a^2$

But $b \leq a^2$ is not true

$\therefore\ aRb \not\Rightarrow bRa$

For example, $a = 2$, $b = 5$, then $2 \leq 5^2$ but $5 \leq 2^2$ is not true.

Hence, R is not symmetric.

Transitive : Let $a, b, c \in R$

Consider aRb and bRc

$aRb \Rightarrow a \leq b^2$ and $bRc \Rightarrow b \leq c^2 \Rightarrow a \leq c^4 \not\Rightarrow aRc$

For example, if $a = 2$, $b = -3$, $c = 1$

$aRb \Rightarrow 2 \le 9$; $bRc \Rightarrow -3 \le 1$; $aRc \Rightarrow 2 \le 1$ is not true.
Hence, R is not transitive.

33. Given that, $\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix} A = \begin{bmatrix} -1 & -8 \\ 1 & -2 \\ 9 & 22 \end{bmatrix}$

Let $X = \begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix}_{3\times 2}$ and $Y = \begin{bmatrix} -1 & -8 \\ 1 & -2 \\ 9 & 22 \end{bmatrix}_{3\times 2}$

As order of X is 3×2, then A should be of order 2×2, so that we get Y matrix of order 3×2.

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

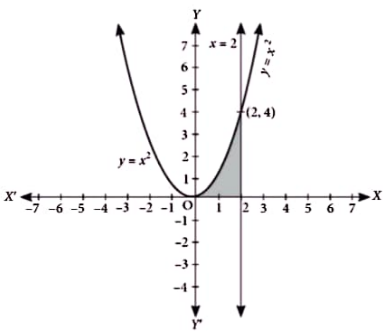
Now, $\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} -1 & -8 \\ 1 & -2 \\ 9 & 22 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2a-c & 2b-d \\ a+0 & b+0 \\ -3a+4c & -3b+4d \end{bmatrix} = \begin{bmatrix} -1 & -8 \\ 1 & -2 \\ 9 & 22 \end{bmatrix}$

On comparing the corresponding elements, we get
$2a - c = -1$ ...(i) ; $2b - d = -8$; ...(ii) ; $a = 1$ ...(iii)
and $b = -2$ ...(iv)
Substituting $a = 1$ in equation (i), we get
$c = 3$ and substituting $b = -2$ in (ii), we get, $d = 4$

So, $A = \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}$

34. The graph of given region is;

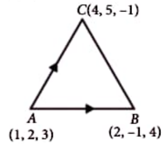


The points of intersection of the parabola $y = x^2$ and the line $x = 2$ is given by
$y = 2^2 = 4$

So, point of intersection of the curve $y = x^2$ and $x = 2$ is (2, 4).

$\therefore$ Required area $= \int_0^2 x^2 dx = \left[\frac{x^3}{3}\right]_0^2 = \frac{8}{3}$

35. Here, $\overrightarrow{AB} = (2-1)\hat{i} + (-1-2)\hat{j} + (4-3)\hat{k}$
$= \hat{i} - 3\hat{j} + \hat{k}$
and $\overrightarrow{AC} = (4-1)\hat{i} + (5-2)\hat{j} + (-1-3)\hat{k}$
$= 3\hat{i} + 3\hat{j} - 4\hat{k}$



Now, $\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 1 \\ 3 & 3 & -4 \end{vmatrix}$

$= \hat{i}(12-3) - \hat{j}(-4-3) + \hat{k}(3+9)$
$= 9\hat{i} + 7\hat{j} + 12\hat{k}$

Also, $|\overrightarrow{AB} \times \overrightarrow{AC}| = \sqrt{9^2 + 7^2 + 12^2}$
$= \sqrt{81+49+144} = \sqrt{274}$ ...(i)

Since, area of $\Delta ABC = \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}|$

$= \frac{1}{2}\sqrt{274}$ sq. units [Using (i)]

OR

Given, $l + m + n = 0$ ...(i)
and $2\,mn + 2\,ml - nl = 0$ ...(ii)
From equation (i), we get $n = -(l + m)$
Putting $n = -(l + m)$ in equation (ii), we get
$-2\,m\,(l + m) + 2\,ml + (l + m)\,l = 0$
$\Rightarrow -2\,ml - 2\,m^2 + 2\,ml + l^2 + ml = 0$
$\Rightarrow l^2 + ml - 2\,m^2 = 0$

$\Rightarrow \left(\frac{l}{m}\right)^2 + \left(\frac{l}{m}\right) - 2 = 0$ [Dividing by m^2]

$\Rightarrow \frac{l}{m} = \frac{-1 \pm \sqrt{1+8}}{2} = \frac{-1 \pm 3}{2} = 1, -2$

Case I : When $\frac{l}{m} = 1$, in this case $m = l$

From equation (i), we get
$2l + n = 0 \Rightarrow n = -2l$
$\therefore$ $l : m : n = 1 : 1 : -2$
$\therefore$ Direction ratios of the line are 1, 1, –2.
$\therefore$ Direction cosines of the line are

$\pm \frac{1}{\sqrt{(1)^2 + (1)^2 + (-2)^2}}, \pm \frac{1}{\sqrt{(1)^2 + (1)^2 + (-2)^2}},$

$\mp \frac{2}{\sqrt{(1)^2+(1)^2+(-2)^2}}$, *i.e.*, $\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}}$ or $\frac{-1}{\sqrt{6}}, \frac{-1}{\sqrt{6}}, \frac{2}{\sqrt{6}}$

Case II : When $\frac{l}{m} = -2$, in this case, $l = -2m$

From (i), we get $-2m + m + n = 0 \Rightarrow n = m$

$\therefore \quad l : m : n = -2 : 1 : 1$

$\therefore$ Direction ratios of the line are $-2, 1, 1$.

Direction cosines of the line are

$\mp \frac{2}{\sqrt{(-2)^2+(1)^2+(1)^2}}, \frac{1}{\sqrt{(-2)^2+(1)^2+(1)^2}},$

$\pm \frac{1}{\sqrt{(-2)^2+(1)^2+(1)^2}}$

i.e., $-\frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}$ or $\frac{2}{\sqrt{6}}, -\frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}}$

36. **(i)** Here, P.V. of $A = 4\hat{i} + 10\hat{j}$ and P.V. of $C = 4\hat{i} + 2\hat{j}$

$\therefore \quad \overrightarrow{AC} = (4-4)\,\hat{i} + (2-10)\,\hat{j} = -8\,\hat{j}$

(ii) Here, $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\therefore \ |\vec{a}| = \sqrt{1^2+2^2+3^2} = \sqrt{1+4+9} = \sqrt{14}$

$\therefore \ \hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{\hat{i}+2\hat{j}+3\hat{k}}{\sqrt{14}} = \frac{1}{\sqrt{14}}\hat{i} + \frac{2}{\sqrt{14}}\hat{j} + \frac{3}{\sqrt{14}}\hat{k}$

(iii) We have, $\vec{a} = 4\hat{i} + 3\hat{j}$ and $\vec{b} = 3\hat{i} + 4\hat{j}$

$\therefore \ |\vec{a}| = \sqrt{4^2+3^2} = \sqrt{16+9} = \sqrt{25} = 5$

and $|\vec{b}| = \sqrt{3^2+4^2} = \sqrt{9+16} = \sqrt{25} = 5$

Thus, $|\vec{a}| + |\vec{b}| = 5 + 5 = 10$

OR

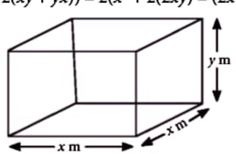
$\vec{a} = 2\hat{i} + 2\hat{j}$ and $\vec{b} = \hat{i} + \hat{j}$

$\vec{a} - \vec{b} = \hat{i} + \hat{j} \quad \therefore \ |\vec{a} - \vec{b}| = \sqrt{1^2+1^2} = \sqrt{2}$

37. **(i)** Since, the tank is open from the top, therefore the total surface area is

= (Outer + Inner) surface area

$= 2(x \times x + 2(xy + yx)) = 2(x^2 + 2(2xy) = (2x^2 + 8xy)\ \text{m}^2$



(ii) Since, volume of tank should be 32000 *l*.

$\therefore \quad x^2y\ \text{m}^3 = 32000\ l = 32\ \text{m}^3 \qquad [\because 1 \text{ litre} = 0.001\ \text{m}^3]$

So, $x^2y = 32$

(iii) Let S be the outer surface area of tank.

Then, $S = x^2 + 4xy$

$\Rightarrow \ S(x) = x^2 + 4x \cdot \frac{32}{x^2} = x^2 + \frac{128}{x} \qquad [\because x^2y = 32]$

$\Rightarrow \ \frac{dS}{dx} = 2x - \frac{128}{x^2}$ and $\frac{d^2S}{dx^2} = 2 + \frac{256}{x^3}$

For maximum or minimum values of S, consider $\frac{dS}{dx} = 0$

$\Rightarrow \ 2x = \frac{128}{x^2} \Rightarrow x^3 = 64 \Rightarrow x = 4$ m

At $x = 4$, $\frac{d^2S}{dx^2} = 2 + \frac{256}{4^3} = 2 + 4 = 6 > 0$

$\therefore \quad S$ is minimum when $x = 4$

OR

Now as $x^2y = 32$, therefore $y = 2$. Thus, $x = 2y$

Since, surface area is minimum when $x = 2y$, therefore cost of material will be least when $x = 2y$.

Thus, cost of material will be least when width is equal to twice of its depth.

Since, minimum surface area

$= x^2 + 4xy = 4^2 + 4 \times 4 \times 2 = 48\ \text{m}^2$ and

Cost per m^2 = ₹ 360

$\therefore$ Minimum cost = ₹ (48×360) = ₹ 17280

38. Sample space is given by

{*MFSD, MFDS, MSFD, MSDF, MDFS, MDSF, FMSD, FMDS, FSMD, FSDM, FDMS, FDSM, SFMD, SFDM, SMFD, SMDF, SDMF, SDFM DFMS, DFSM, DMSF, DMFS, DSMF, DSFM*}

$\therefore \quad n(s) = 24$

(i) Let A denotes the event that son is at one end.

$\therefore \quad n(A) = 12$

And B denotes the event that father and mother are in middle.

$\therefore \quad n(B) = 4$

Also, $n(A \cap B) = 4$

$\therefore \ P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{4/24}{4/24} = 1$

(ii) Let A denotes the event that mother is at left end.

$\therefore \quad n(A) = 6$

And B denotes the event that son and daughter are together.

$\therefore \quad n(B) = 12$

Also, $n(A \cap B) = 4$

$\therefore \ P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{4/24}{12/24} = \frac{1}{3}$

◆◆

MONTHLY TEST DRIVE

XII

This specially designed column enables students to self analyse their extent of understanding of specified chapter. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Total Marks : 80

Series 9 : Probability

Time Taken : 60 Min.

Only One Option Correct Type

1. One ticket is selected at random from 50 tickets numbered 00, 01, 02, ... , 49. Then the probability that the sum of the digits on the selected tickets is 8, given that the product of these digits is zero, equals
   (a) 1/14 (b) 1/7
   (c) 5/14 (d) 1/50

2. A pair of fair and ordinary dice is rolled simultaneously. It is found that they show different outcomes. The probability that the sum of the outcomes will be either 6 or 10, is equal to
   (a) 1/5 (b) 1/6 (c) 1/3 (d) 2/3

3. If $P(A) = 0.1$, $P(B|A) = 0.6$ and $P(B \mid A^c) = 0.3$, then $P(A|B)$ is equal to
   (a) 2/11 (b) 4/11 (c) 7/11 (d) 9/11

4. If A and B are independent events associated to some experiment E such that $P(A^c \cap B) = 2/15$ and $P(A \cap B^c) = 1/6$, then $P(B)$ is equal to
   (a) 1/6 or 1/5 (b) 1/6 or 4/5
   (c) 4/5 or 1/5 (d) 4/5 or 5/6

5. For A, B and C the chances of being selected as the manager of a firm are 4 : 1 : 2 respectively. The respective probabilities for them to introduce a radical change in marketing strategy are 0.3, 0.8 and 0.5 respectively. If the change does takes place, find the probability that it is due to the appointment of B.
   (a) 6/15 (b) 4/15 (c) 1/15 (d) 8/15

6. A letter is to come from either LONDON or CLIFTON. The postal mark on the letter legibly shows consecutive letters "ON". The probability that the letter has come from LONDON is
   (a) 12/17 (b) 13/17
   (c) 5/17 (d) 4/17

One or More than One Option(s) Correct Type

7. A and B are two independent events. The probability that both A and B occur is 1/6 and the probability that neither of them occurs is 1/3 . The probability of occurrence of A is
   (a) 1/2 (b) 1/3 (c) 1/4 (d) 1/6

8. A lot contains 50 defective and 50 non defective bulbs. Two bulbs are drawn at random one at a time with replacement. The events A, B, C are defined as the first bulb is defective, the second bulb is non-defective, the two bulbs are both defective or non defective, respectively. Then
   (a) A, B, C are pairwise independent
   (b) A, B, C are pairwise not independent
   (c) A, B, C are independent
   (d) A, B, C are not independent

9. There are three coins C_1, C_2 and C_3, C_1 is a fair coin painted blue on the head side and white on the tail side. C_2 and C_3 are biased coins so that the probability of a head is p. They are painted blue on the tail side and red on the head side. Two of the three coins are selected at random and tossed. If the probability that both the coins land up with sides of the same colour is 29/96, then the possible value(s) of p can be
   (a) 1/8 (b) 3/8 (c) 5/8 (d) 7/8

10. Let X and Y be two events such that $P(X/Y)=\frac{1}{2}$, $P(Y/X)=\frac{1}{3}$ and $P(X \cap Y)=\frac{1}{6}$. Which of the following is(are) correct?
   (a) $P(X \cup Y) = 2/3$
   (b) X and Y are independent
   (c) X and Y are not independent
   (d) $P(X^c \cap Y) = 1/3$

11. A wire of length a units cut into three pieces, the probability that three pieces form a triangle is

(a) 1/3 (b) 2/3 (c) 1/4 (d) 1/2

12. Which of the following is/are true?

(a) A and B are two independent events. If the probability that both A and B occur is 1/12 and the probability that neither A nor B occurs is 1/2, then $P(A - B) = 1/6$. (Given : $P(A) < P(B)$)

(b) A fair coin is tossed repeatedly. If the head appears on the first three tosses, then the probability that the tail appearing on the fourth toss equals 1/2.

(c) If the letters of the word "FREEDOM" are written down at random in a row, the probability that no two E's occur next to each other is 6/7.

(d) For two given events A and B, $P(A) + P(B) - 1 \le P(A \cap B) \le P(A) + P(B)$.

13. Let us define the events A and B as

A : An year chosen at random contains 29 February.

B : An year chosen at random has 52 Fridays.

If $P(E)$ denotes the probability of happening of event E, then

(a) $P(\bar{B}) = \dfrac{2}{7}$ (b) $P(B) = \dfrac{23}{28}$

(c) $P(A|\bar{B}) = \dfrac{2}{5}$ (d) $P(A|B) = \dfrac{5}{23}$

Comprehension Type

Paragraph for Q. No. 14 and 15

In a class of 10 students, probability of exactly i students passing an examination is directly proportional to i^2. Then answer the following questions:

14. The probability that exactly 5 students passing an examination is

(a) 1/11 (b) 5/77 (c) 25/77 (d) 10/77

15. If a student is selected at random, then the probability that he has passed the examination is

(a) 1/7 (b) 11/35

(c) 11/14 (d) None of these

Matrix Match Type

16. Sixteen players $S_1, S_2, ..., S_{16}$ play in a tournament. They are divided into eight pairs at random. From each players a winner is decided on the basis of a game played between the two players of the pair. Assume that all the players are of equal strength. The probability that

	Column-I		Column-II
(P)	The players S_1 is among eight winners	(1)	8/15
(Q)	Exactly one of S_1 and S_2 is among eight winners	(2)	7/30
(R)	Both S_1 and S_2 are among the eight winners	(3)	1/2

	P	Q	R
(a)	3	3	2
(b)	2	2	3
(c)	1	2	3
(d)	3	1	2

Numerical Answer Type

17. In a sequence of independent trials, the probability of success is 1/4. If p denotes the probability that the second success occurs on the fourth trial or later trial, then the value of 32 p is ________.

18. An urn contains six white and four black balls. A fair die is rolled and that number of balls are drawn from the urn. If the probability that the balls selected are white is p, then the value of $15p$ is ________.

19. The probability of at least one double-six being thrown in n throws with two ordinary dice is greater than 99 percent. The least numerical value of n is ______.

20. A sample of size 4 is drawn with replacement (without replacement) from an urn containing 12 balls, of which 8 are white. The conditional probability that the ball drawn on the third draw was white, given that the sample contains 3 white balls is $3/s$, where s is equal to________.

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